Instabilities

Marmottant and Villermaux (2004)

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LFMI
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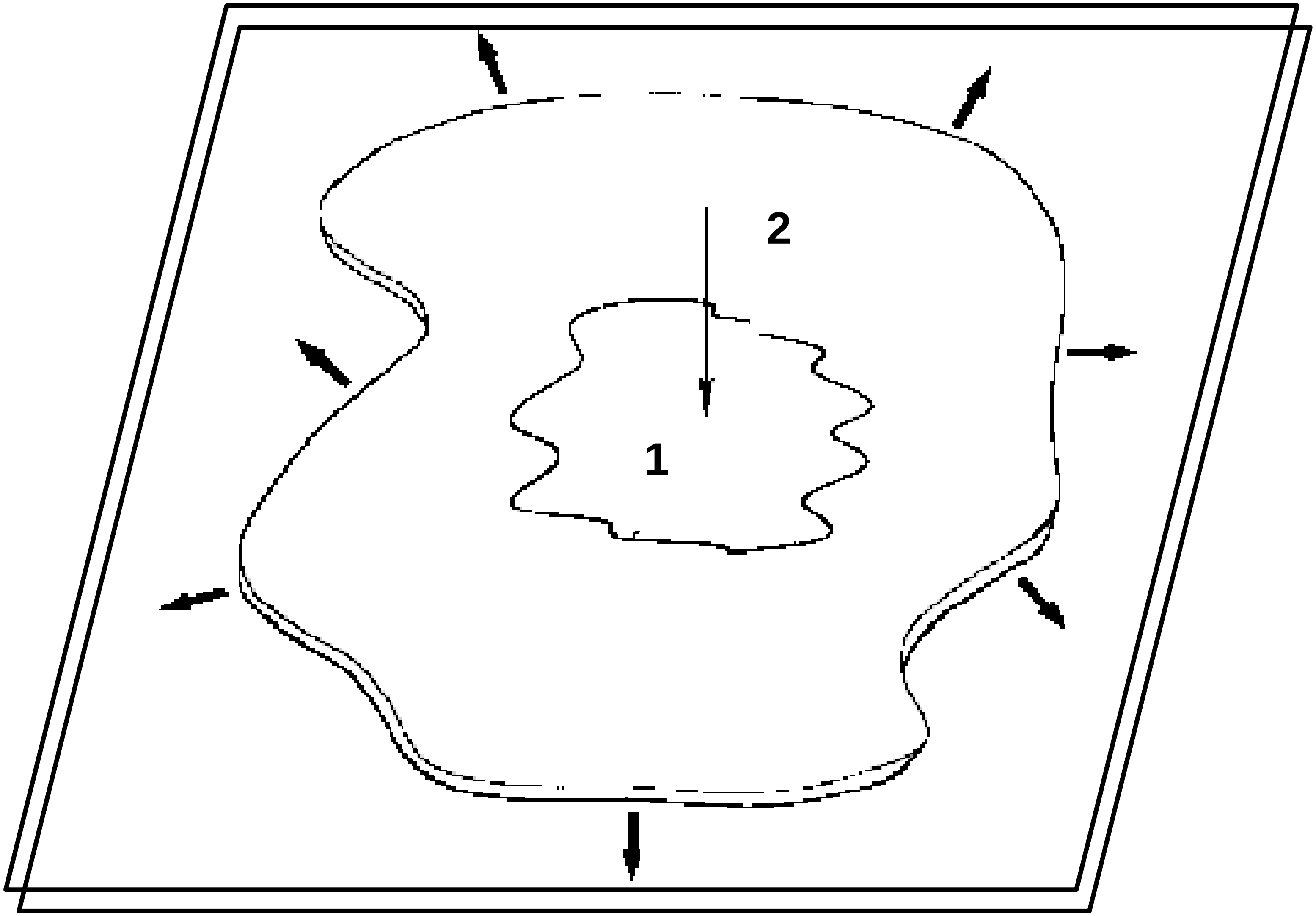
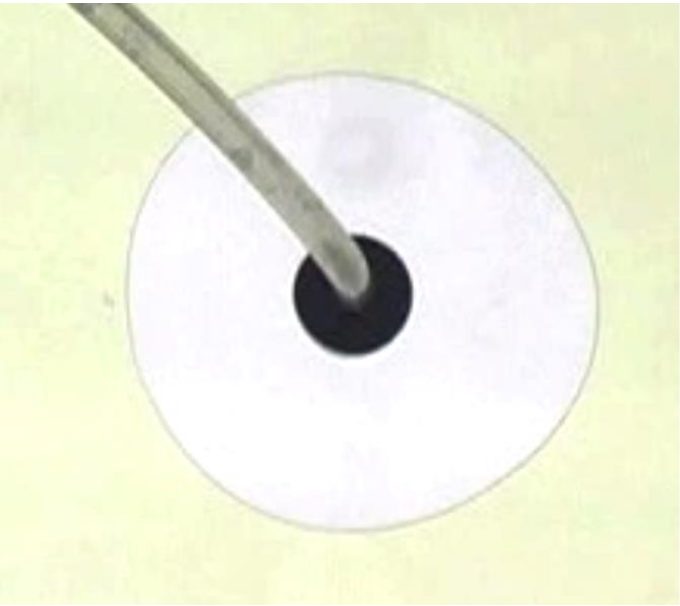
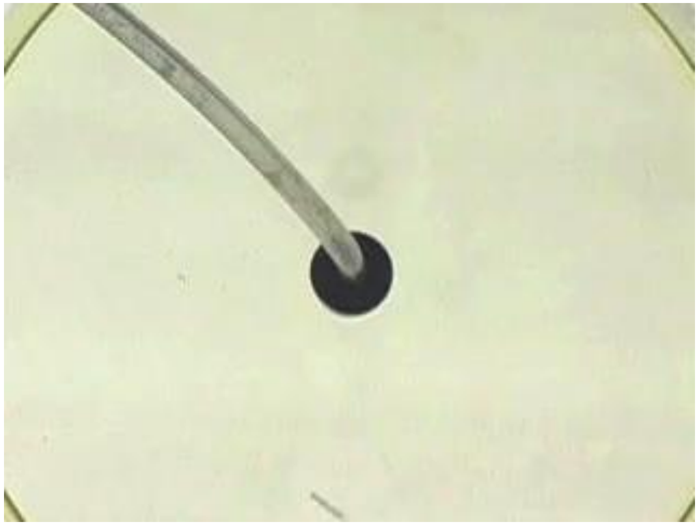
Injection of fluid 1 in fluid 2 in a Hele-Shaw cell



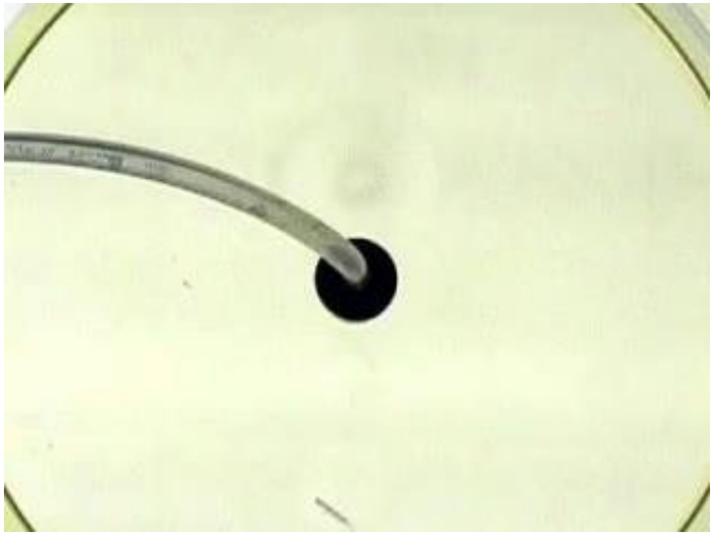
Stable?



These 2 fluids are water and oil, what is fluid 1?



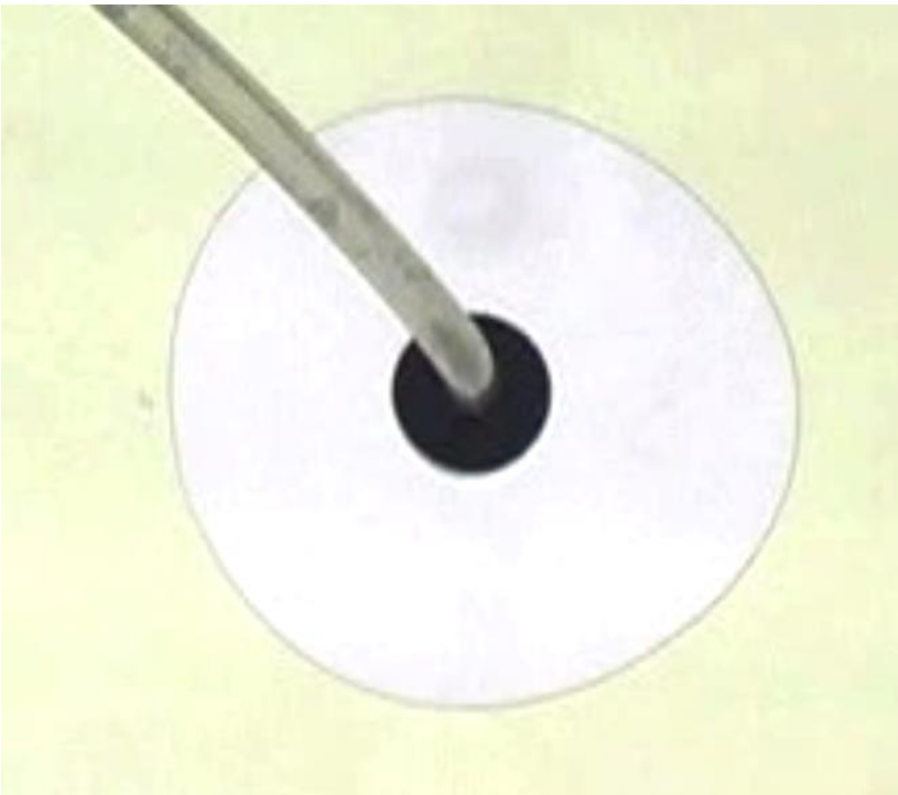
Fluid 1 is colored



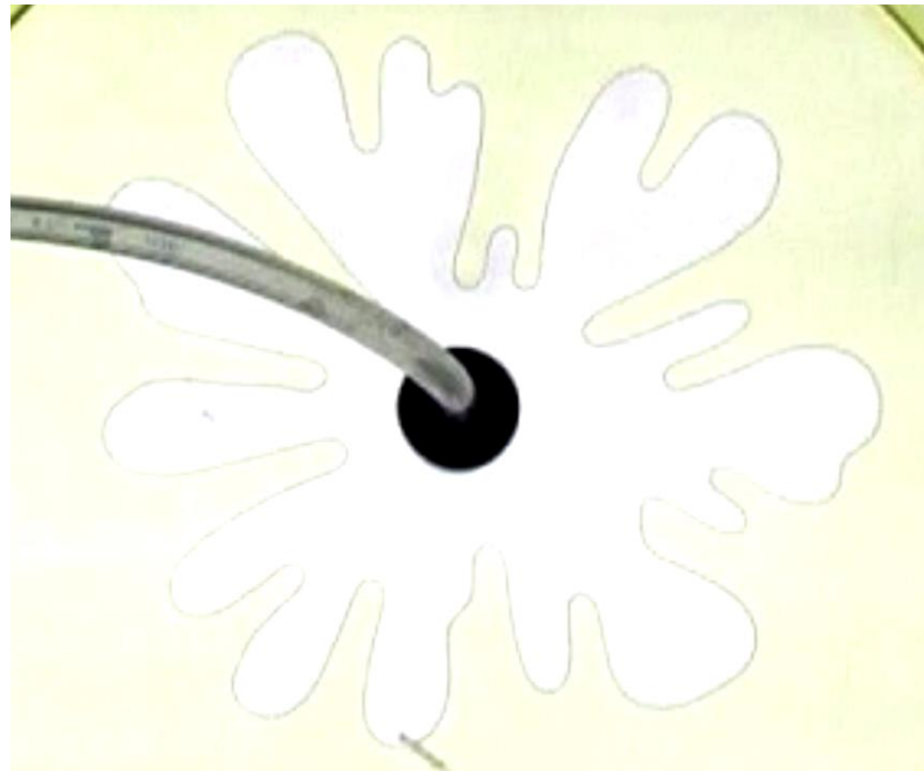
Fluid 2 is colored

The important parameter is the viscosity ratio

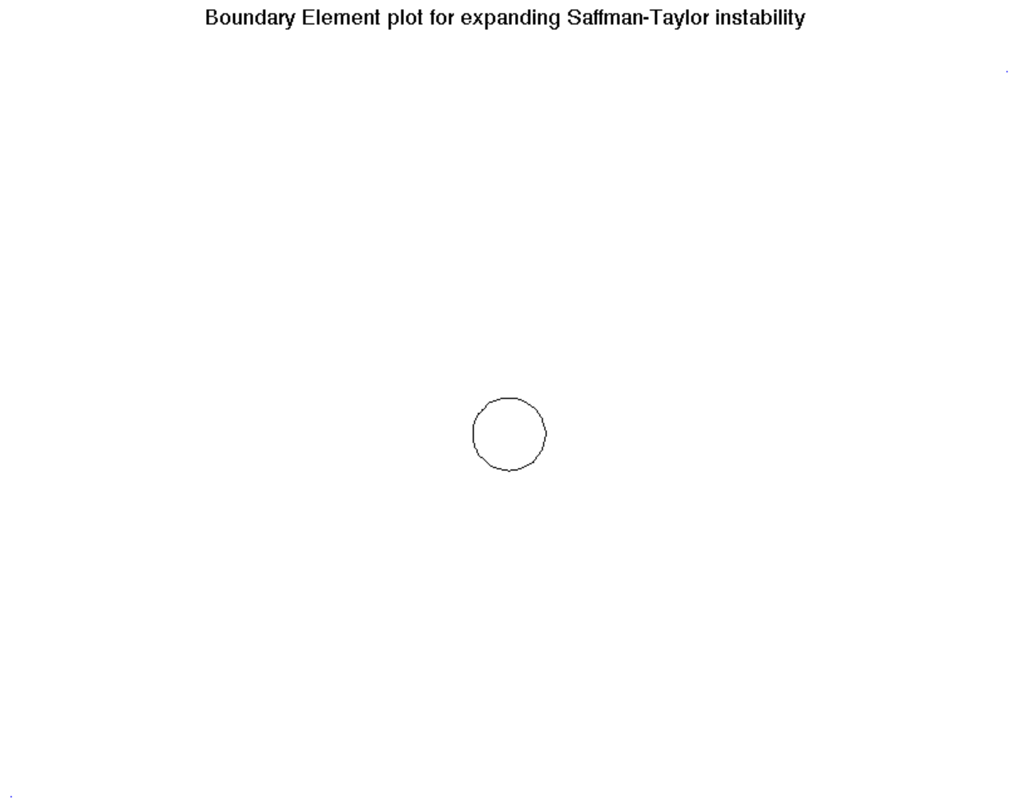
Oil in water is stable



Water in oil is unstable



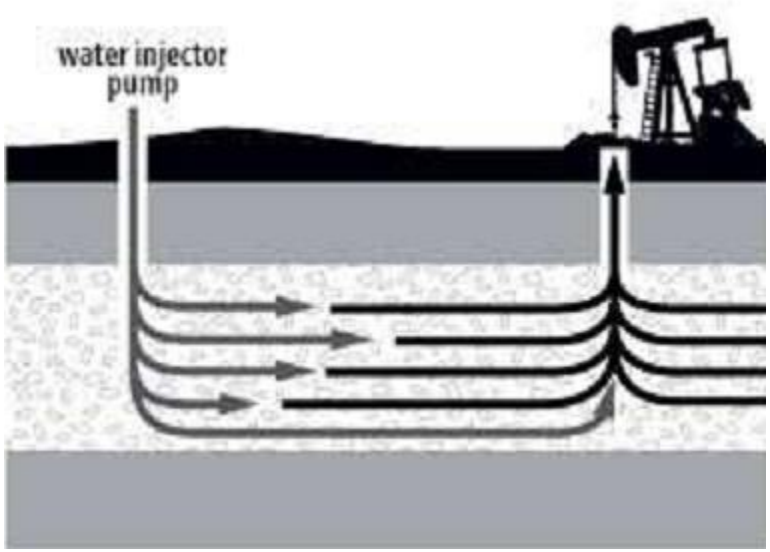
Saffman-Taylor instability
Numerical Simulation
Dr. Mathias Nagel, LFMI

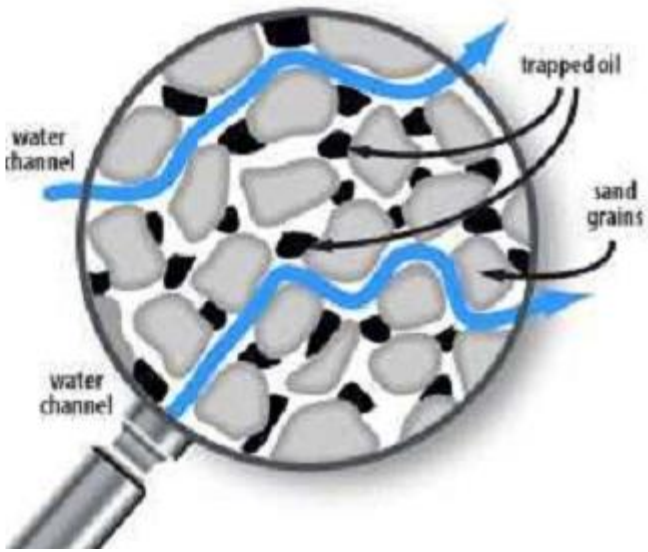


Viscous fingering

An example of Saffman-Taylor fingering. The interface here is affected by surface tension, which leads to a characteristic length scale over which the dendrites occur (adapted from *Perspectives in Fluid Dynamics*, CUP (2000))

Viscous fingering





Most flows are unstable...

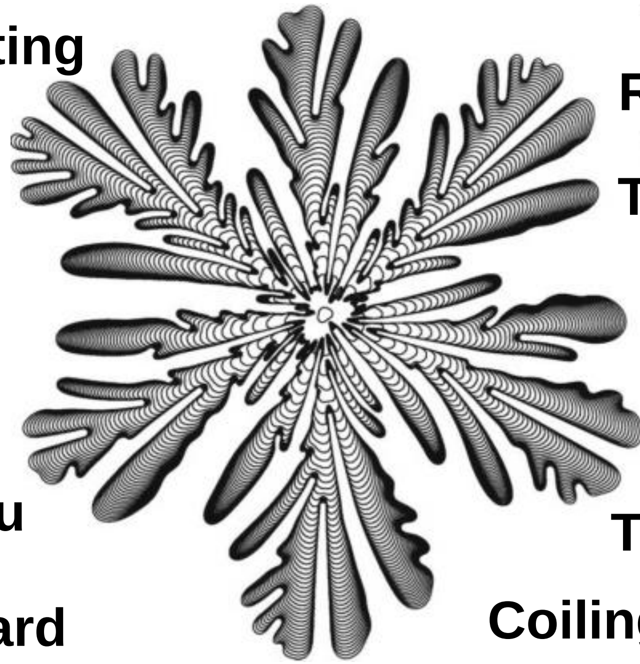
Vortex shedding

Saffman-Taylor

Flow separation

Tollmien-Schlichting

Rayleigh-Taylor



Lift-up and Streaks

Traffic waves

Meandering instability

Gravito-capillary waves

Taylor-Couette

Rayleigh-Plateau

Tearing instability

Coiling instability

Rayleigh-Benard

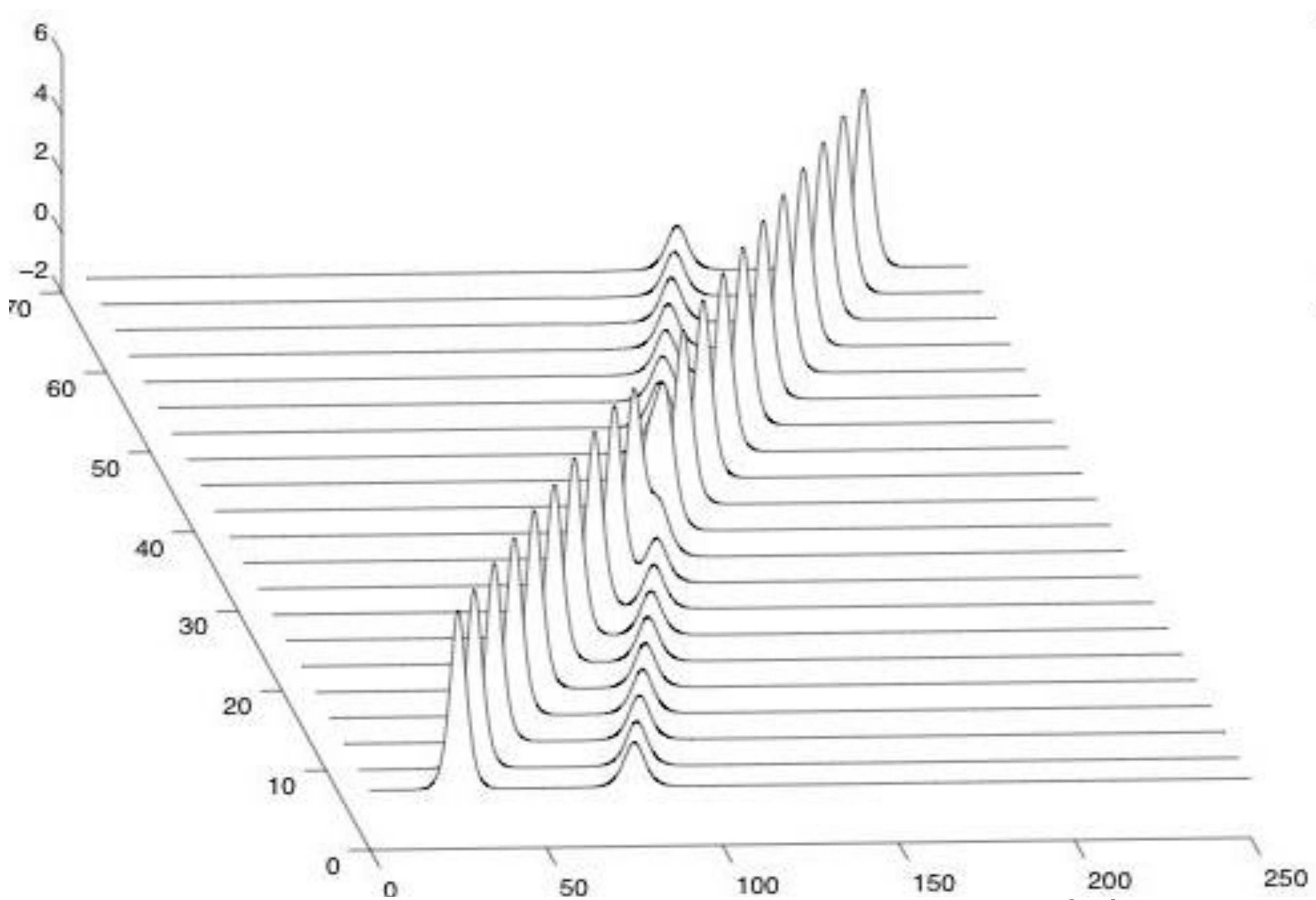
Benard-Marangoni

Kelvin-Helmholtz

But some are very stable

Garcia & Chomaz 2005

Collision of tsunamis



Trefethen 2000

Solitons are very stable solutions of the nonlinear water-wave equations

Instabilities

Friday 10h15-13h

Homework:

Exercise or scientific article

Exam: Instabilities
Written exam

Books:
- **Drazin P.G. , Introduction to hydrodynamic instability, CUP, 2000**
- **F. Charru, Instabilités hydrodynamiques, EDP sciences, 2007**

Instabilities

NEBIS

Compte utilisateur | Préférences | Recherche guidée | Aide | Se déconnecter

Liste des résultats | Historique | Liste | Panier

Recherche | **Recherche avancée** | **Recherche en mode expert** | **Parcourir >**

Titres sélectionnés: Voir la sélect. | Sauvegarder/E-mail | Sous-ensemble

Ensemble des résultats: Tout sélect. | Supprimer sélect. | Modifier la recherch

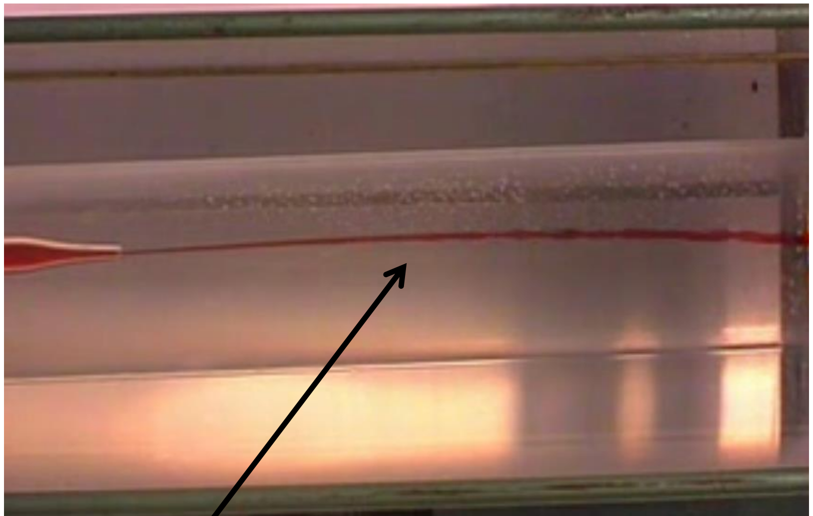
Résultats pour Mots= charru

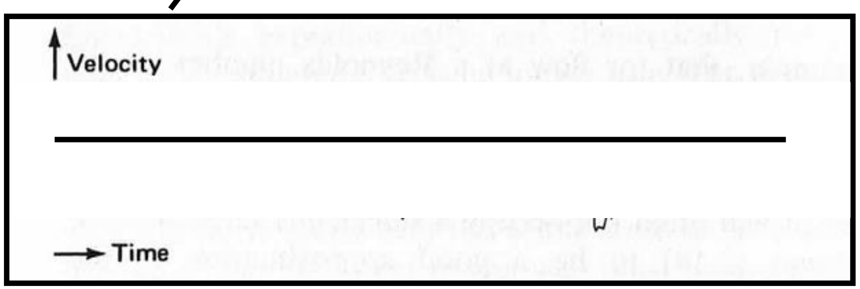
Trié par : Année (desc), puis Auteur

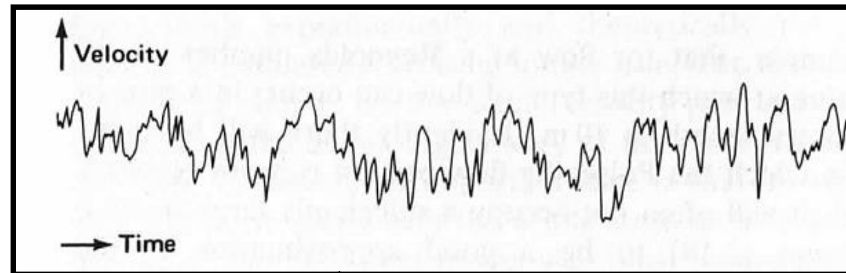
Titre 1 - 7 à 7 (Limite d'affichage et de triage : 20000/1000)

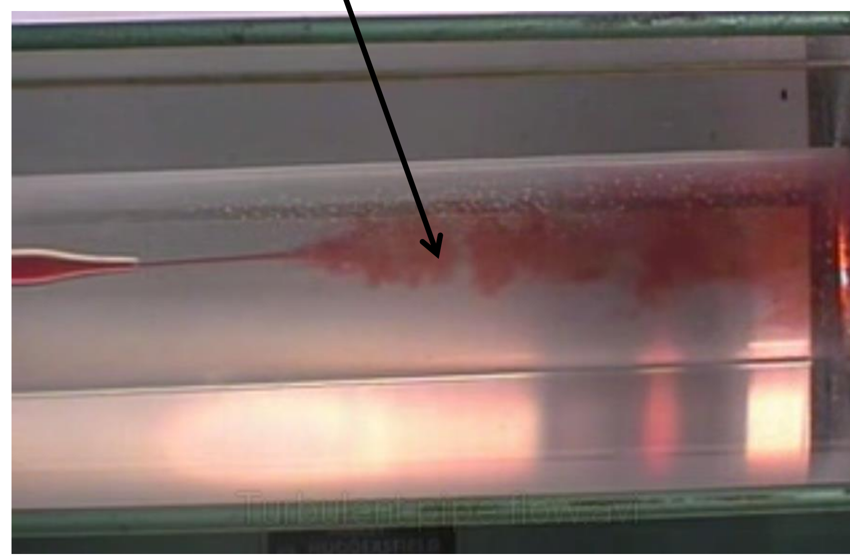
#		Typ	Auteur	Titre
1	☐	Livre & en ligne	Charru, François	Hydrodynamic instabilities

Laminar/turbulent









Instabilities and turbulence

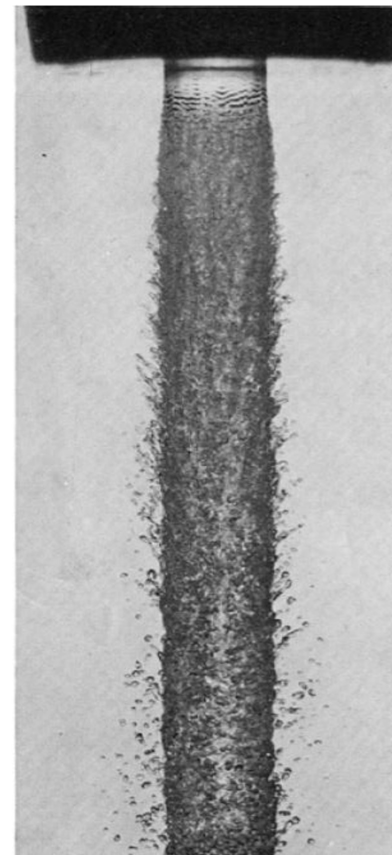
Laminar ⇨ Instability ⇨ Disorder/Pattern/Chaos ⇨ Turbulence

Transition

Marmottant and Villermaux (2004)

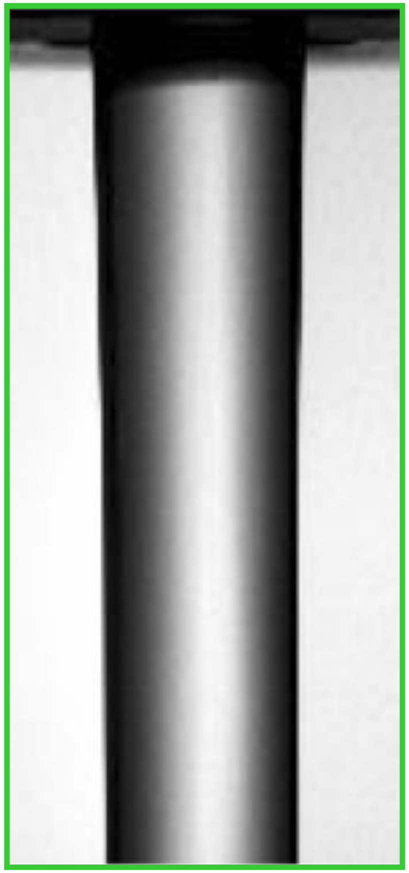
Rayleigh (1891)

Marmottant and Villermaux (2004)



Hoyt and Taylor (1977)

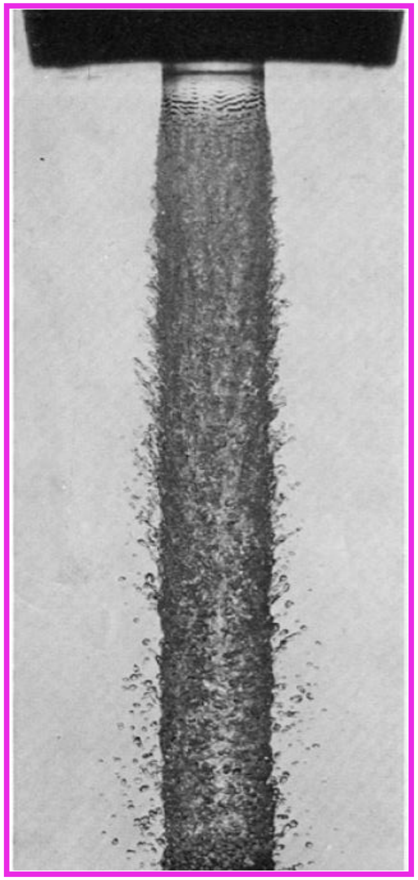
Laminar ⇨ **Instability** ⇨ **Pattern** ⇨ **Turbulence**



Marmottant and Villermaux (2004)

Rayleigh (1891)

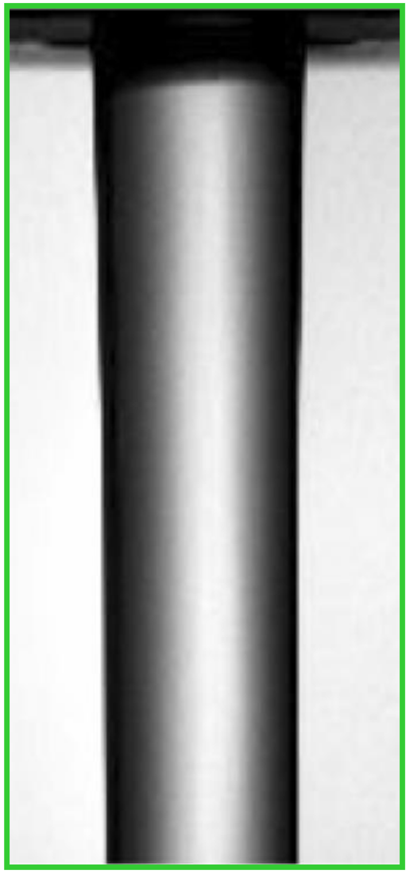
Marmottant and Villermaux (2004)



Hoyt and Taylor (1977)

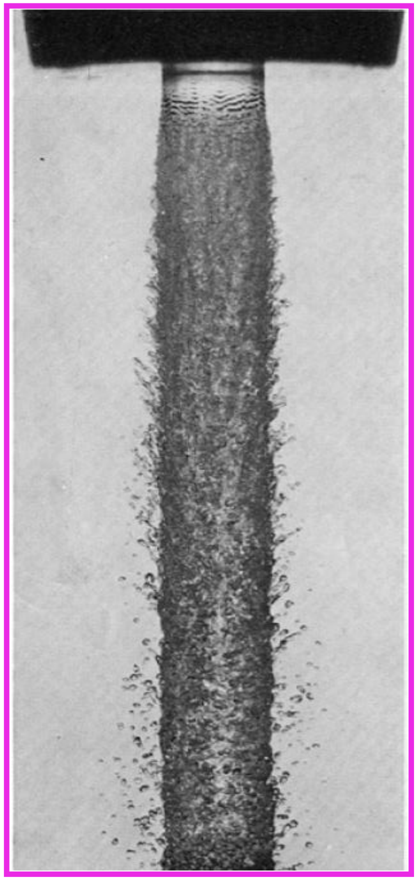
no fluctuation	⇨	**ε**	⇨	**1**	⇨	**fluctuation > average**

Laminar ⇨ Instability ⇨ Pattern ⇨ Turbulence


Marmottant and Villermaux (2004)

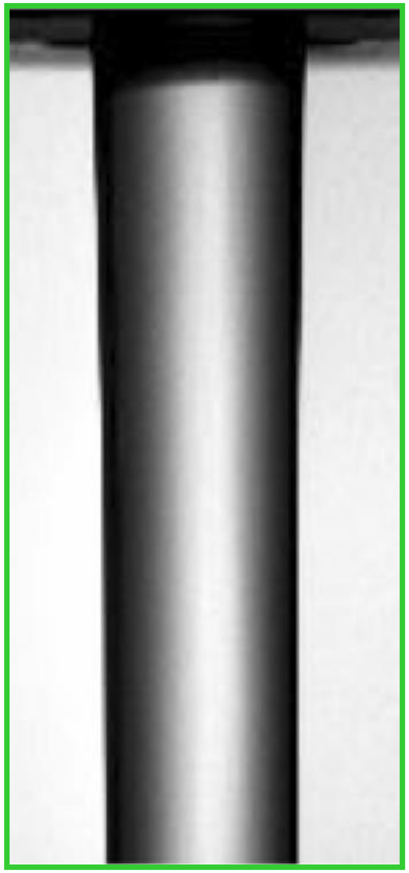

Rayleigh (1891)


Marmottant and Villermaux (2004)


Hoyt and Taylor (1977)

R ⇨ R, λ ⇨ R, λ_1, λ_2 ⇨ full spectrum of lengscales

Laminar ⇨ **Instability** ⇨ **Pattern** ⇨ **Turbulence**



Marmottant and Villermaux (2004)

Rayleigh (1891)

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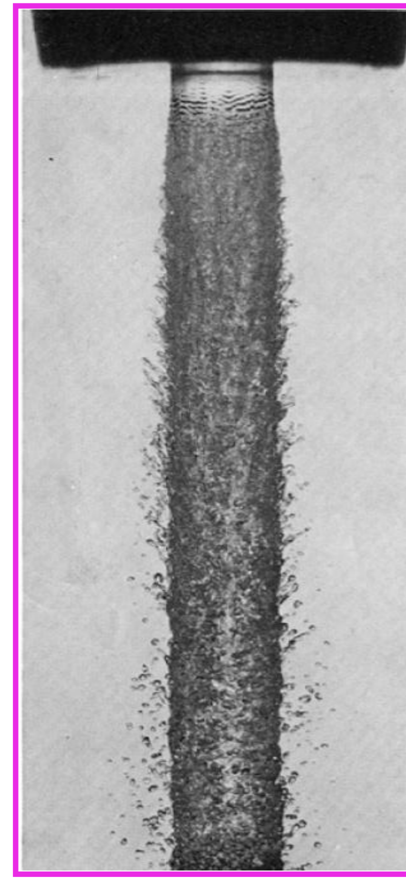
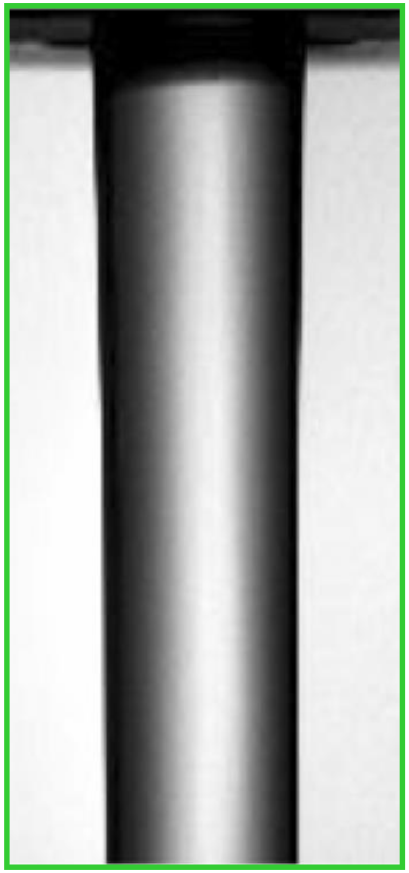


Hoyt and Taylor (1977)

steady ⇨ ω ⇨ ω_1, ω_2 ⇨ **full frequency spectrum**

Laminar ⇨ Instability ⇨ Pattern ⇨ Turbulence



Marmottant and Villermaux (2004)

Rayleigh (1891)

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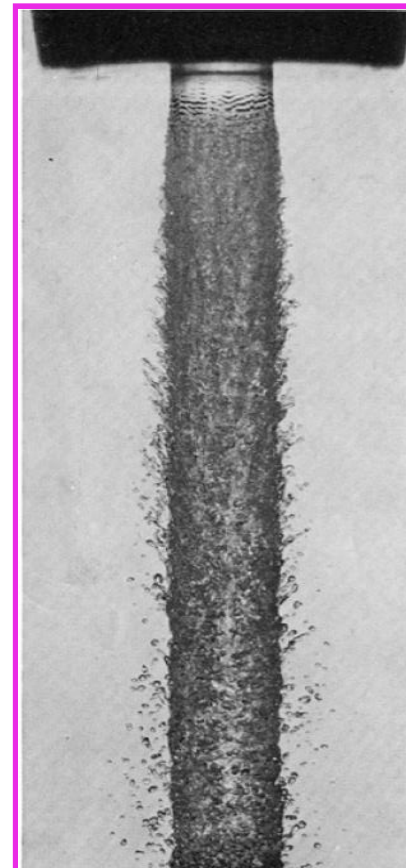
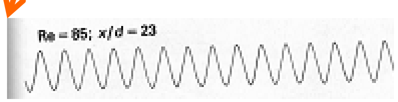
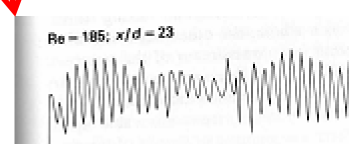


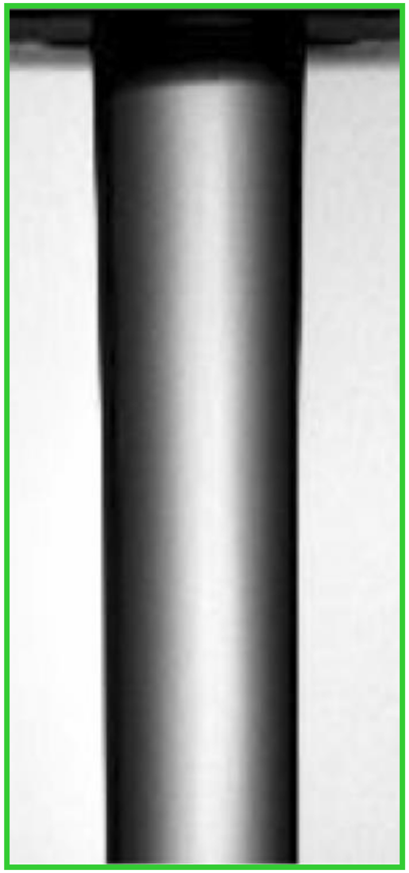
Hoyt and Taylor (1977)

steady ⇨ ω ⇨ ω_1, ω_2 ⇨ full frequency spectrum





Laminar ⇨ Instability ⇨ Pattern ⇨ Turbulence



Marmottant and Villermaux (2004)

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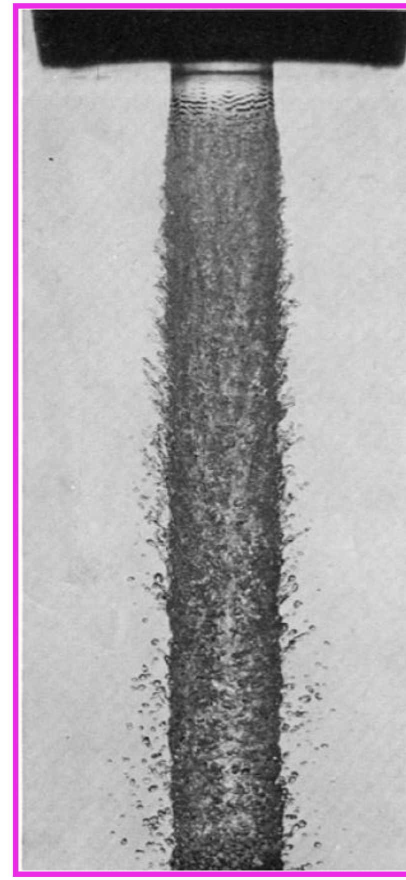


Hoyt and Taylor (1977)

no fluctuation	⇨	ε	⇨	1	⇨	fluctuation > average
R	⇨	R, λ	⇨	R, λ_1, λ_2	⇨	full spectrum of lengscales
steady	⇨	ω	⇨	ω_1, ω_2	⇨	full frequency spectrum

Laminar ⇨ Instability ⇨ Disorder/Pattern/Chaos ⇨Turbulence

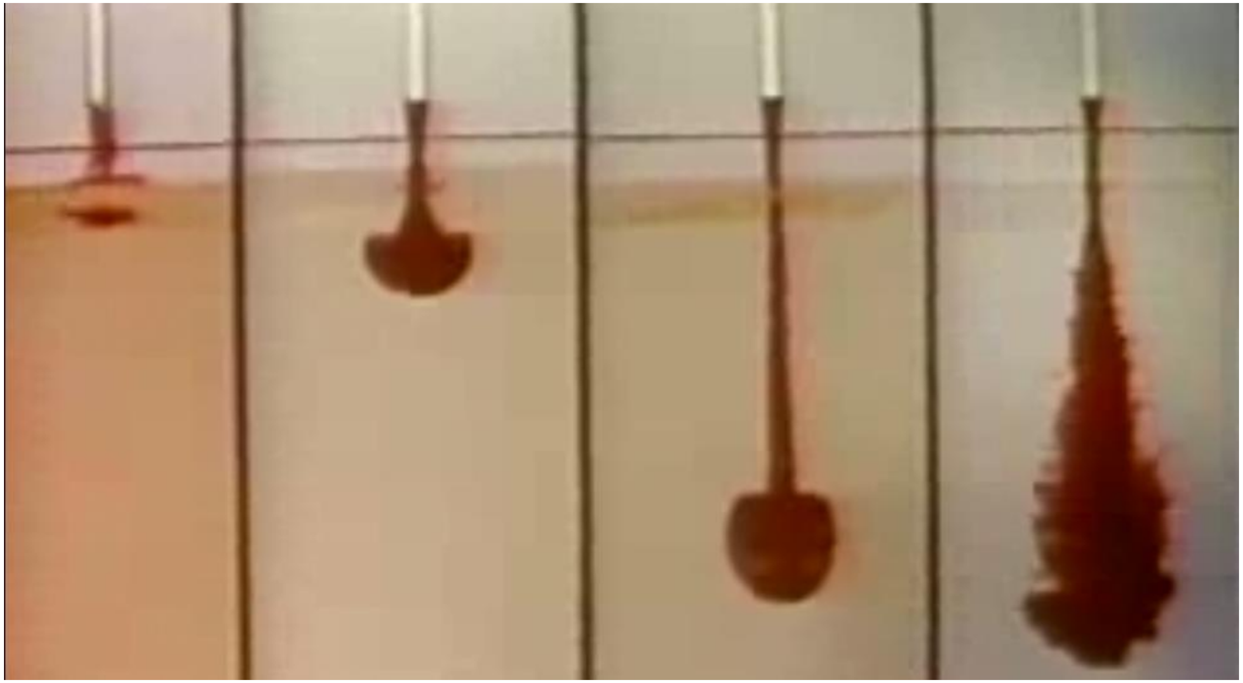
$$\underbrace{\hspace{12em}}_{\textbf{Transition}}$$

- ⬀ **fluctuations amplitude**
- ⬂ **lengthscales**
- ⬀ **frequency spectrum**

Loss of predictability
Sensitivity to initial condition (butterfly effect)

Laminar ⇨ Instability ⇨ Disorder/Pattern/Chaos ⇨Turbulence

$$\underbrace{\text{Instability} \Rightarrow \text{Disorder/Pattern/Chaos}}_{\text{Transition}}$$



Beware: instability without turbulence!

Laminar ⇨ Instability ⇨ Disorder/Pattern/Chaos ⇨Turbulence

Transition

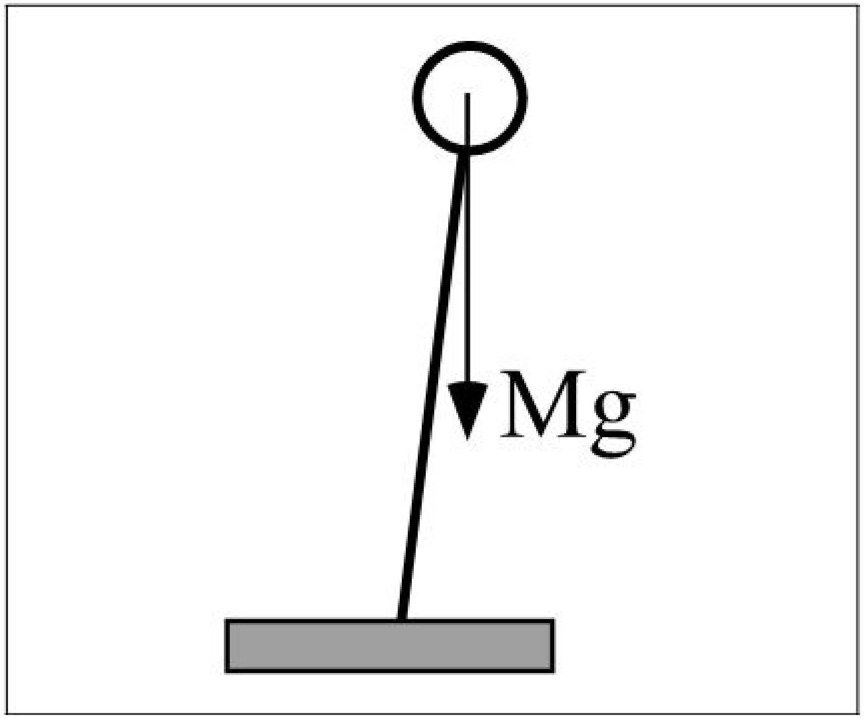
Beware: turbulence without linear instability!

Instabilities

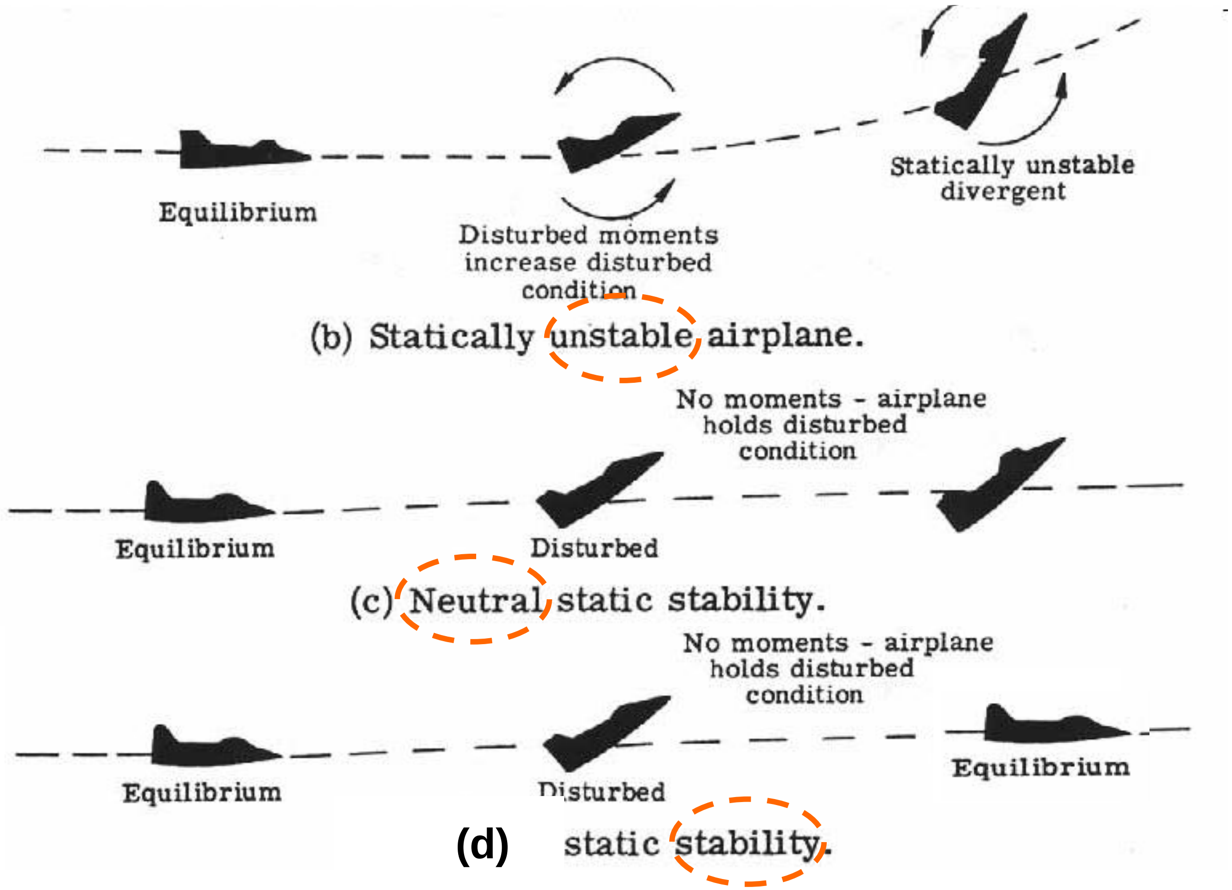
Today:

1. **Dynamical system instability (an example)**
2. **Definitions**

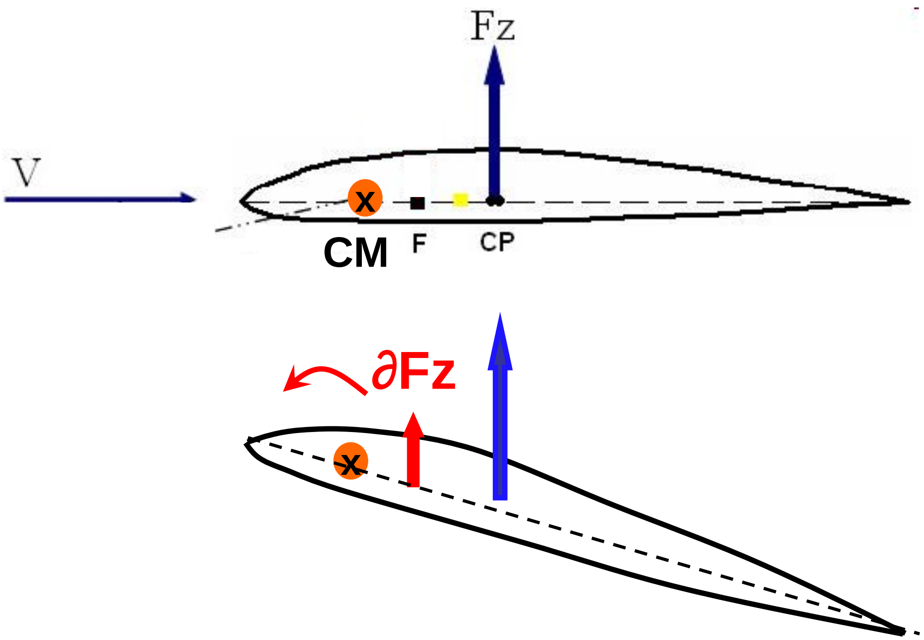
A dynamical system approach



Longitudinal static stability of an airplane

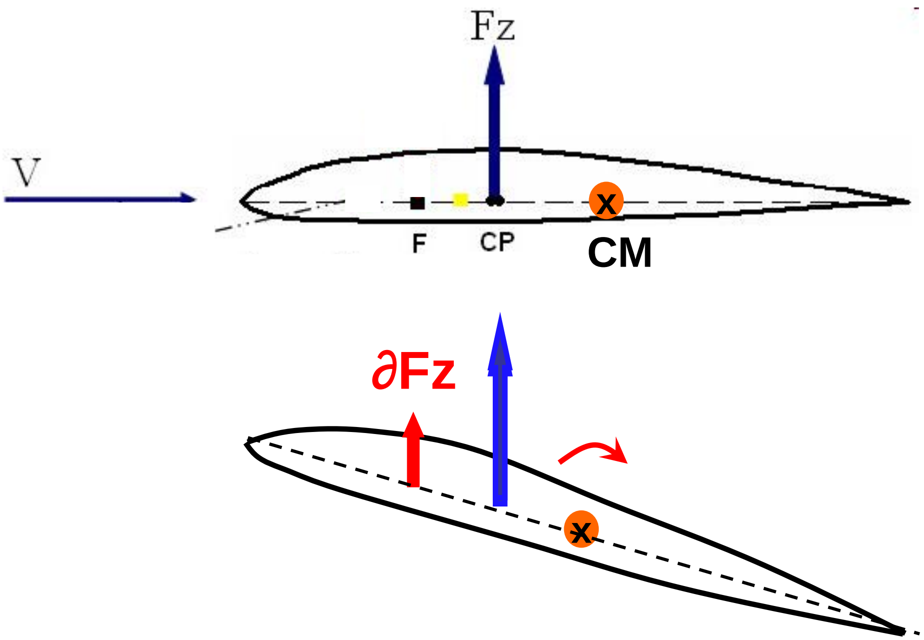


A stable wing



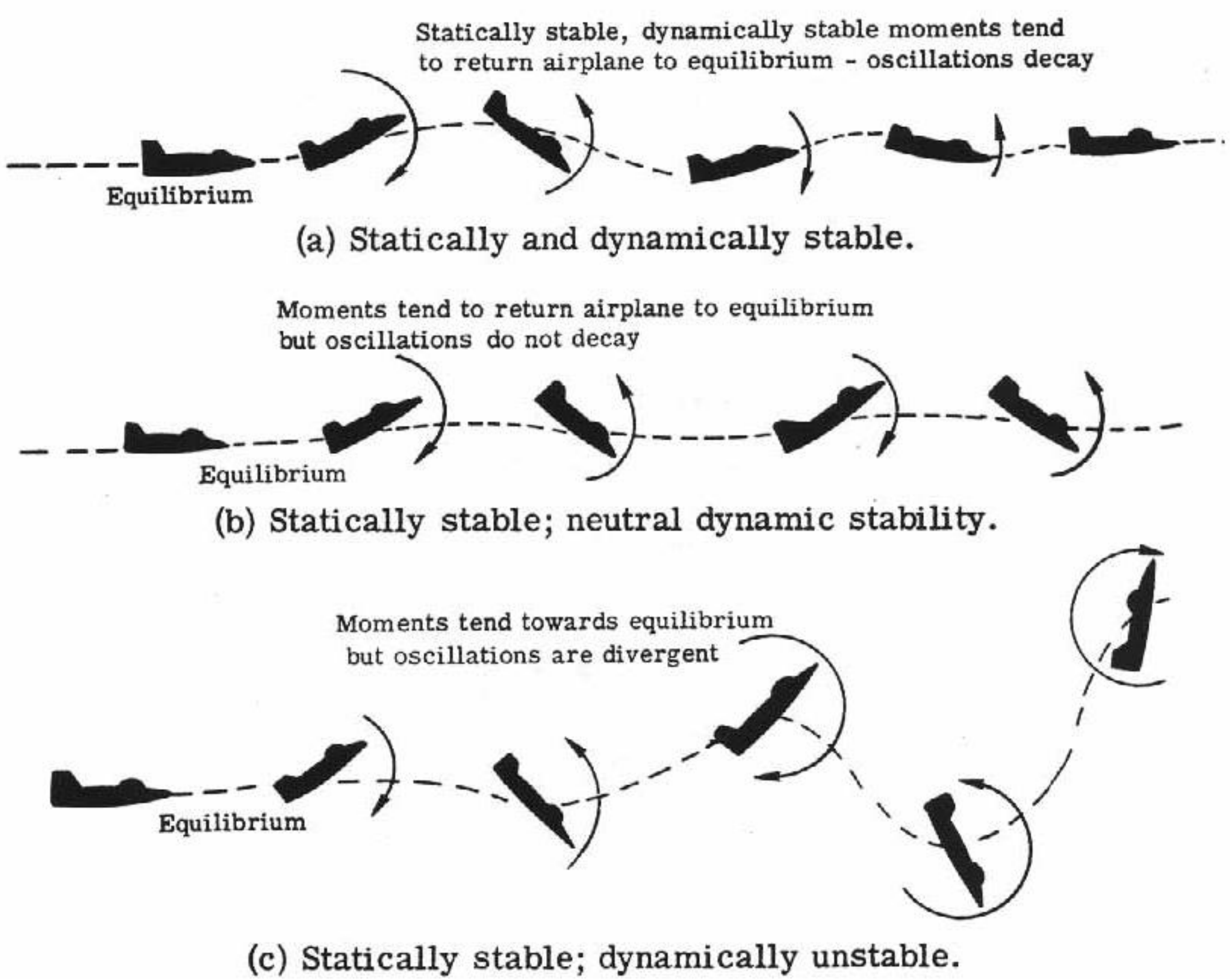
F: focal point
CM: center of mass
CP: center of lift

Unstable wing



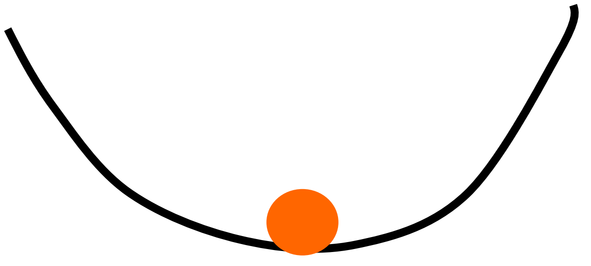
F: focal point
CM: center of mass
CP: center of lift

Dynamical longitudinal stability of an airplane



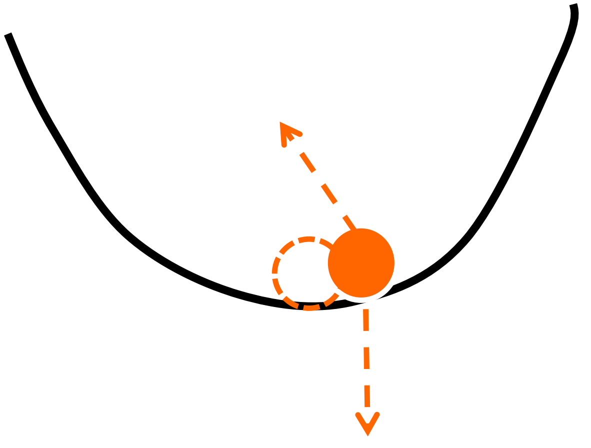
Write equations for temporal evolution:
Dynamical system approach

A dynamical system approach



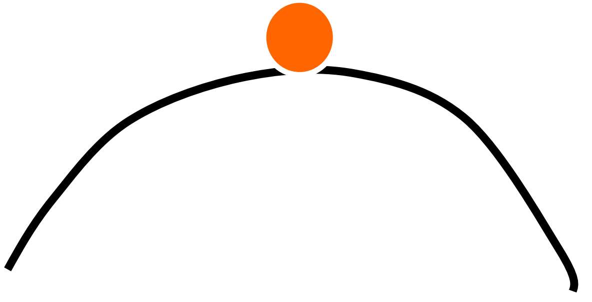
?

A dynamical system approach



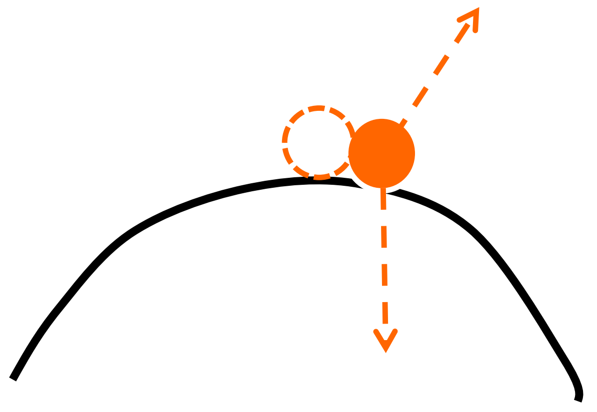
Stable

A dynamical system approach



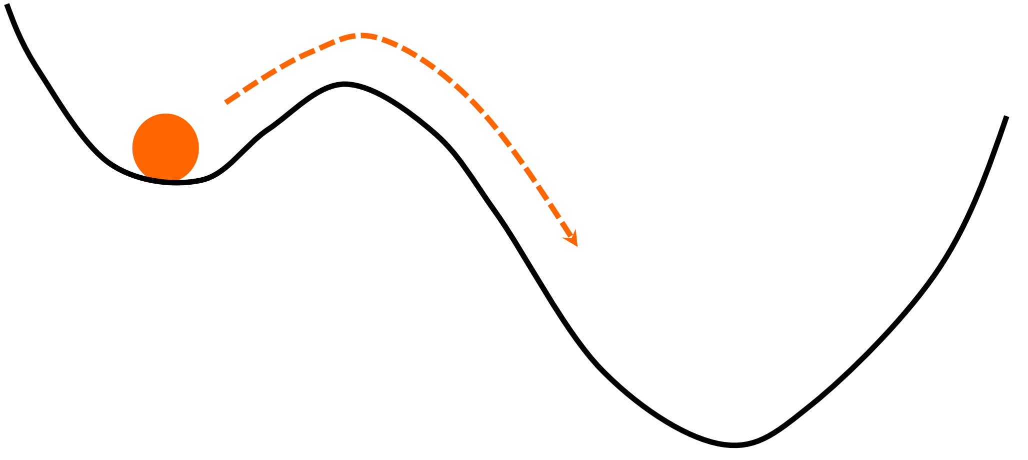
?

A dynamical system approach



Unstable

A dynamical system approach

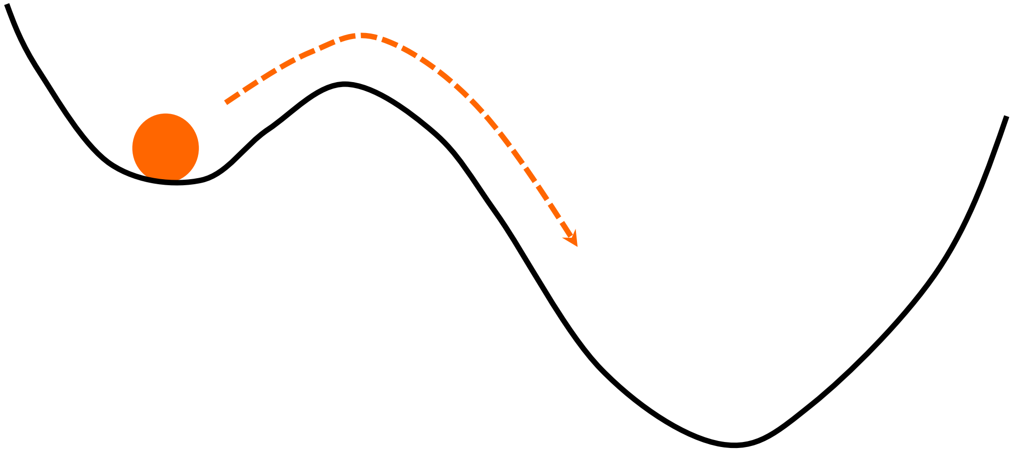


Local /Global energy minimum

Linearly stable but nonlinearly unstable

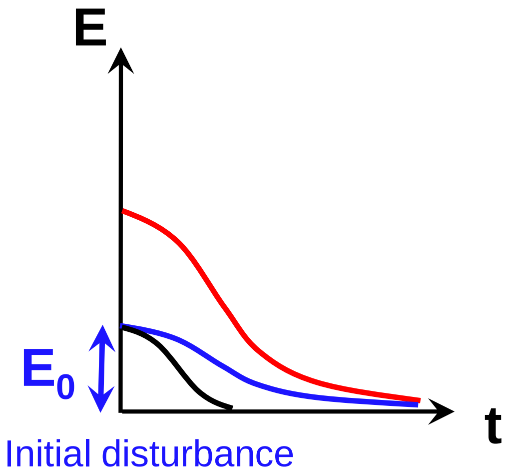
Concept of bassin of attraction

A dynamical system approach



Beware: fluid dynamics is not a conservative system!

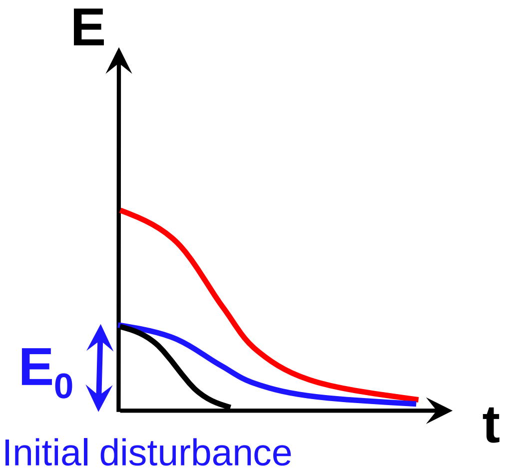
Evolution of disturbance energy



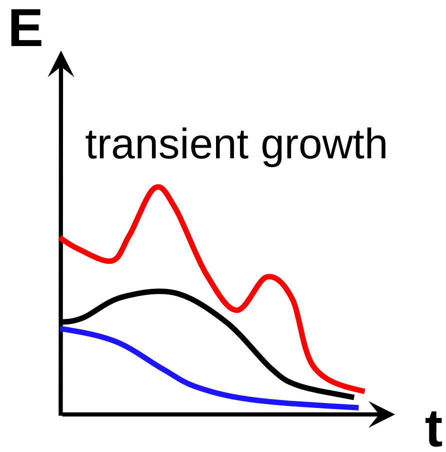
Monotonous stability

E is an integrated quantity:
different disturbance flows can share the same E !

Evolution of disturbance energy

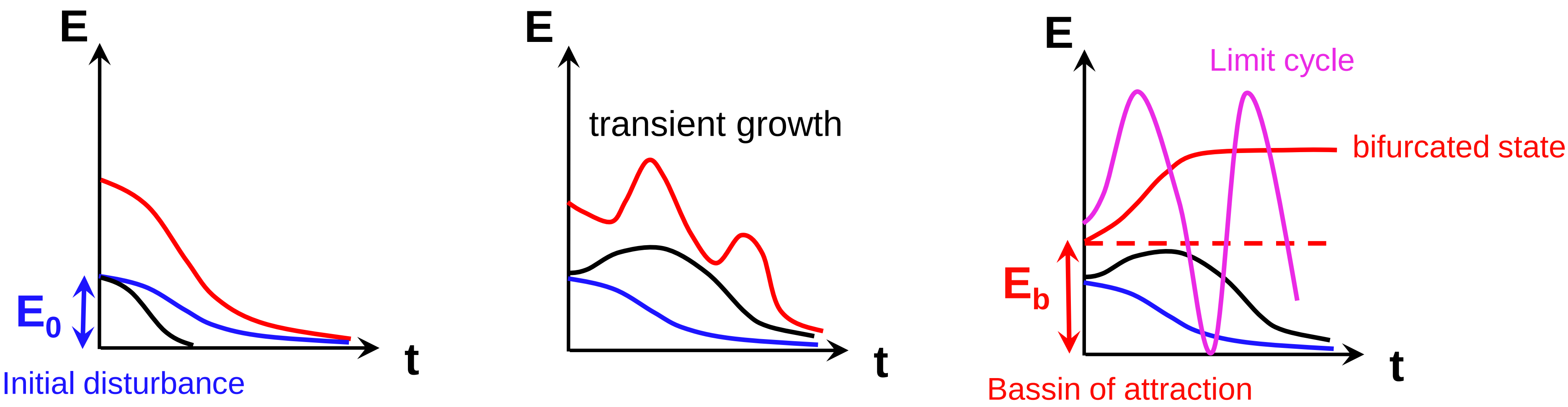


Monotonous stability



Asymptotic stability

Evolution of disturbance energy



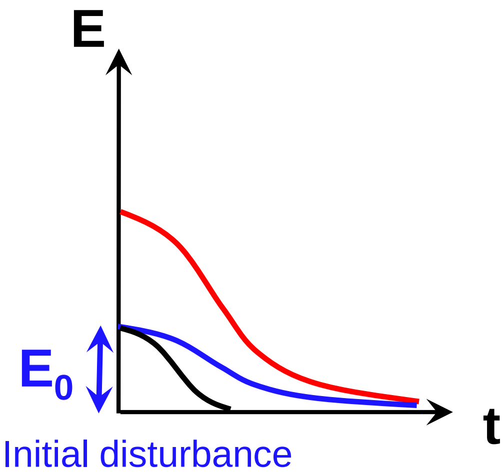
Monotonous stability

Asymptotic stability

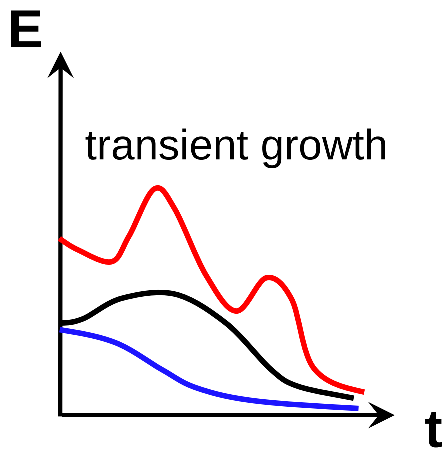
Conditional stability

Lyapunov (global) stability

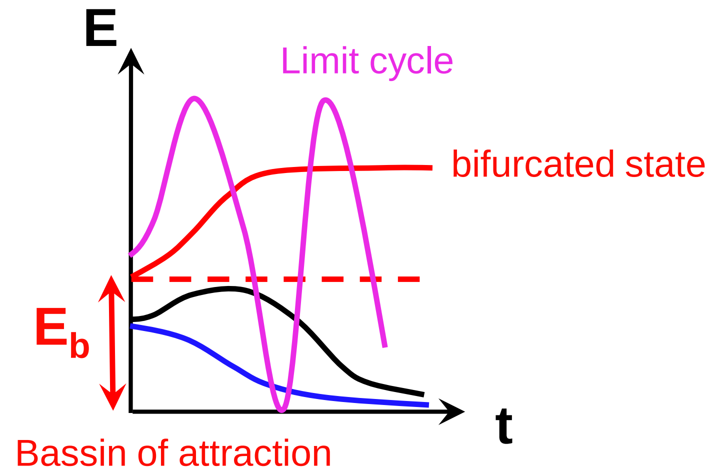
Evolution of disturbance energy



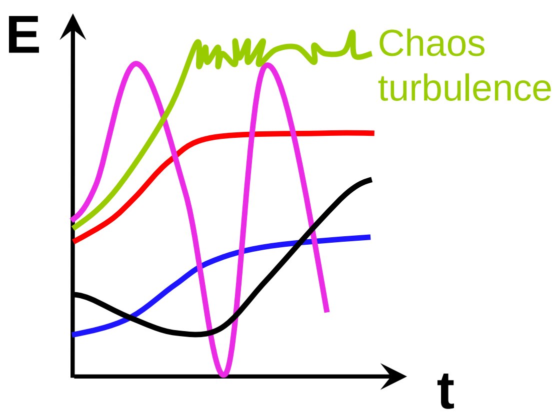
Monotonous stability



Asymptotic stability

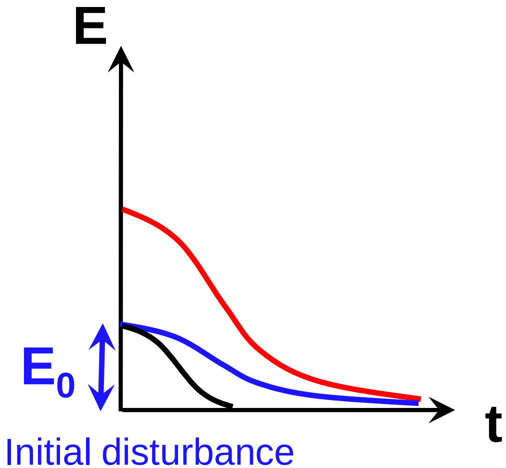


Conditional stability

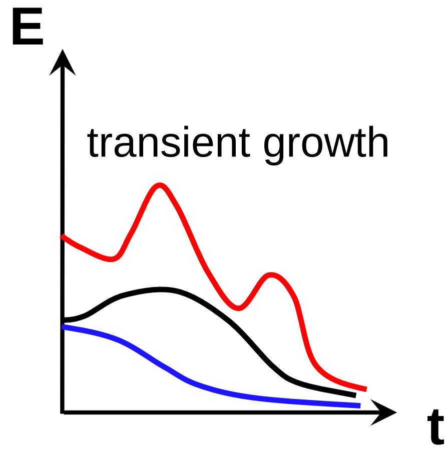


Inconditional instability

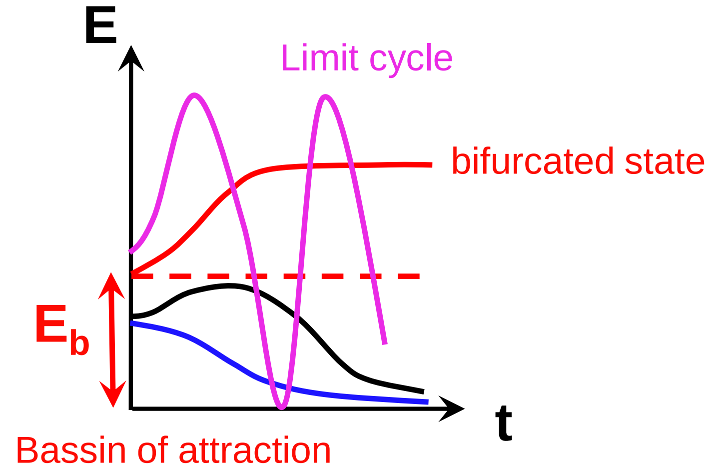
Evolution of disturbance energy



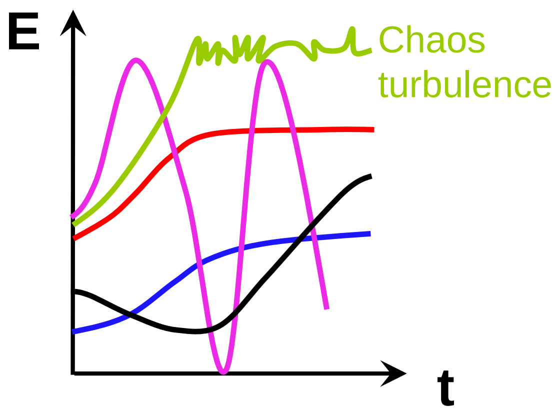
Monotonous
stability



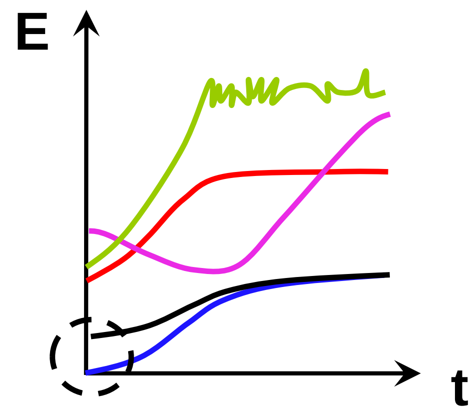
Asymptotic
stability



Conditional
stability



Inconditional
instability

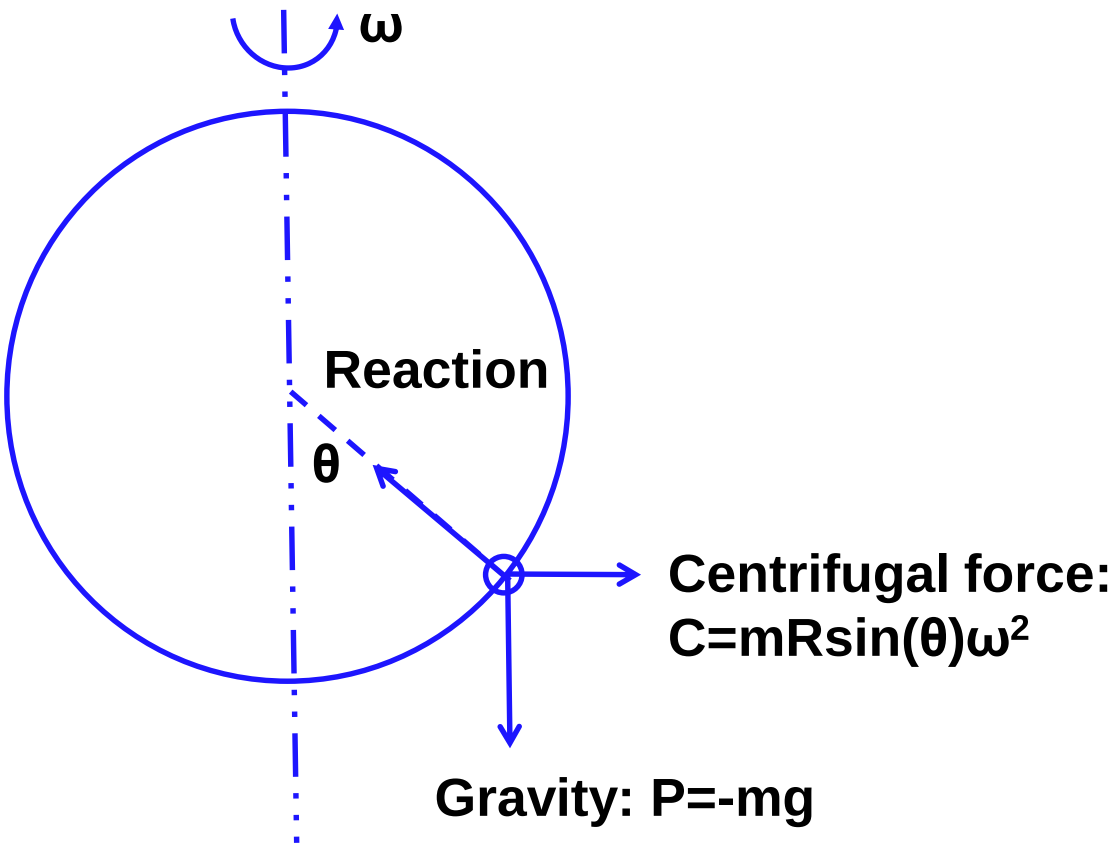


Linear
instability

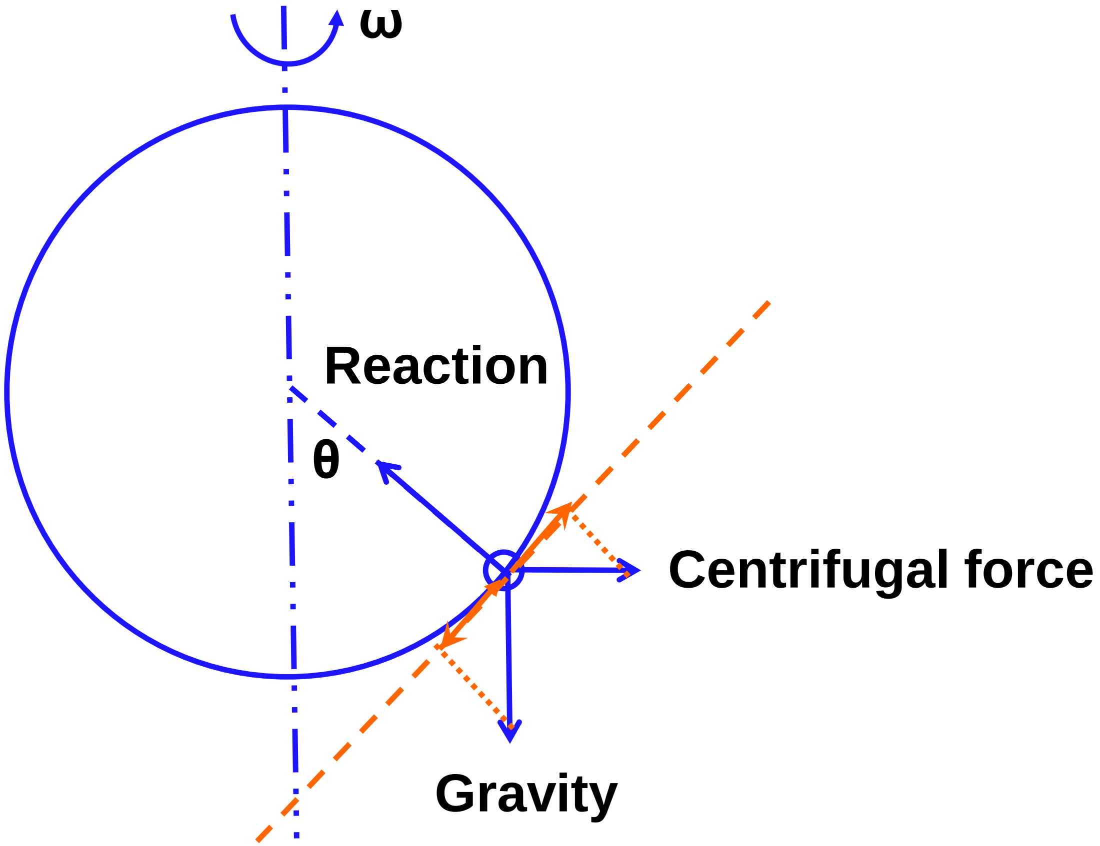
Important concepts

- Bassin of attraction
- Bifurcation
- Subcritical/Supercritical Bifurcation

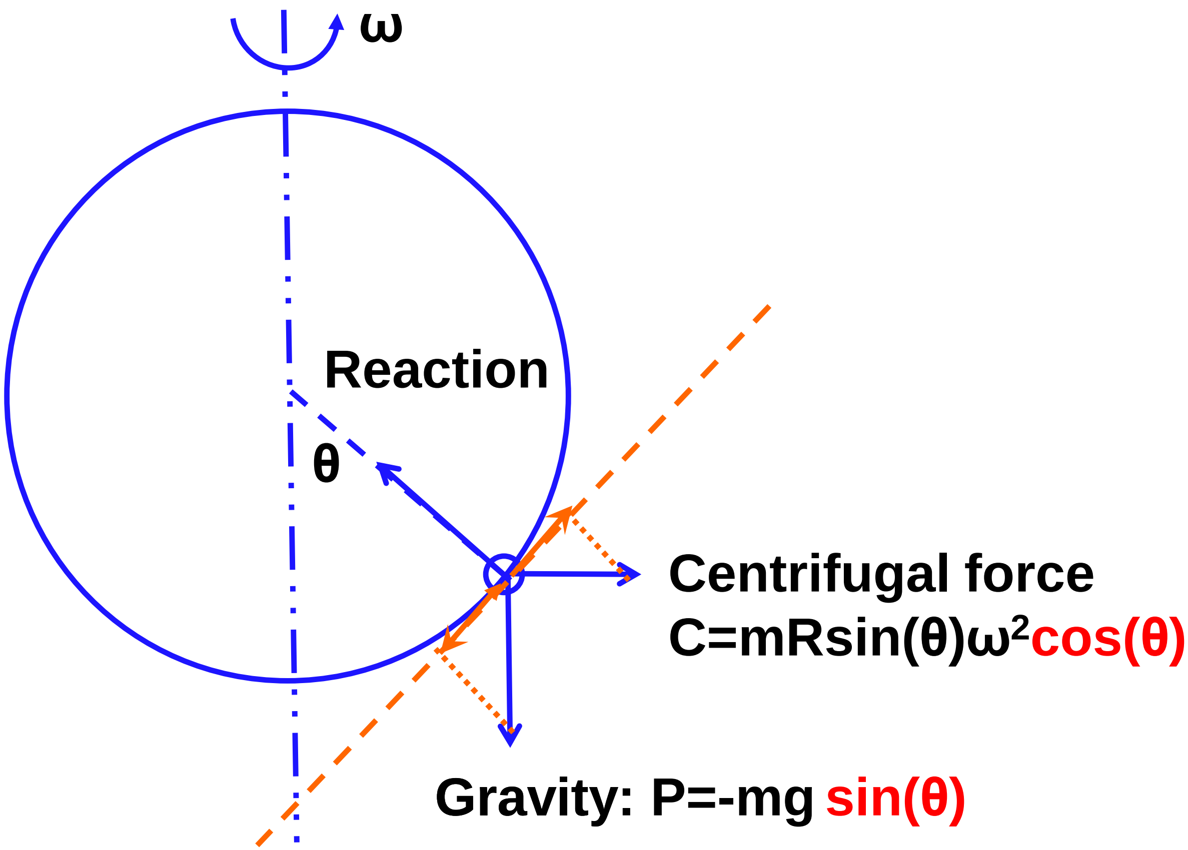
Bifurcations and amplitude equations



Bifurcations and amplitude equations



Bifurcations and amplitude equations



$$mR\ddot{\theta}=-mg\sin(\theta)+mR\sin(\theta)\omega^2\cos(\theta)$$

Governing equations

$$\boxed{\ddot{\theta}=-\omega_0^2\sin(\theta)+\omega^2\sin(\theta)\cos(\theta)}$$

$\omega_0^2=g/R$

Pendulum frequency

Governing equations

$$\boxed{\ddot{\theta}=-\omega_0^2\sin(\theta)+\omega^2\sin(\theta)\cos(\theta)}$$

$\omega_0^2=g/R$

Pendulum frequency

Base 'flow'

$\theta=0$

Governing equations

$$\boxed{\ddot{\theta}=-\omega_0^2\sin(\theta)+\omega^2\sin(\theta)\cos(\theta)}$$

$\omega_0^2=g/R$

Pendulum frequency

Base ‘flow’

$\theta=0$

Small perturbations

$\theta=0+\varepsilon\,\theta'$

Governing equations

$$\boxed{\ddot{\theta}=-\omega_0^2\sin(\theta)+\omega^2\sin(\theta)\cos(\theta)}$$

$\omega_0^2=g/R$
Pendulum frequency

Base 'flow'

$\theta=0$

Small perturbations

$\theta=0+\varepsilon\,\theta'$

Linearized equations

$$\boxed{\ddot{\theta}'=-\omega_0^2\theta'+\omega^2\theta'}$$

Linearized equations

$$\boxed{\ddot{\theta}' = -\omega_0^2 \theta' + \omega^2 \theta'}$$

Normal mode

$$\theta' = A \exp(st)$$

Dispersion relation

$$\boxed{s^2 = \omega^2 - \omega_0^2}$$

$$\boxed{\omega^2 < \omega_0^2}$$

$$\boxed{\omega^2 > \omega_0^2}$$

$$\boxed{s = \pm i(\omega_0^2 - \omega^2)^{1/2}}$$

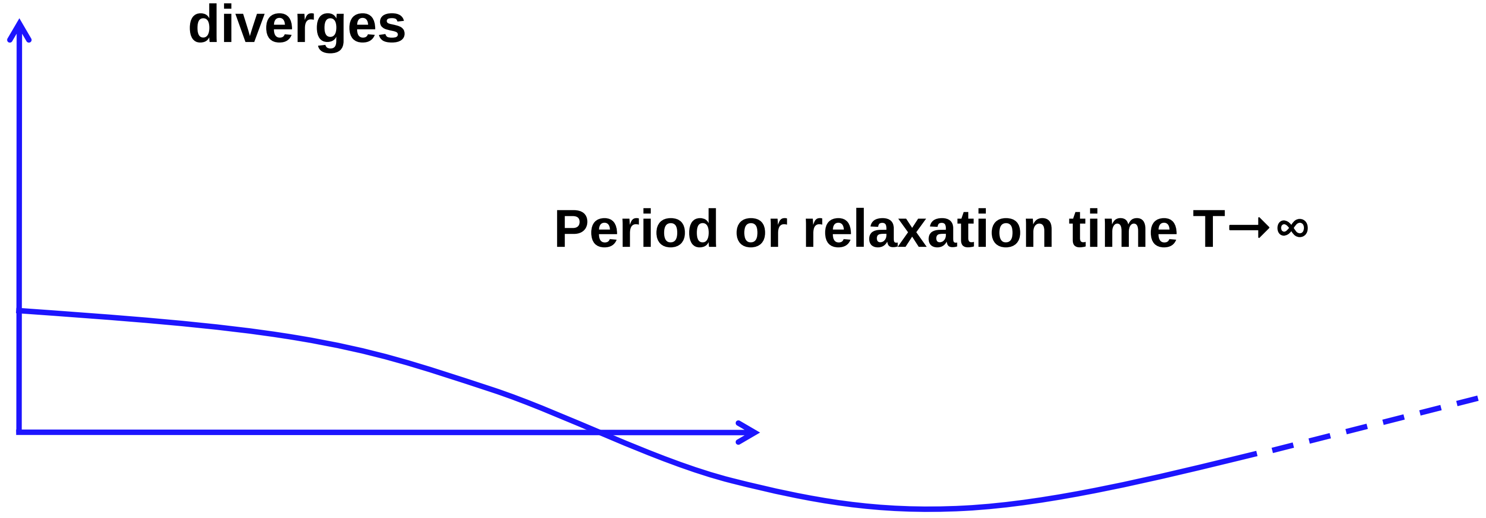
STABLE

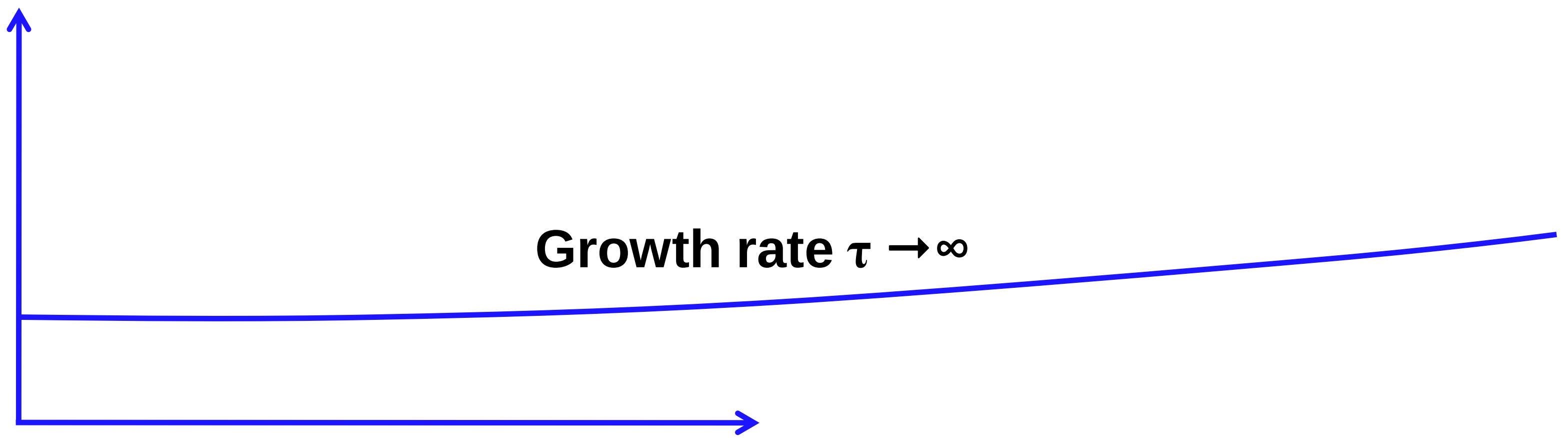
$$\boxed{s = \pm(\omega^2 - \omega_0^2)^{1/2}}$$

UNSTABLE

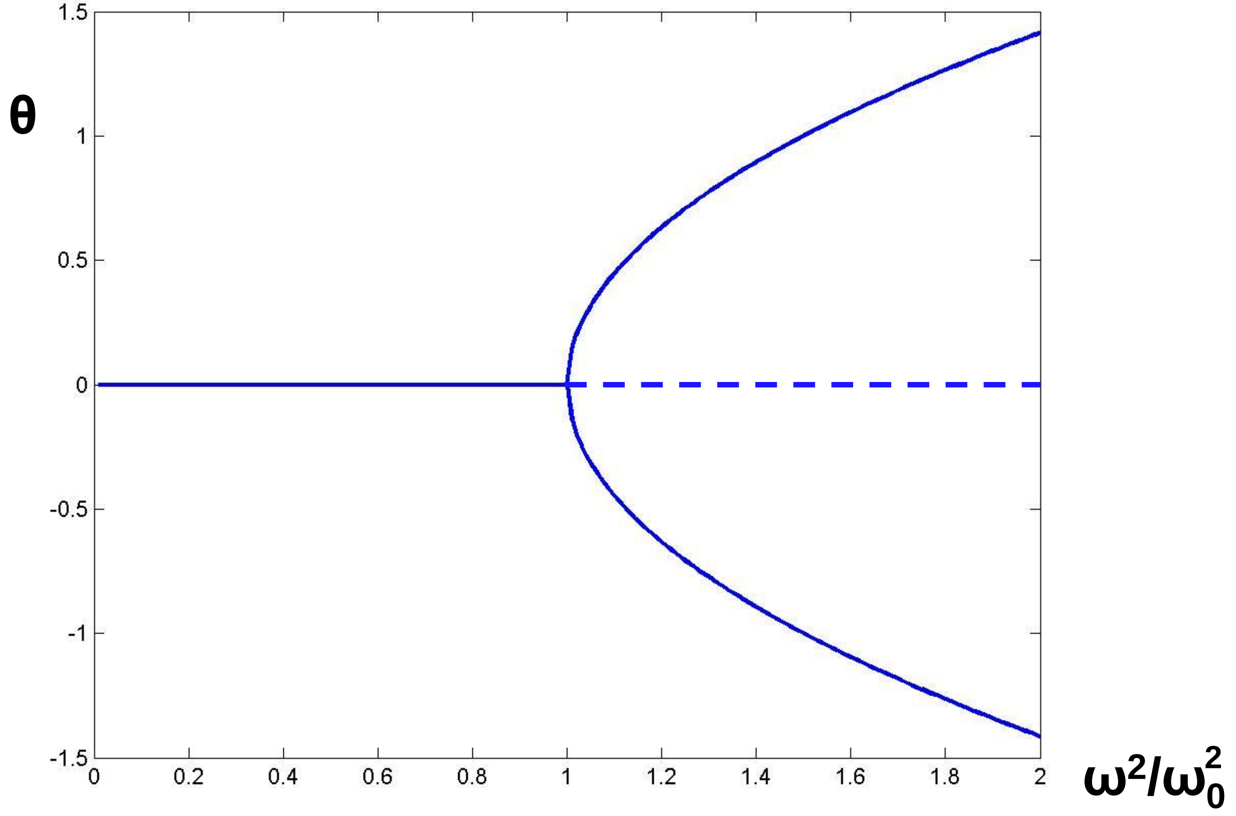
Important concept: critical slowing

When $\omega^2 \sim \omega_0^2$, the characteristic time $\tau = 1/|s|$ diverges





What about nonlinearities?



What about nonlinearities?

But recall the full nonlinear equation

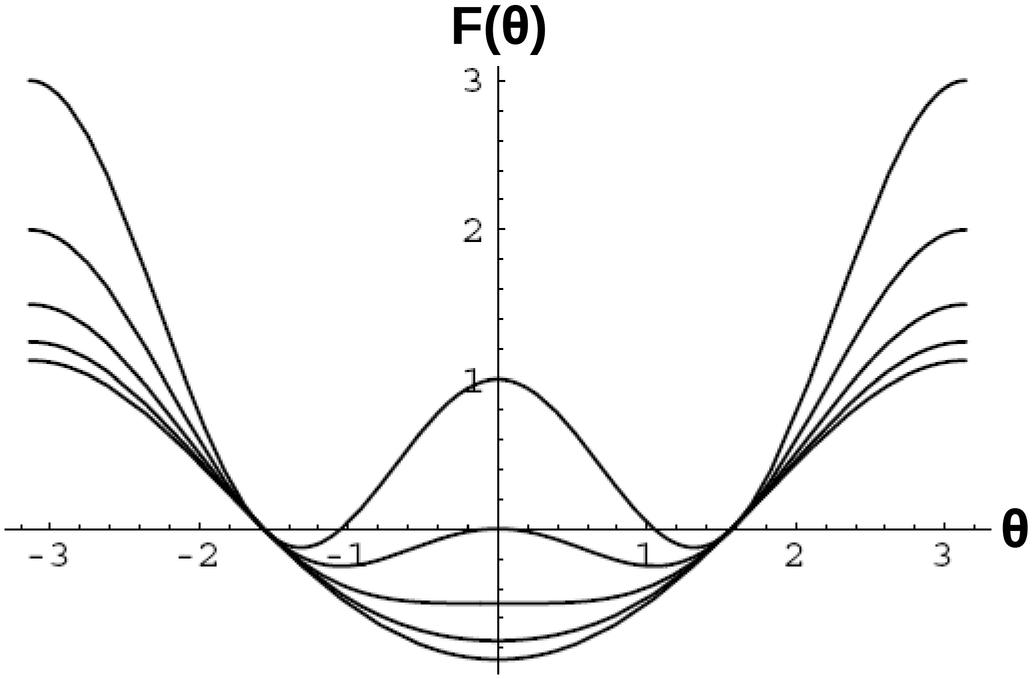
$$\boxed{\ddot{\theta} = -\omega_0^2 \sin(\theta) + \omega^2 \sin(\theta)\cos(\theta)}$$

It has another 2 steady solutions

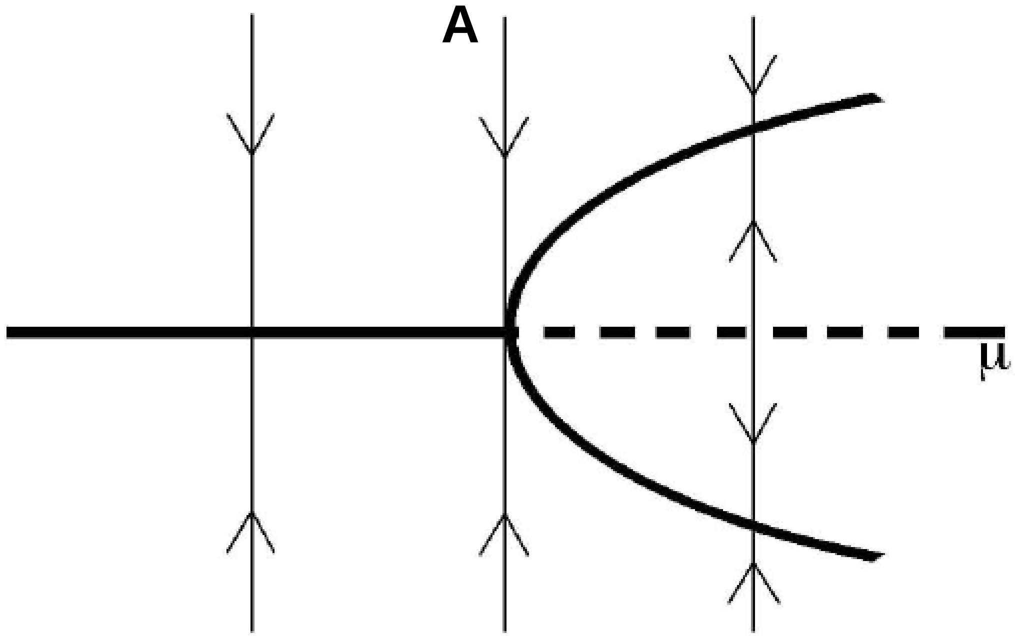
$$\theta_s = \arccos((\omega_0/\omega)^2)$$

Physical interpretation (Potential)

$$\boxed{m\ddot{\theta} = F'(\theta)}$$



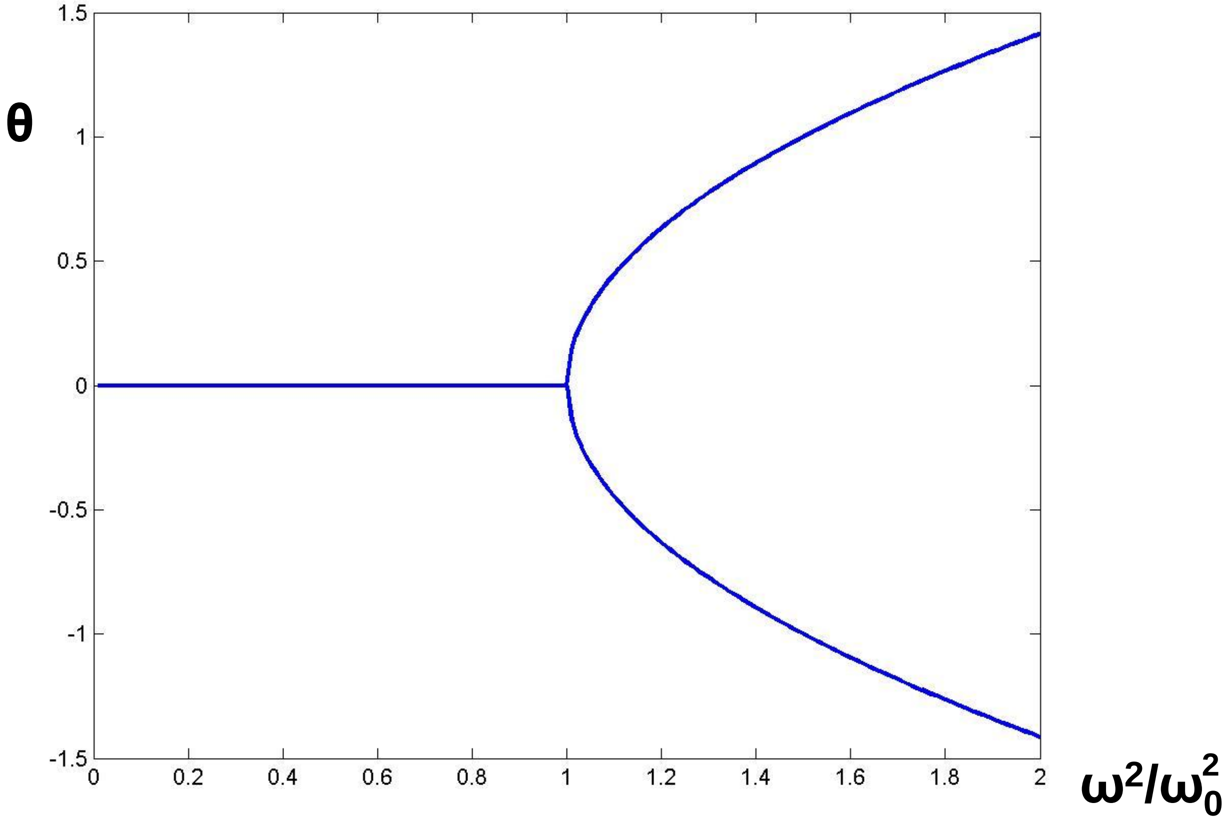
What about nonlinearities?



Supercritical fork bifurcation

— stable
--- unstable

Simple exercise for next week



Demonstrate the dynamic stability of the bifurcated states