

Instabilities



Marmottant and Villermaux (2004)

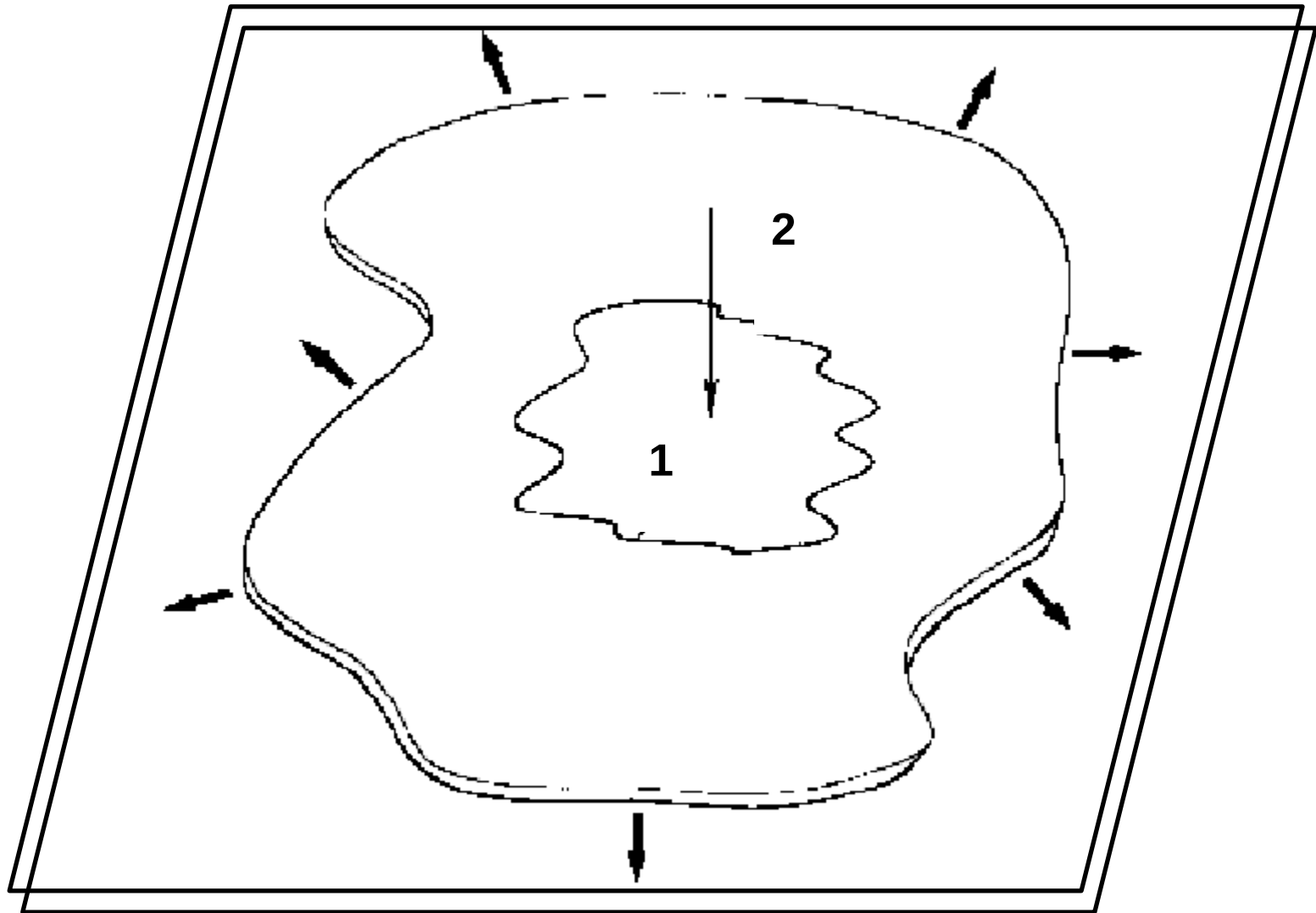
Pr. Francois Gallaire

LFMI

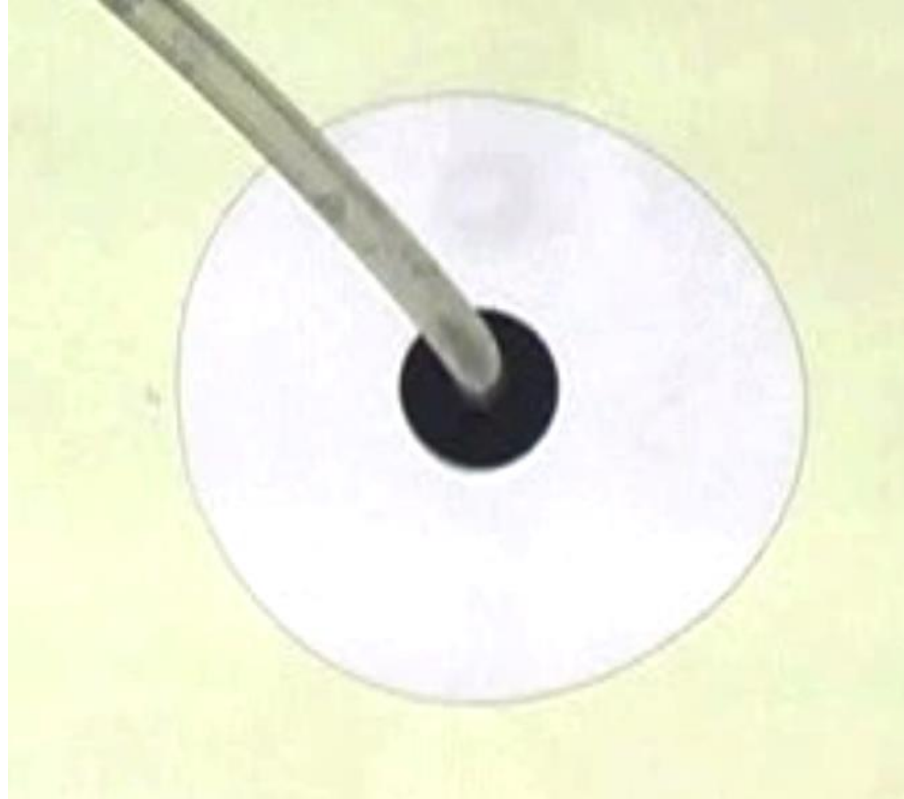
MED 2 2926

Email: francois.gallaire@epfl.ch

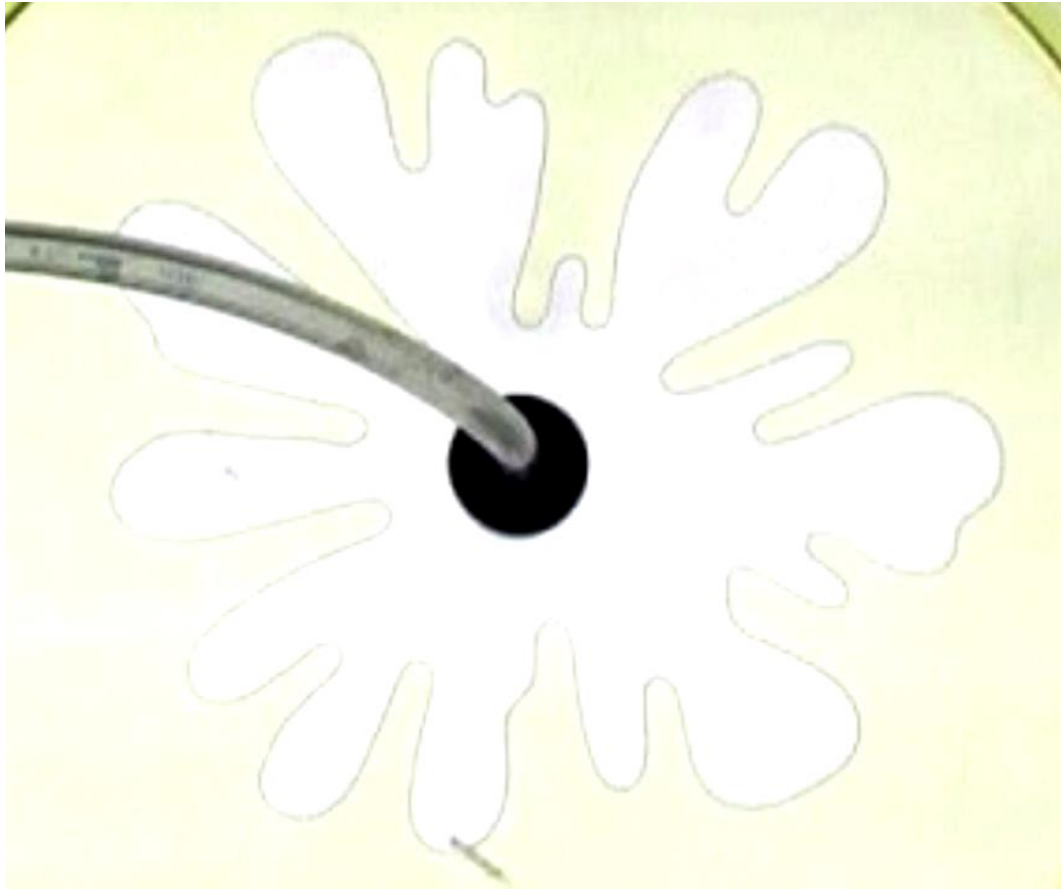
Injection of fluid 1 in fluid 2 in a Hele-Shaw cell



Stable?

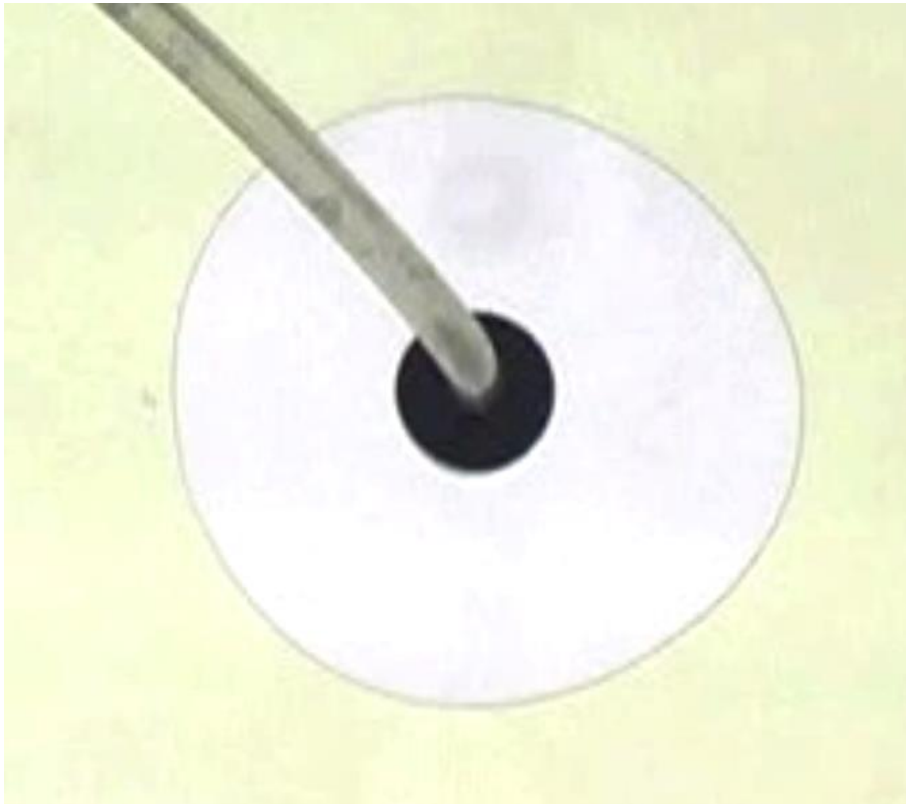


Unstable?

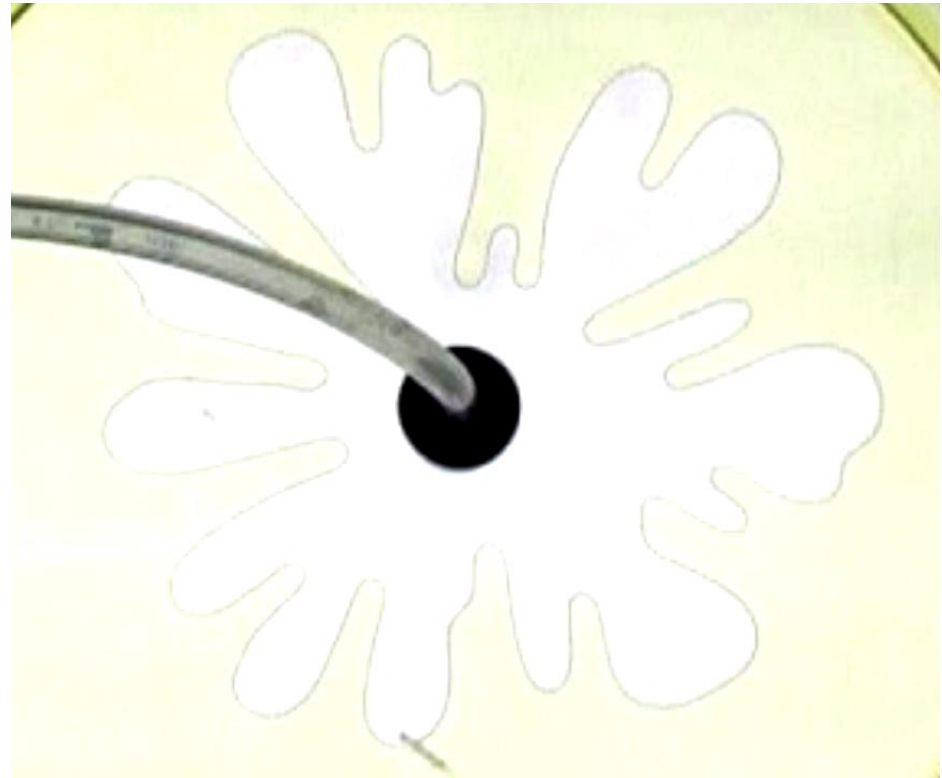


The important parameter is the viscosity ratio

Oil in water is stable



Water in oil is unstable



Saffman-Taylor instability

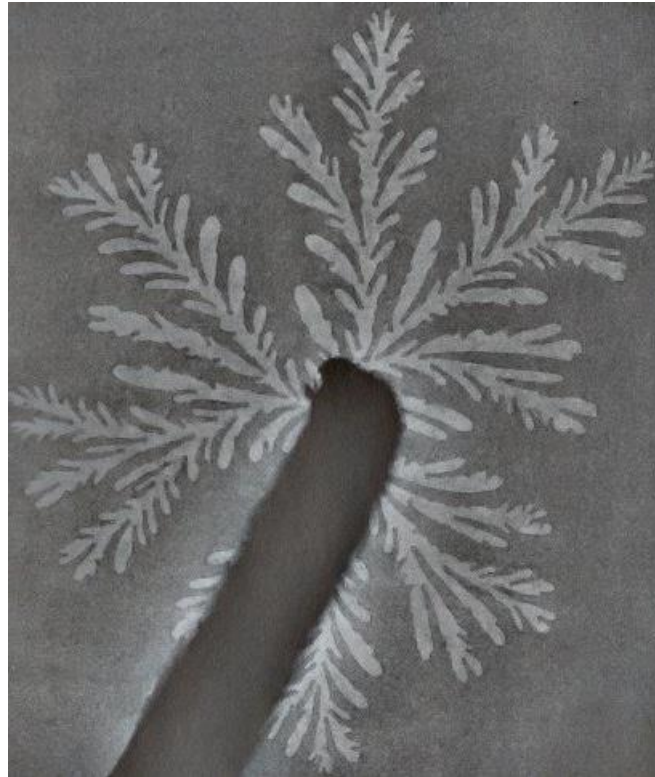
Numerical Simulation

Dr. Mathias Nagel, LFMI

Boundary Element plot for expanding Saffman-Taylor instability

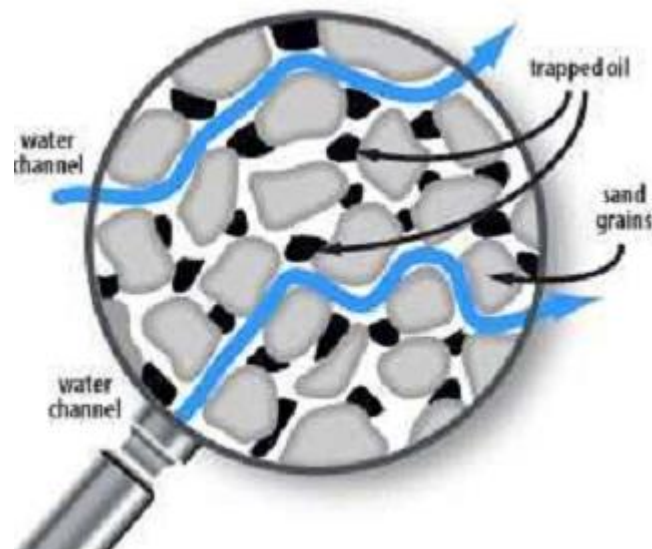
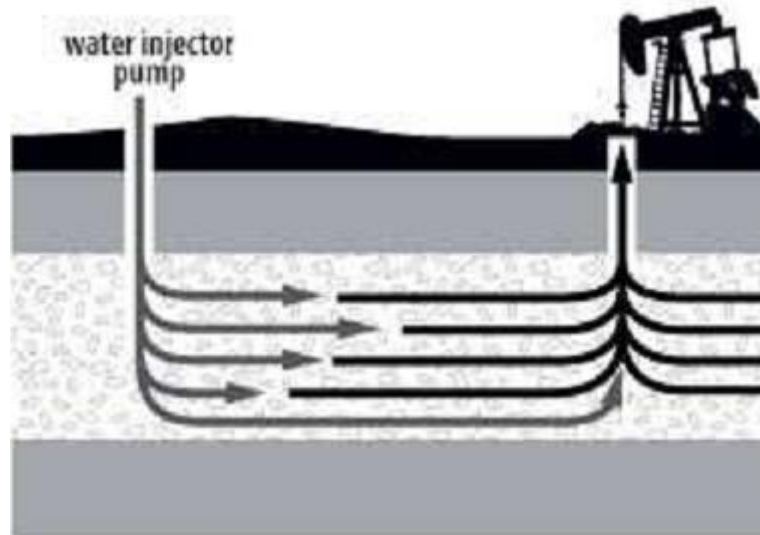


Viscous fingering

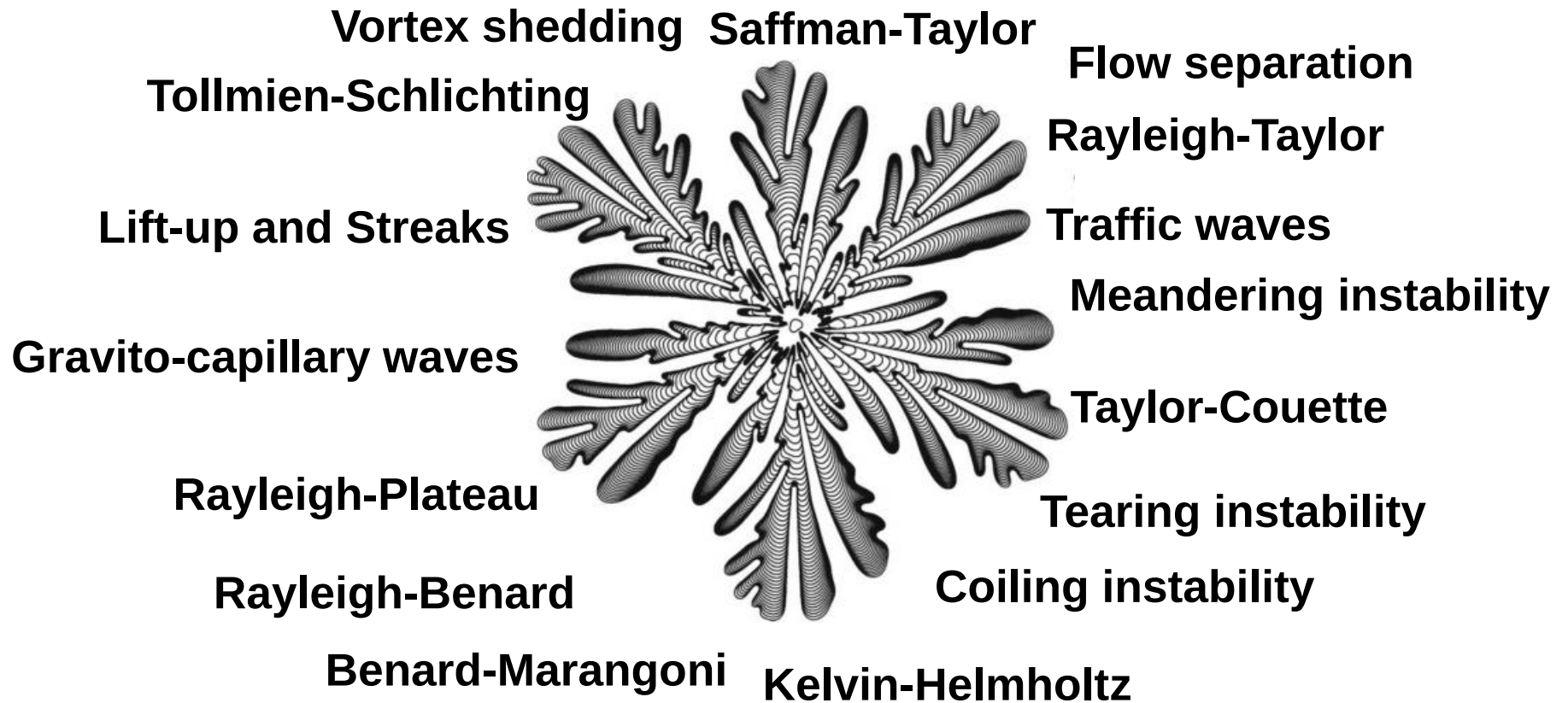


An example of Saffman-Taylor fingering. The interface here is affected by surface tension, which leads to a characteristic length scale over which the dendrites occur (adapted from *Perspectives in Fluid Dynamics*, CUP (2000))

Viscous fingering



Most flows are unstable...

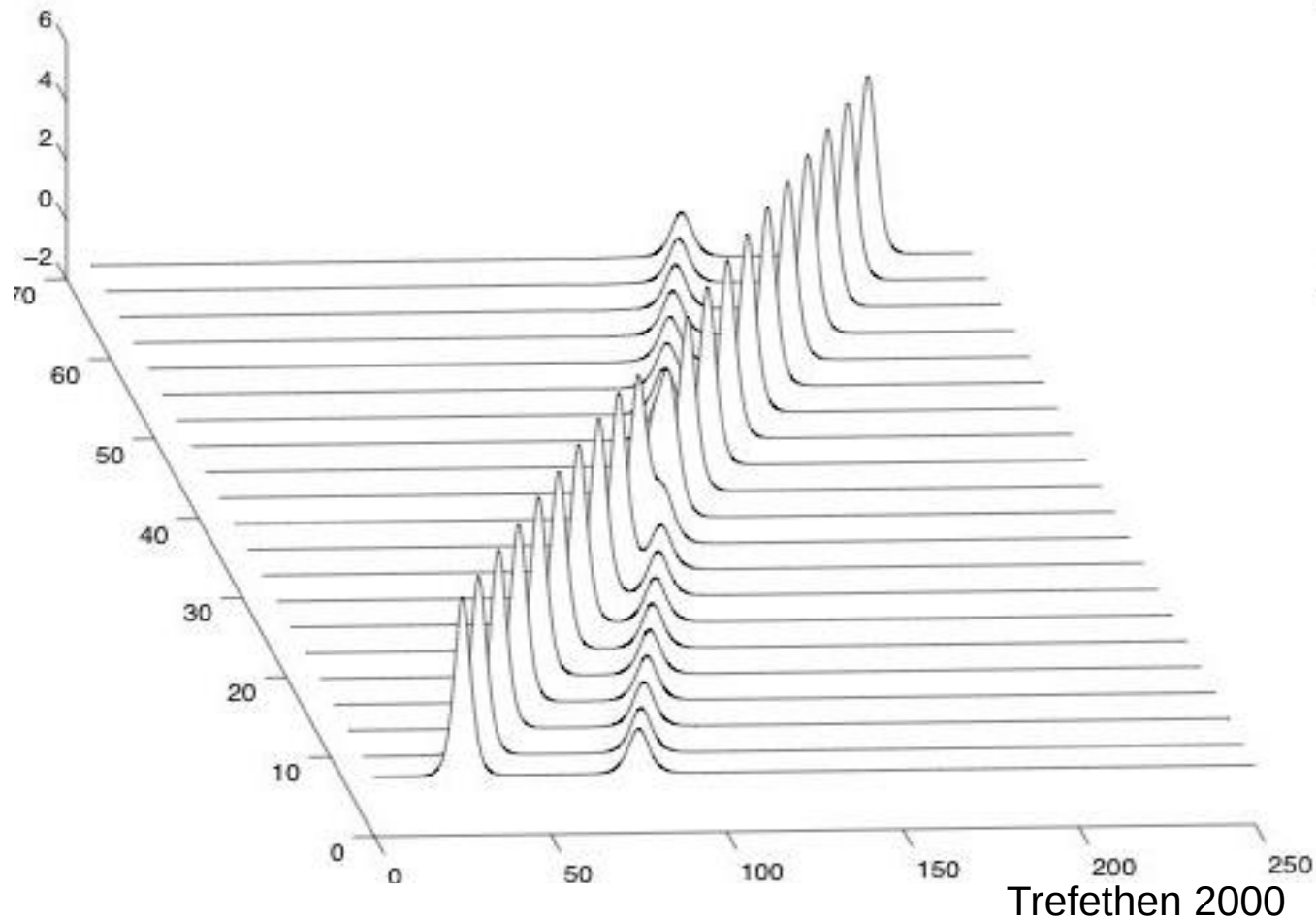


But some are very stable



Garcia & Chomaz 2005

Collision of tsunamis



Solitons are very stable solutions of the nonlinear water-wave equations

Instabilities

Friday 10h15-13h

Homework:

Exercise or scientific article

Exam: Instabilities

Oral: presentation of article

And written exam

Books:

- **Drazin P.G. , Introduction to hydrodynamic instability, CUP, 2000**
- **F. Charru, Instabilités hydrodynamiques, EDP sciences, 2007**

Instabilities



Compte utilisateur | [Préférences](#) | [Recherche guidée](#) | [Aide](#) | [Se déconnecter](#)

[Liste des résultats](#) | [Historique](#) | [Liste](#) | [Panier](#)

Recherche | **Recherche avancée** | **Recherche en mode expert** | **Parcourir >**

Titres sélectionnés: [Voir la sélect.](#) | [Sauvegarder/E-mail](#) | [Sous-ensemble](#)

Ensemble des résultats: [Tout sélect.](#) | [Supprimer sélect.](#) | [Modifier la recherche](#)

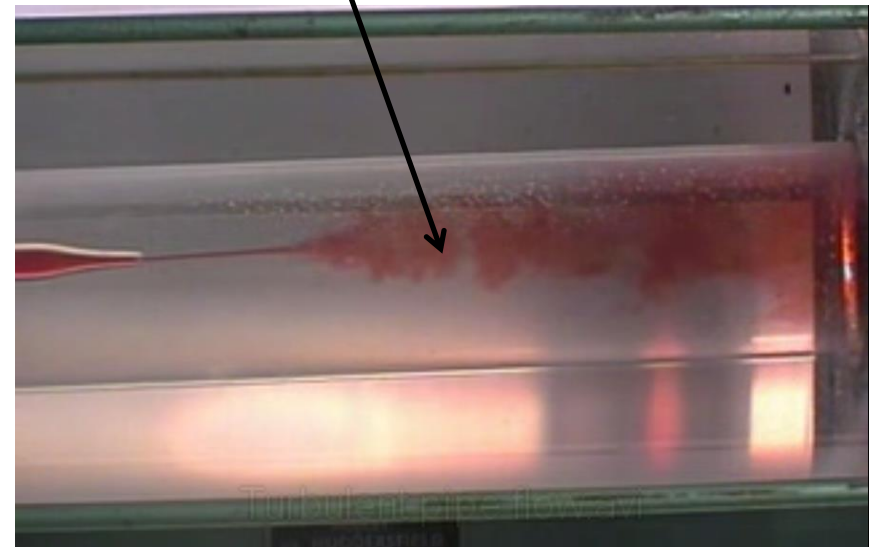
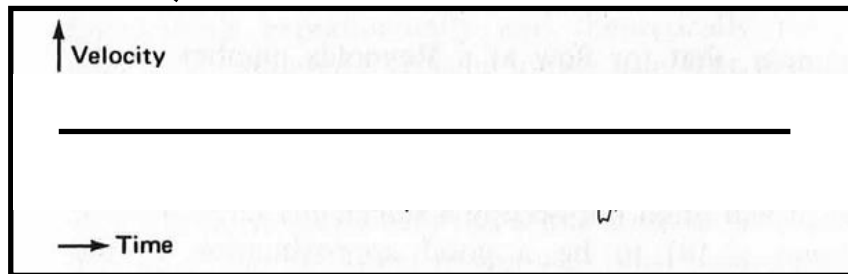
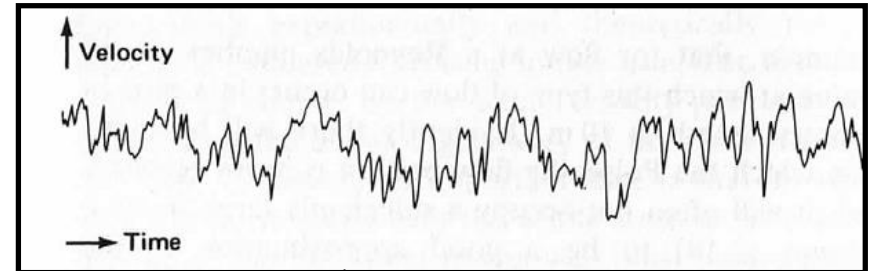
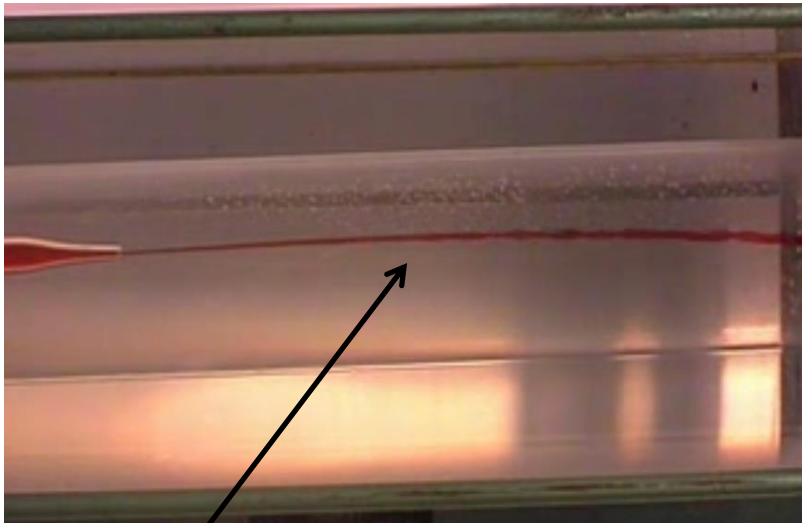
Résultats pour Mots= charru

Trié par : Année (desc), puis Auteur

Titre 1 - 7 à 7 (Limite d'affichage et de triage : 20000/1000)

#	Typ	Auteur	Titre
1	 Livre & en ligne	Charru, François	Hydrodynamic instabilities

Laminar/turbulent



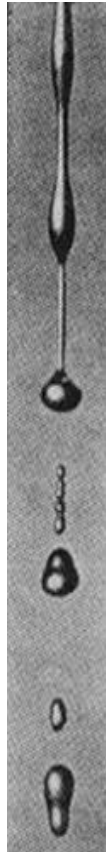
Instabilities and turbulence

Laminar $\Rightarrow$ Instability $\Rightarrow$ Disorder/Pattern/Chaos $\Rightarrow$ Turbulence

Transition



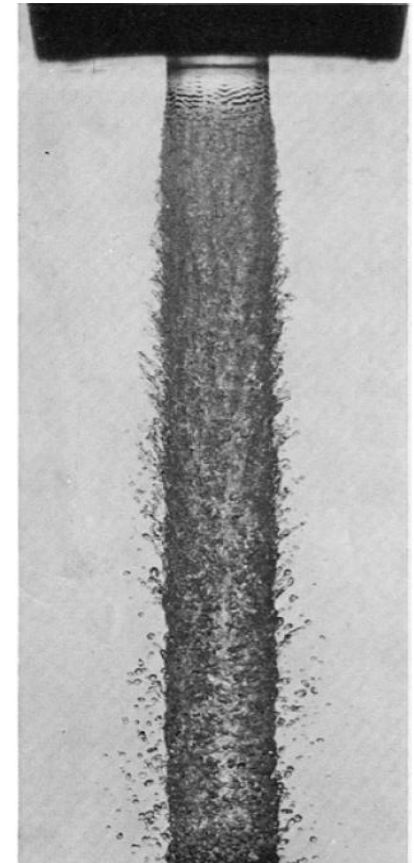
Marmottant and
Villermaux (2004)



Rayleigh (1891)



Marmottant and
Villermaux (2004)



Hoyt and Taylor (1977)¹⁵

Laminar

⇒ Instability ⇒ Pattern ⇒

Turbulence



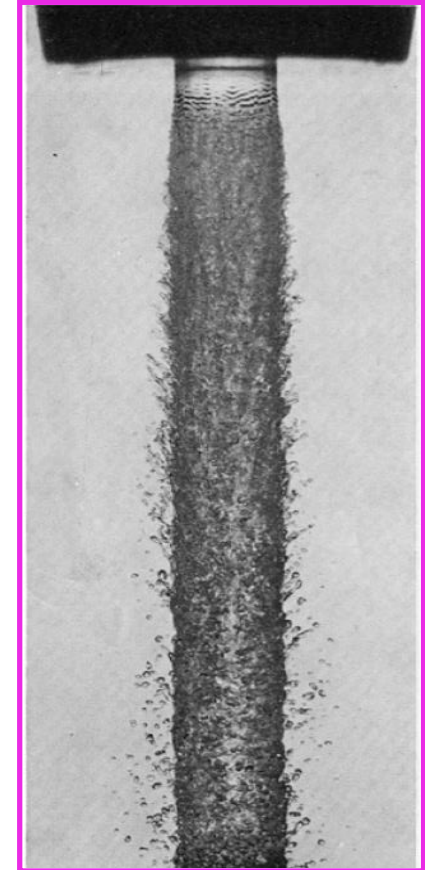
Marmottant and
Villermaux (2004)



Rayleigh (1891)



Marmottant and
Villermaux (2004)



Hoyt and Taylor (1977)

no fluctuation

⇒

ε

⇒

1

⇒

fluctuation > average

Laminar

⇒ Instability ⇒ Pattern ⇒

Turbulence



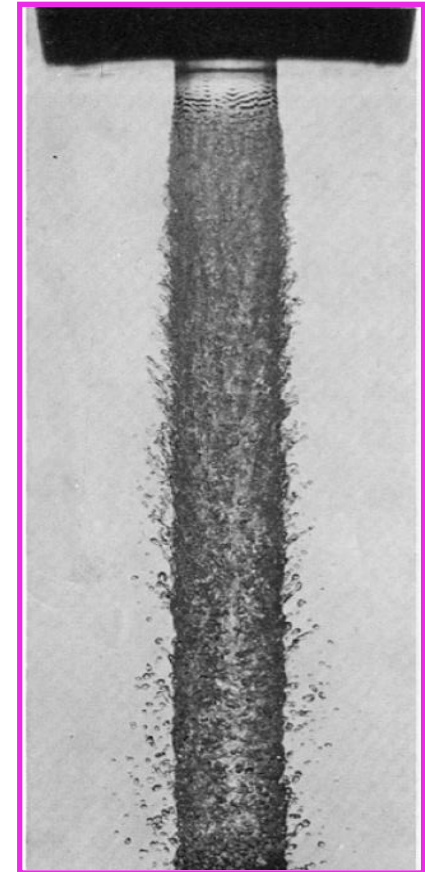
Marmottant and
Villermaux (2004)



Rayleigh (1891)



Marmottant and
Villermaux (2004)



Hoyt and Taylor (1977)

R

⇒

R, λ

⇒

R, λ_1, λ_2

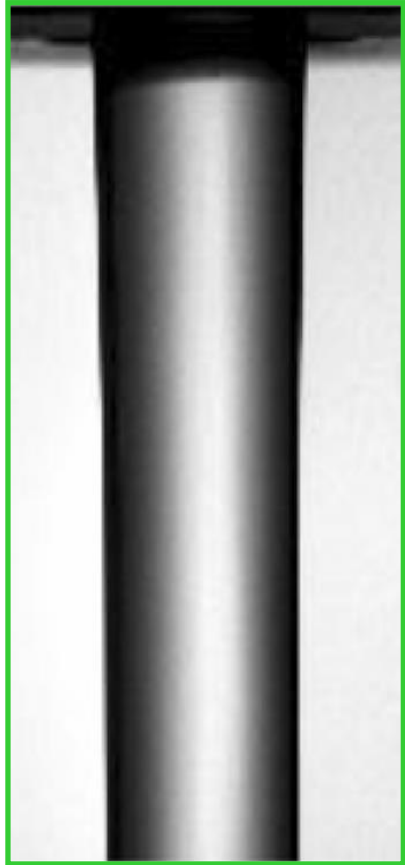
⇒

full spectrum of lengscales

Laminar

⇒ Instability ⇒ Pattern ⇒

Turbulence



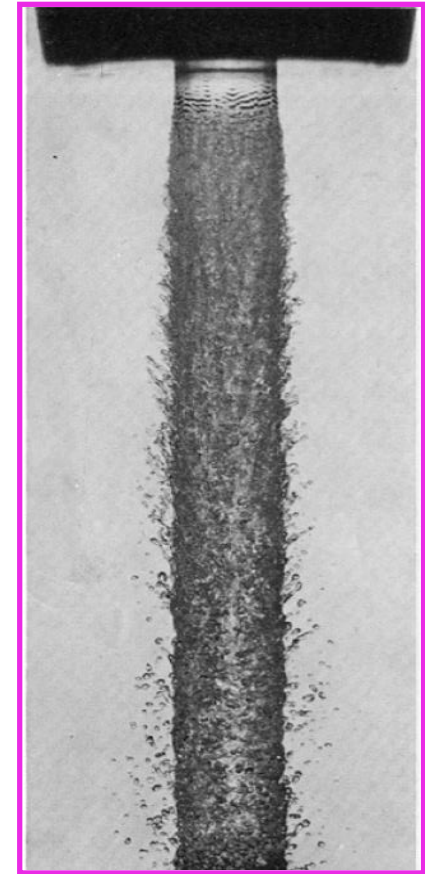
Marmottant and
Villermaux (2004)



Rayleigh (1891)



Marmottant and
Villermaux (2004)



Hoyt and Taylor (1977)

steady

⇒

ω

⇒

ω_1, ω_2

⇒

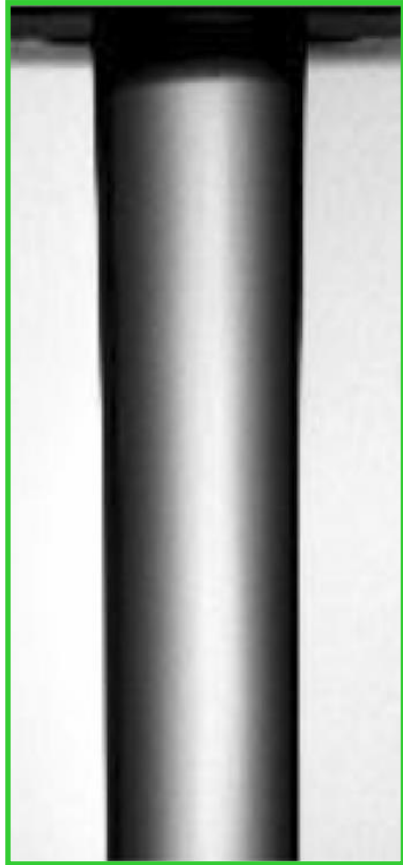
full frequency spectrum

Laminar

⇒ Instability ⇒

Pattern ⇒

Turbulence



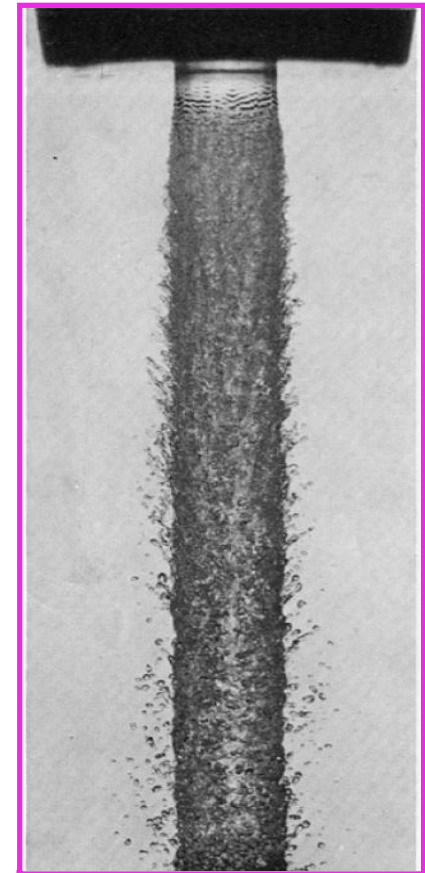
Marmottant and Villiermaux (2004)



Rayleigh (1891)



Marmottant and Villiermaux (2004)



Hoyt and Taylor (1977)

steady

⇒

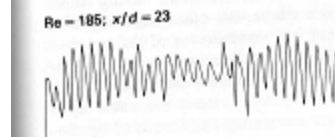
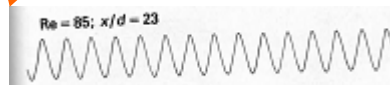
ω

⇒

ω_1, ω_2

⇒

full frequency spectrum



Laminar

⇒ Instability ⇒ Pattern ⇒

Turbulence



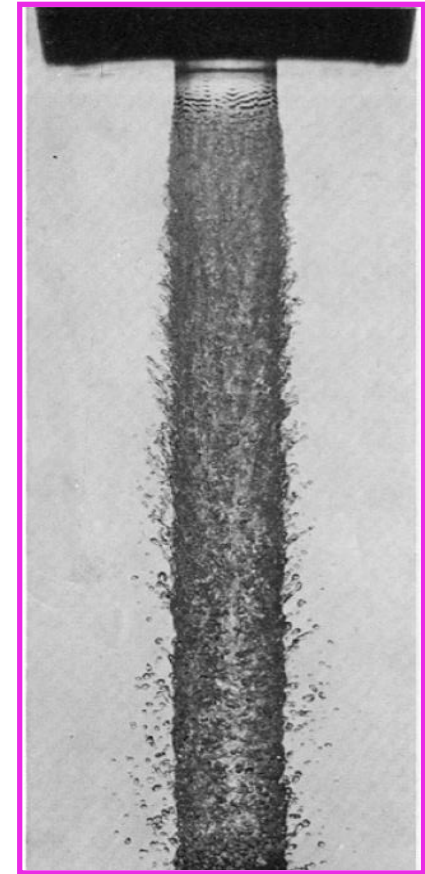
Marmottant and
Villermaux (2004)



Rayleigh (1891)



Marmottant and
Villermaux (2004)



Hoyt and Taylor (1977)

no fluctuation	⇒	ε	⇒	1	⇒	fluctuation > average
R	⇒	R, λ	⇒	R, λ_1, λ_2	⇒	full spectrum of lengscales
steady	⇒	ω	⇒	ω_1, ω_2	⇒	full frequency spectrum ²⁰

Laminar $\Rightarrow$ Instability $\Rightarrow$ Disorder/Pattern/Chaos $\Rightarrow$ Turbulence



Transition

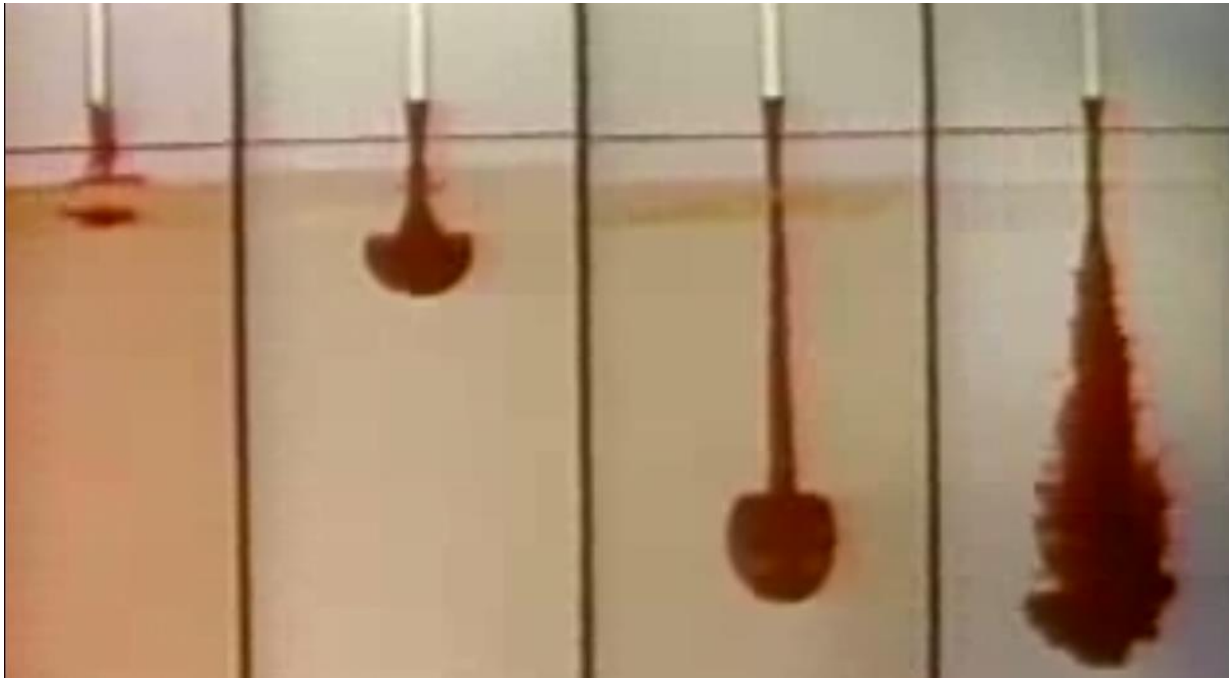
- ∇ fluctuations amplitude
- ∇ lengthscales
- ∇ frequency spectrum

Loss of predictability

Sensitivity to initial condition (butterfly effect)

Laminar $\Rightarrow$ Instability $\Rightarrow$ Disorder/Pattern/Chaos $\Rightarrow$ Turbulence

Transition



Beware: instability without turbulence!

Laminar $\Rightarrow$ Instability $\Rightarrow$ Disorder/Pattern/Chaos $\Rightarrow$ Turbulence



Transition

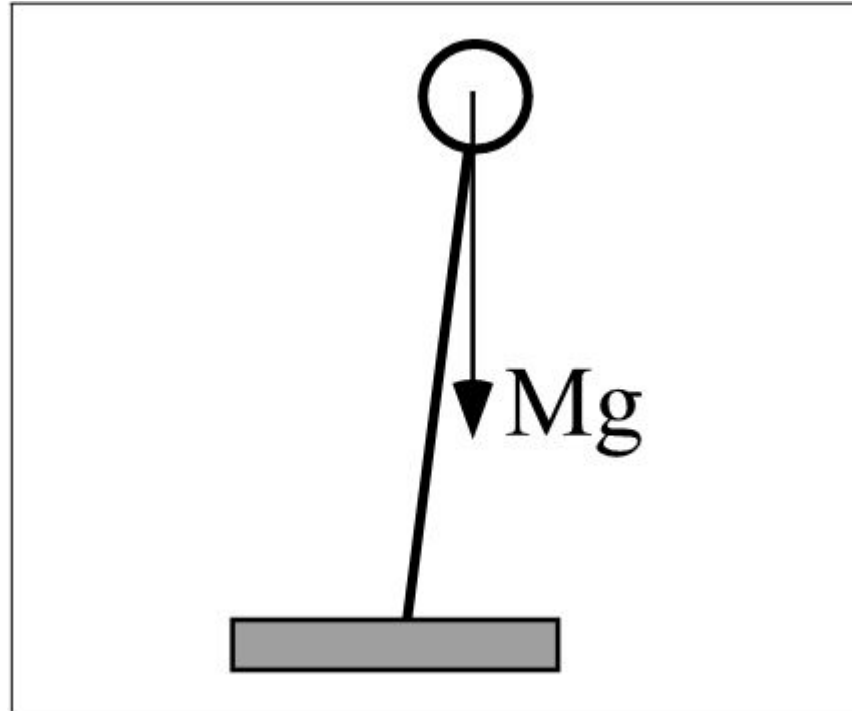
Beware: turbulence without linear instability!

Instabilities

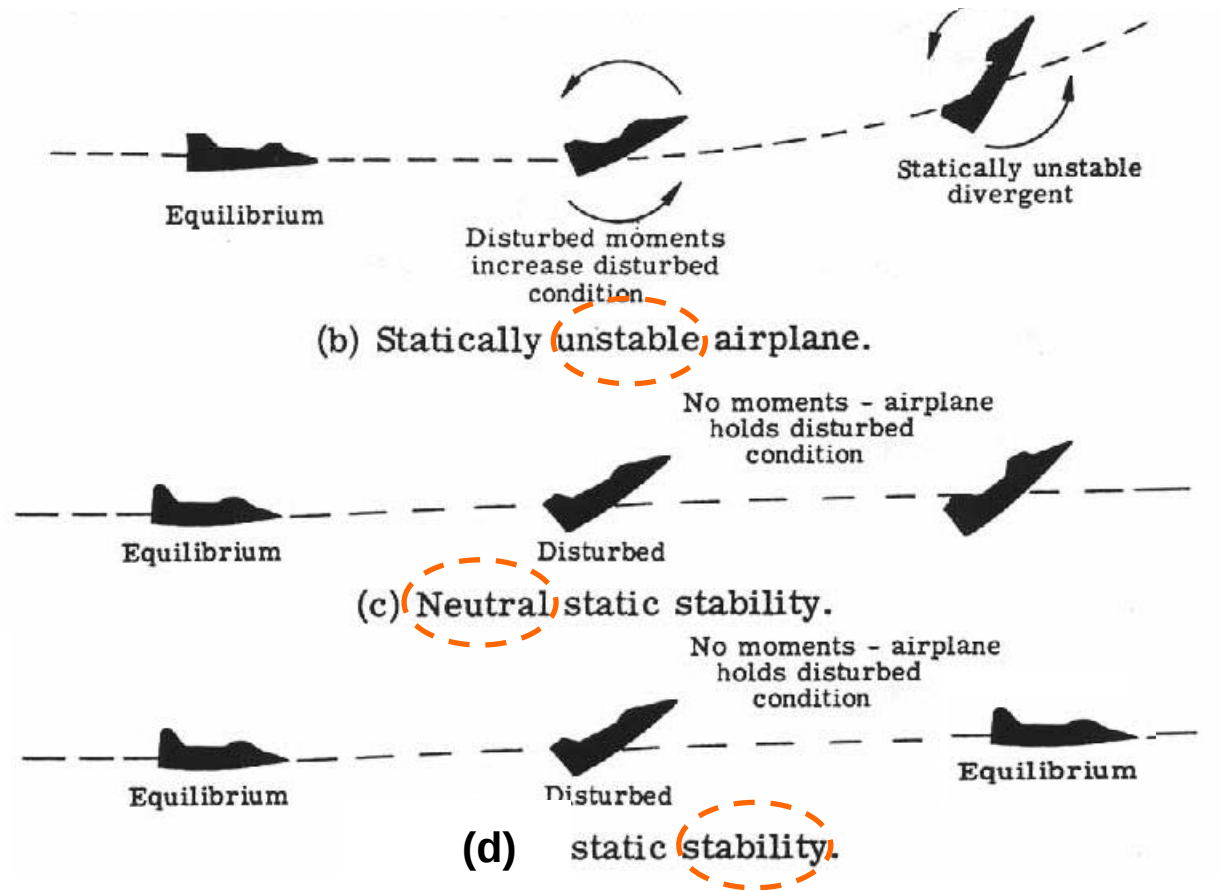
Today:

- 1. Dynamical system instability (an example)**
- 2. Definitions**

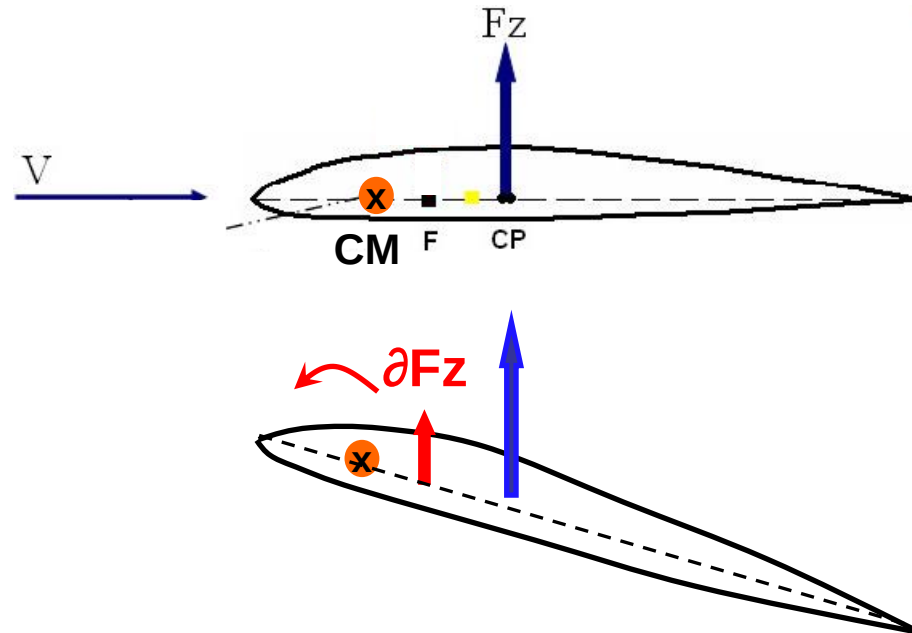
A dynamical system approach



Longitudinal static stability of an airplane

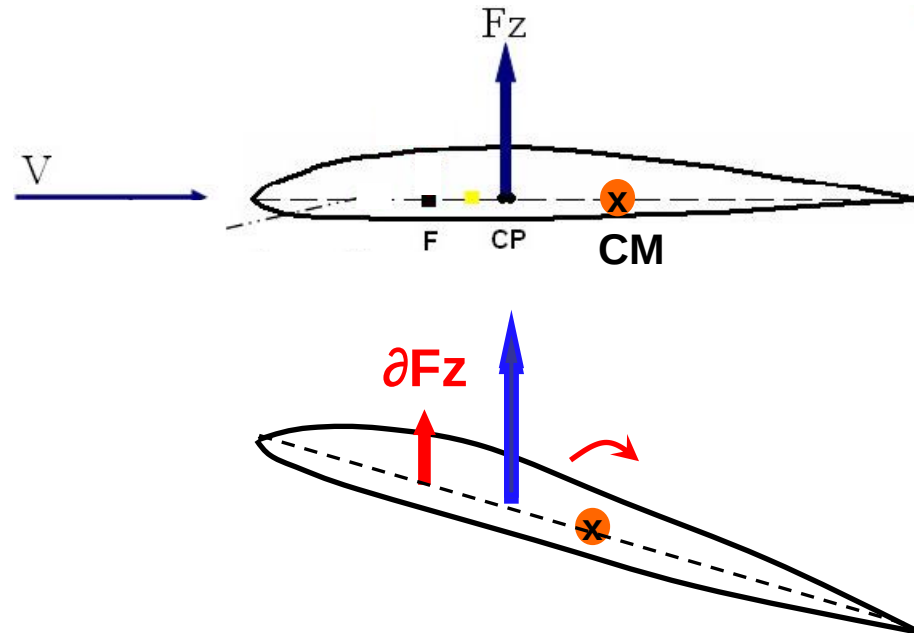


A stable wing



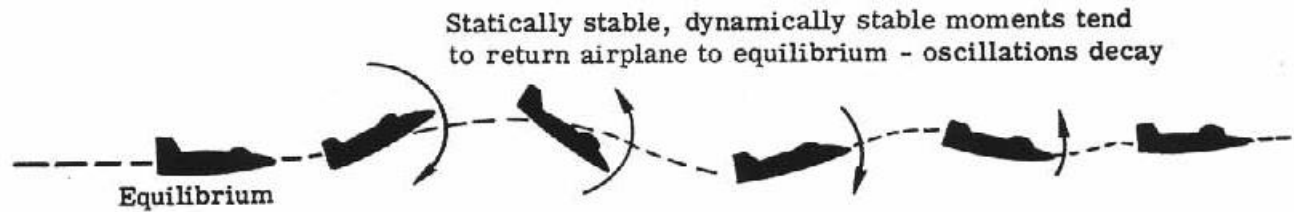
F: focal point
CM: center of mass
CP: center of lift

Unstable wing

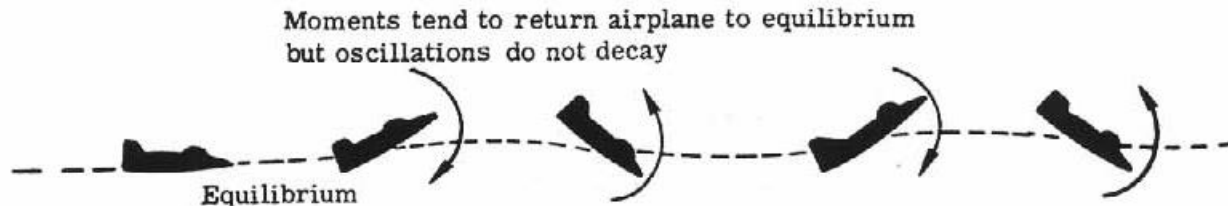


F: focal point
CM: center of mass
CP: center of lift

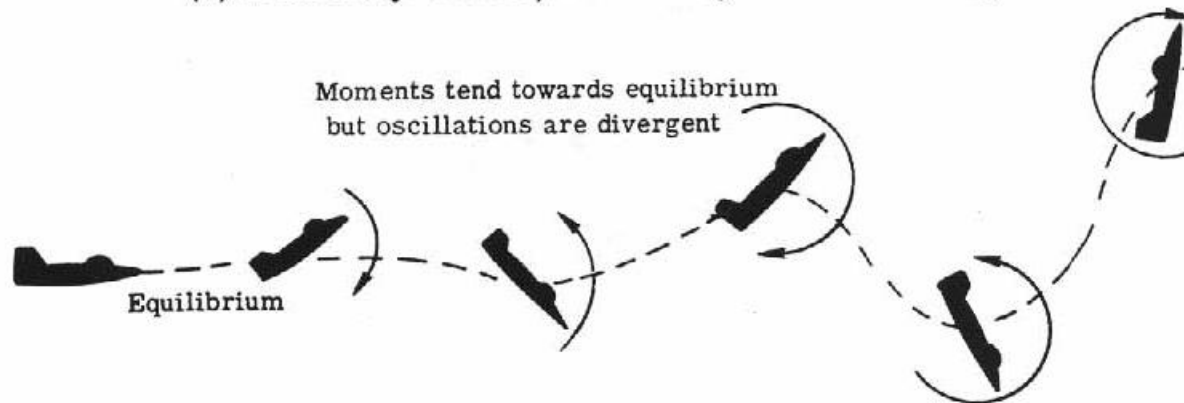
Dynamical longitudinal stability of an airplane



(a) Statically and dynamically stable.



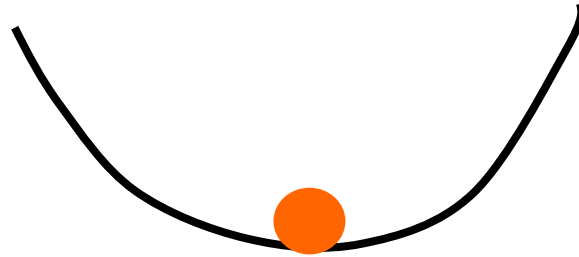
(b) Statically stable; neutral dynamic stability.



(c) Statically stable; dynamically unstable.

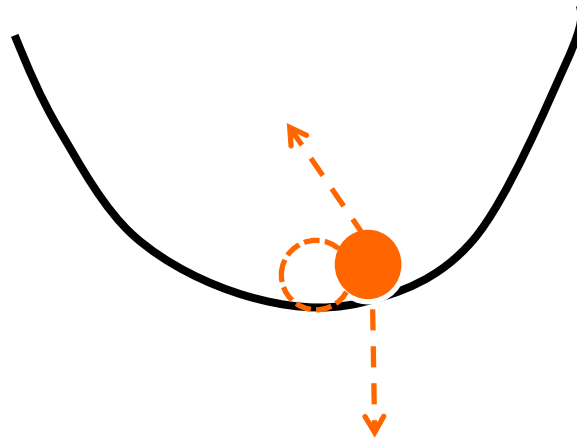
Write equations for temporal evolution:
Dynamical system approach

A dynamical system approach



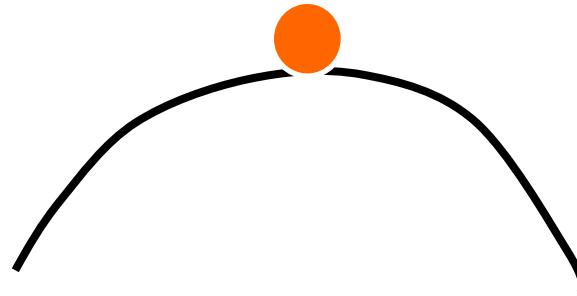
?

A dynamical system approach



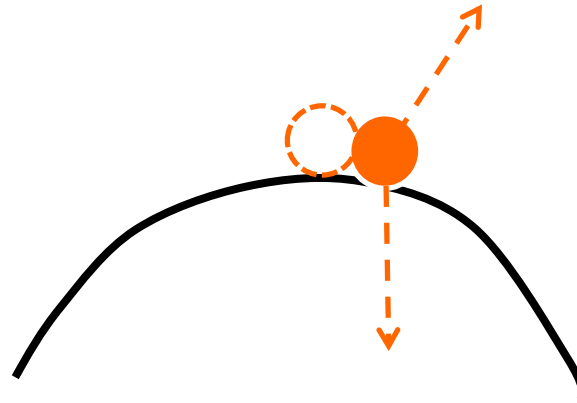
Stable

A dynamical system approach



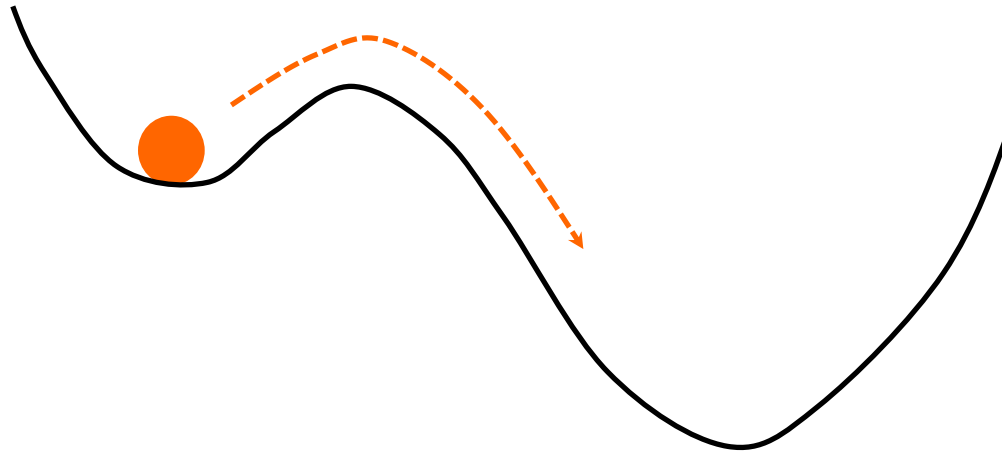
?

A dynamical system approach



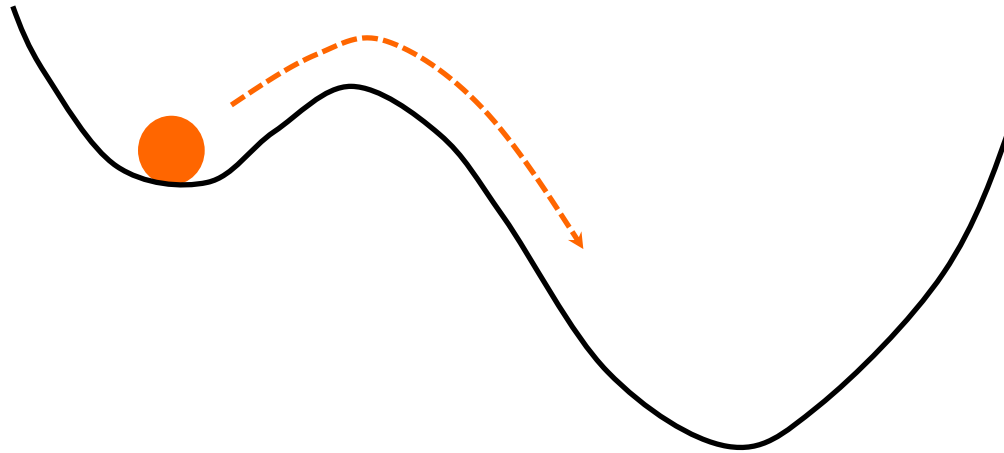
Unstable

A dynamical system approach



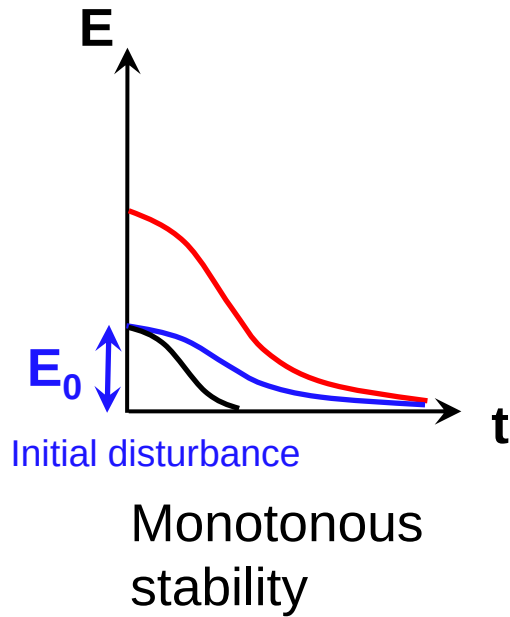
Local /Global energy minimum
Linearly stable but nonlinearly unstable
Concept of basin of attraction

A dynamical system approach



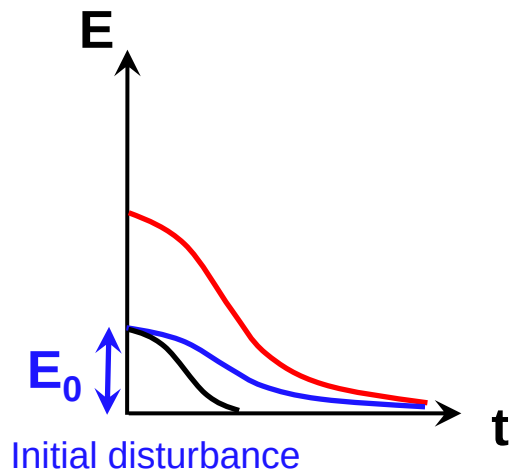
Beware: fluid dynamics is not a conservative system!

Evolution of disturbance energy

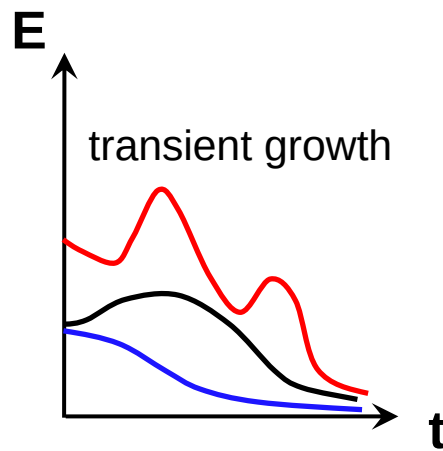


E is an integrated quantity:
different disturbance flows can share the same E !

Evolution of disturbance energy

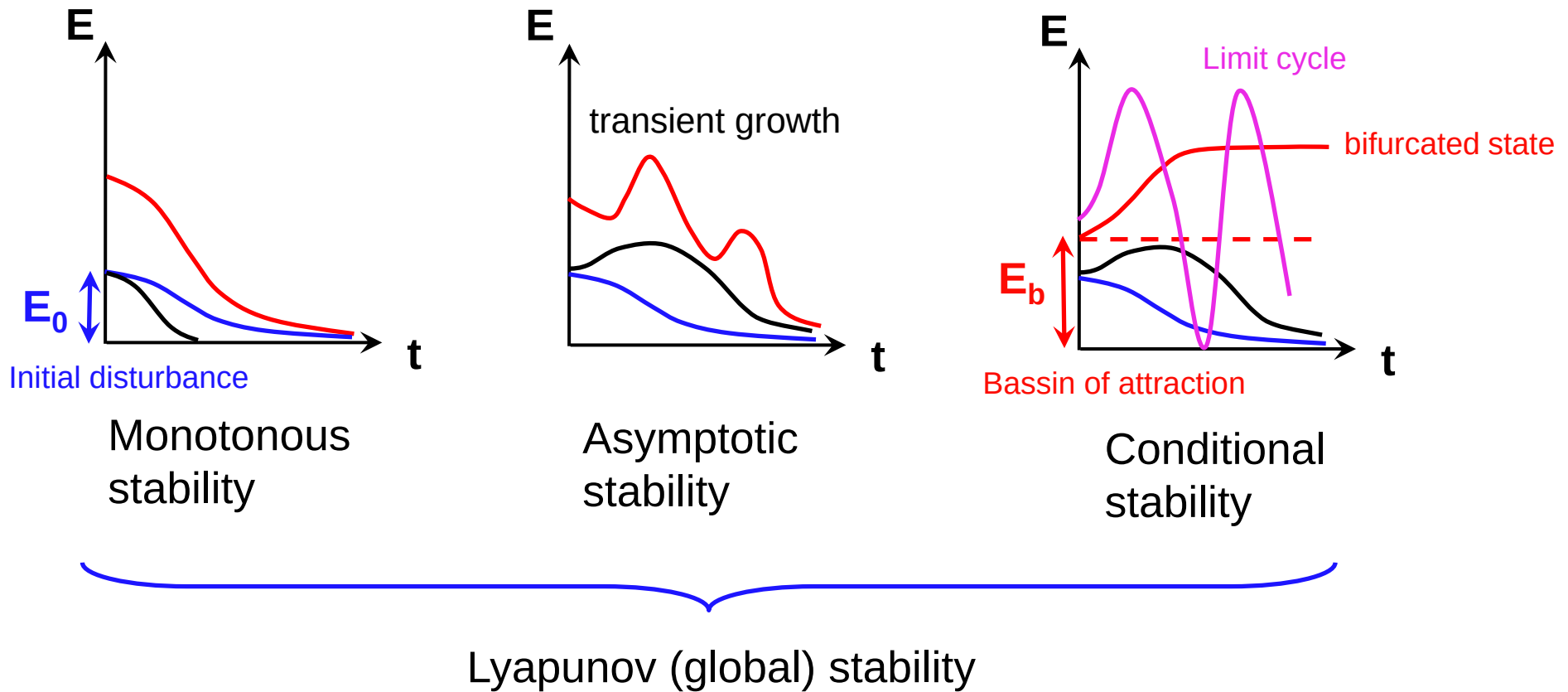


Monotonous
stability

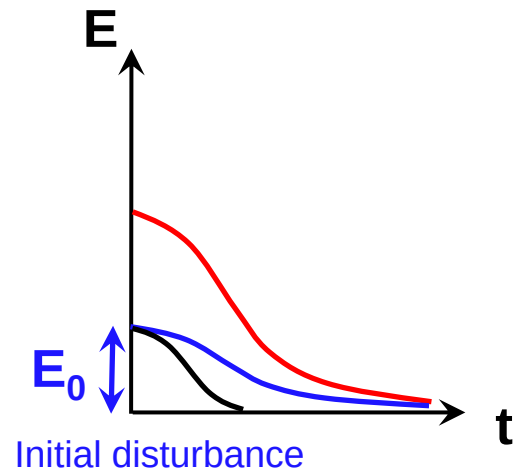


Asymptotic
stability

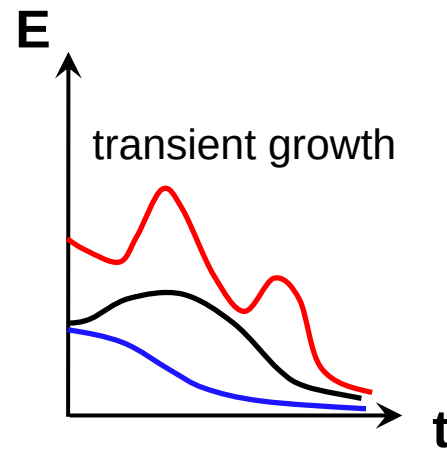
Evolution of disturbance energy



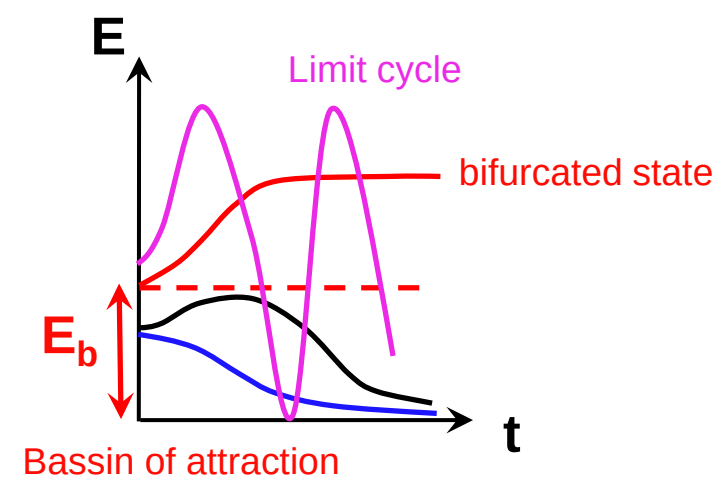
Evolution of disturbance energy



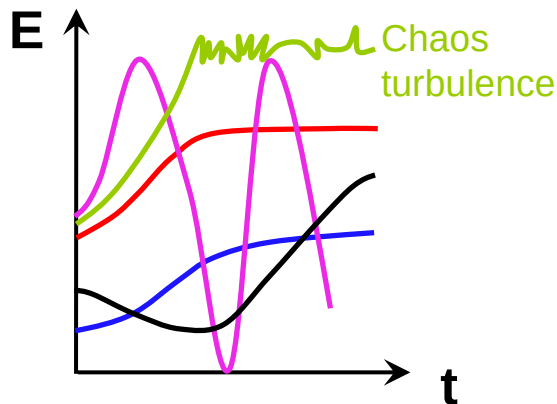
Monotonous stability



Asymptotic stability

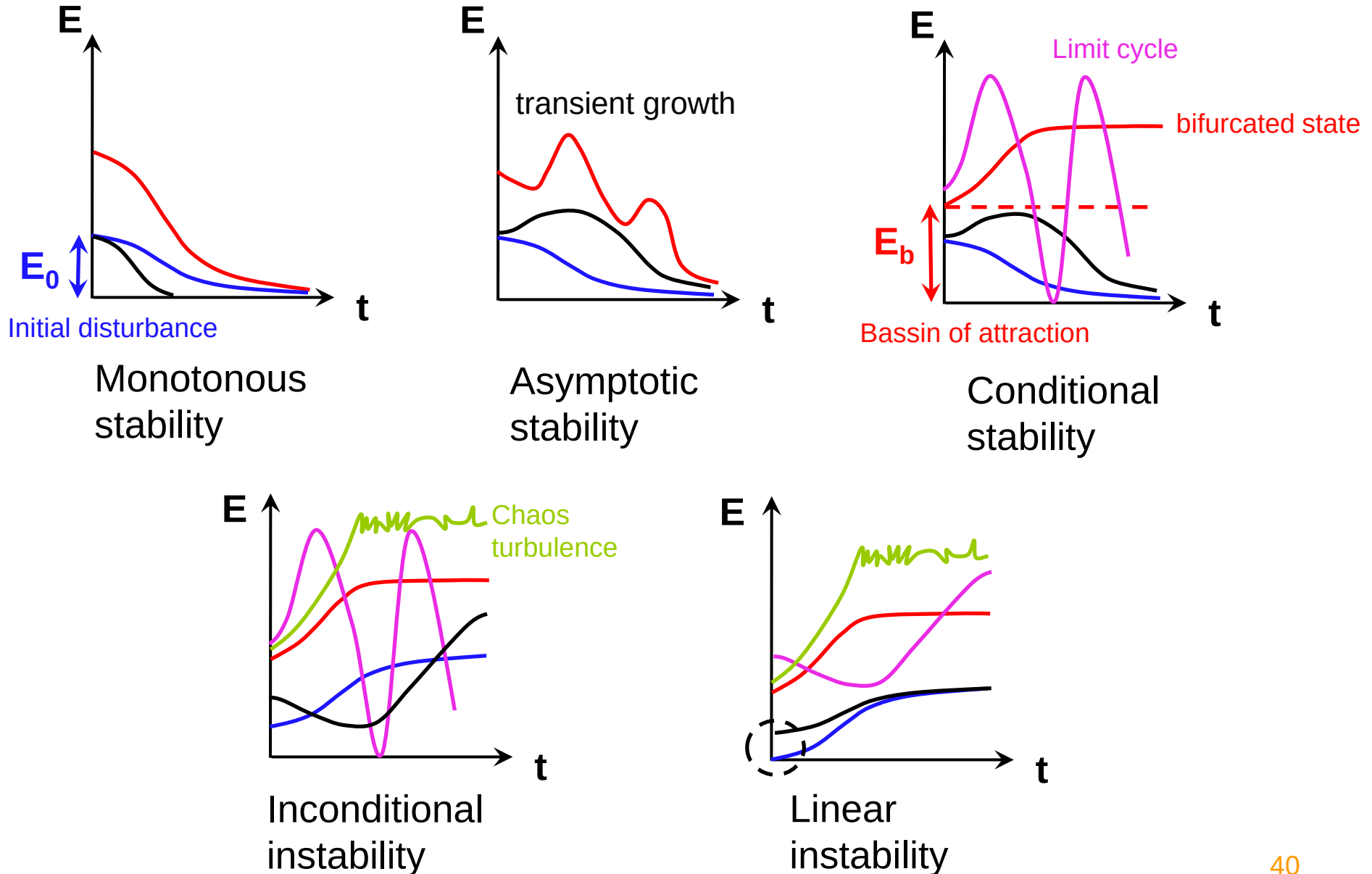


Conditional stability



Inconditional instability

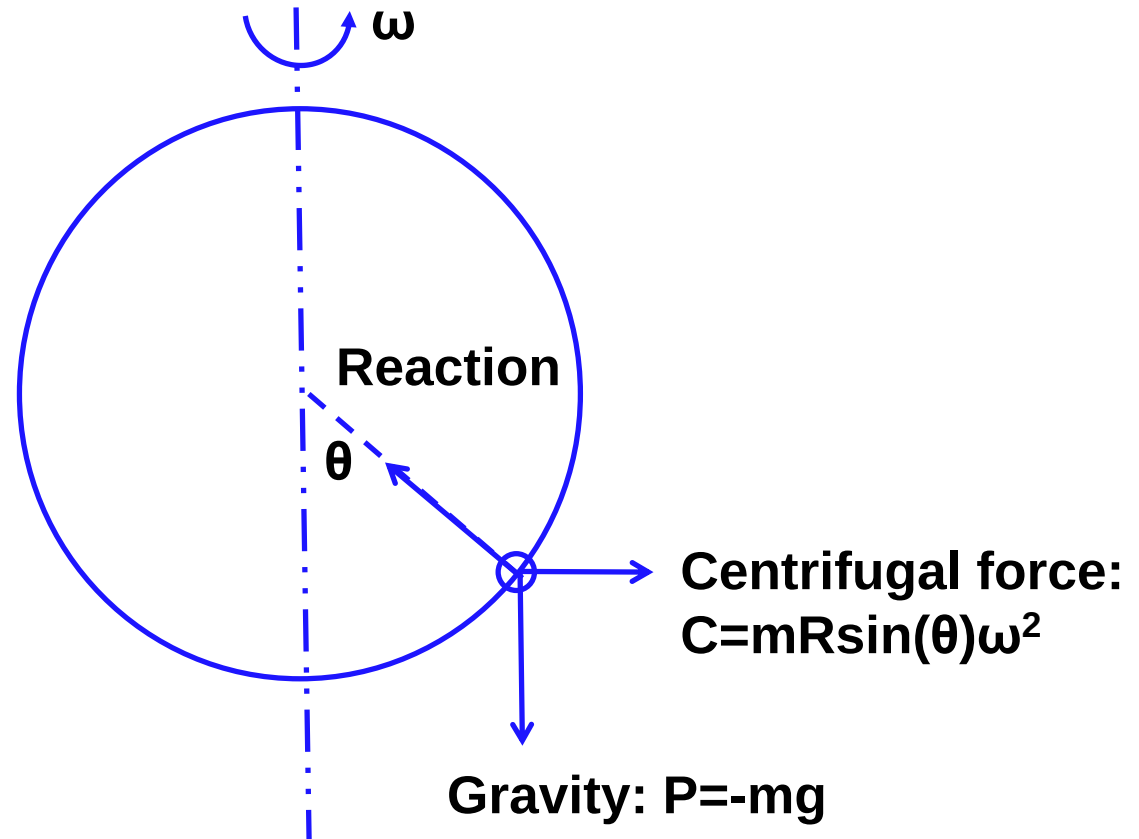
Evolution of disturbance energy



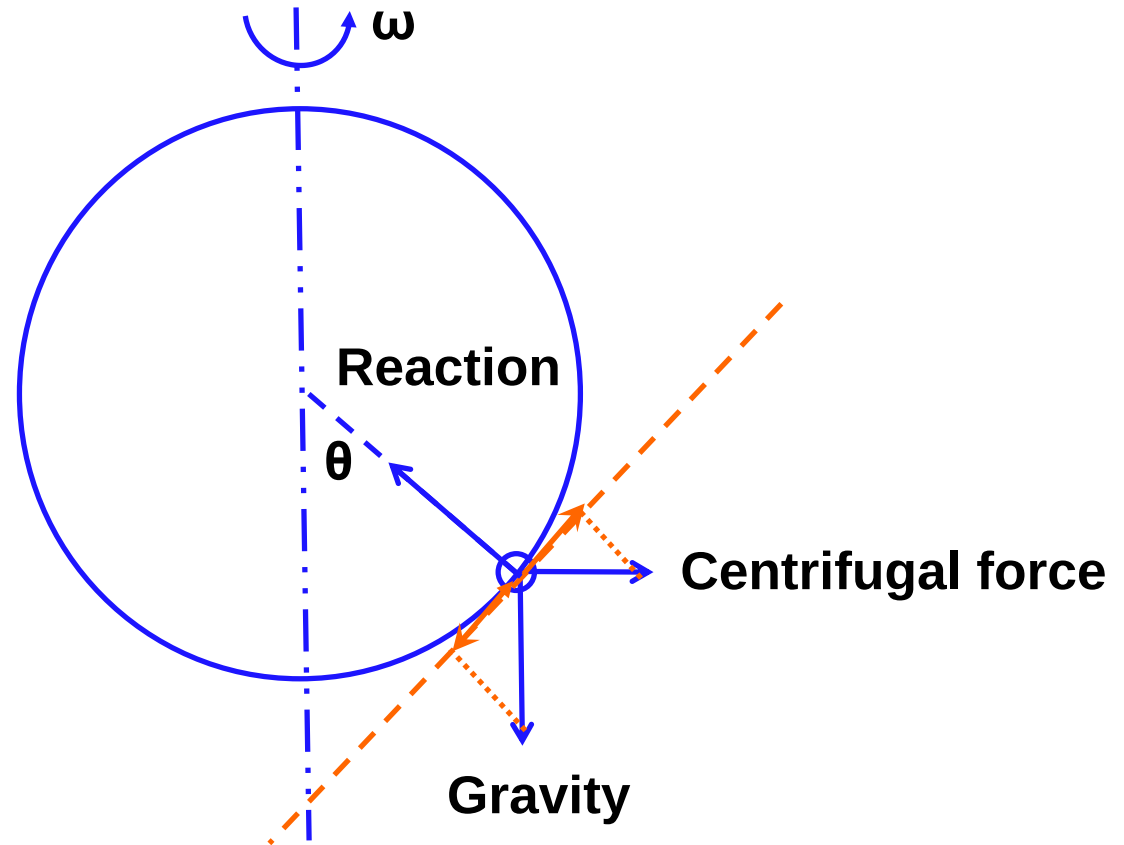
Important concepts

- **Bassin of attraction**
- **Bifurcation**
- **Subcritical/Supercritical Bifurcation**

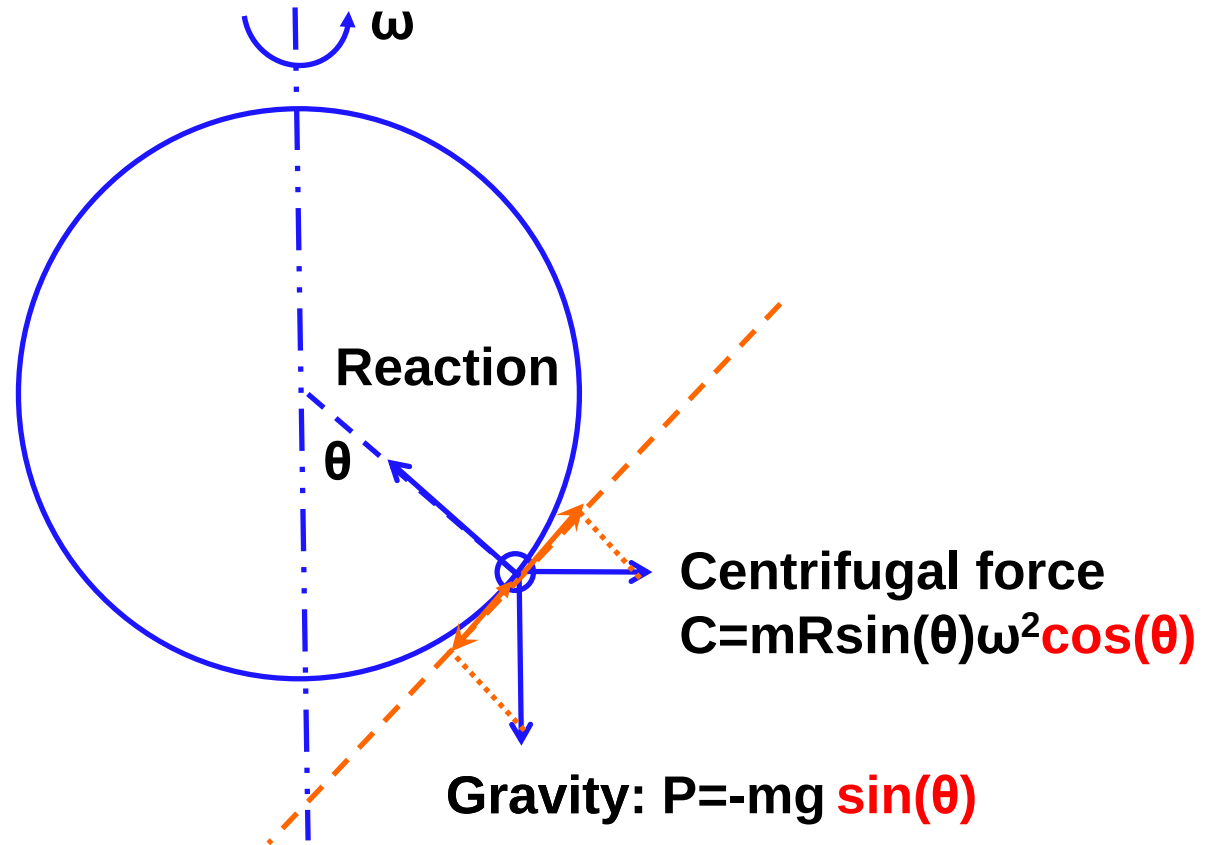
Bifurcations and amplitude equations



Bifurcations and amplitude equations



Bifurcations and amplitude equations



$$mR\ddot{\theta} = -mg \sin(\theta) + mR \sin(\theta) \omega^2 \cos(\theta)$$

Governing equations

$$\ddot{\theta} = -\omega_0^2 \sin(\theta) + \omega^2 \sin(\theta) \cos(\theta)$$

$\omega_0^2 = g/R$
Pendulum frequency

Governing equations

$$\ddot{\theta} = -\omega_0^2 \sin(\theta) + \omega^2 \sin(\theta) \cos(\theta)$$

$\omega_0^2 = g/R$
Pendulum frequency

Base 'flow'

$$\theta = 0$$

Governing equations

$$\ddot{\theta} = -\omega_0^2 \sin(\theta) + \omega^2 \sin(\theta) \cos(\theta)$$

$\omega_0^2 = g/R$
Pendulum frequency

Base 'flow'

$$\theta = 0$$

Small perturbations

$$\theta = 0 + \varepsilon \theta'$$

Governing equations

$$\ddot{\theta} = -\omega_0^2 \sin(\theta) + \omega^2 \sin(\theta) \cos(\theta)$$

$\omega_0^2 = g/R$
Pendulum frequency

Base 'flow'

$$\theta = 0$$

Small perturbations

$$\theta = 0 + \varepsilon \theta'$$

Linearized equations

$$\ddot{\theta}' = -\omega_0^2 \theta' + \omega^2 \theta'$$

Linearized equations

$$\ddot{\theta}' = -\omega_0^2 \theta' + \omega^2 \theta'$$

Normal mode

$$\theta' = A \exp(st)$$

Dispersion relation

$$s^2 = \omega^2 - \omega_0^2$$

$$\omega^2 < \omega_0^2$$

$$\omega^2 > \omega_0^2$$

$$s = \pm i(\omega_0^2 - \omega^2)^{1/2}$$

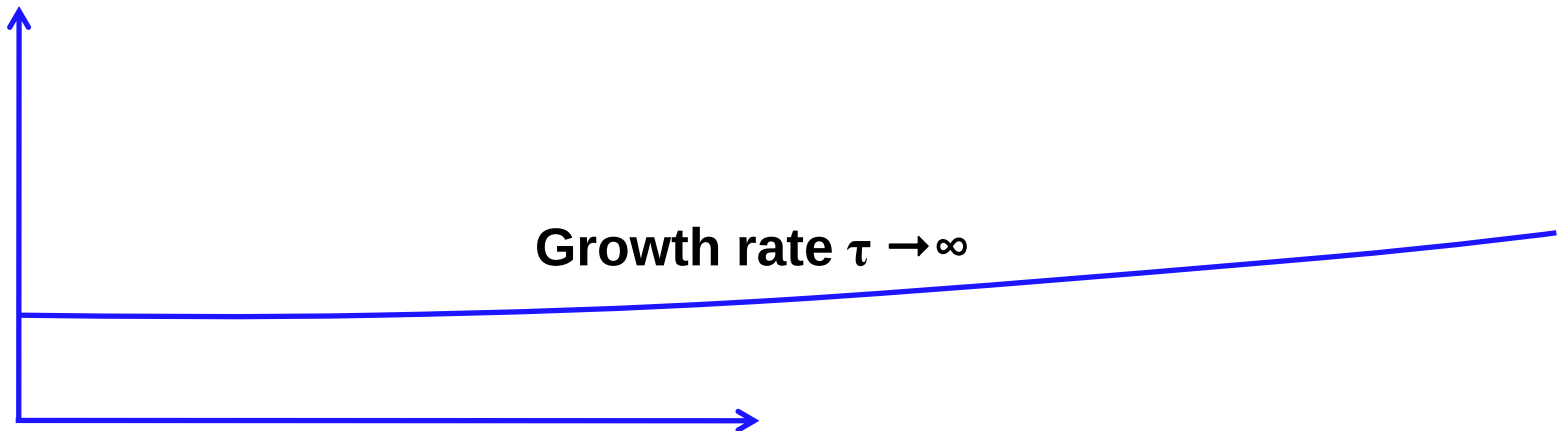
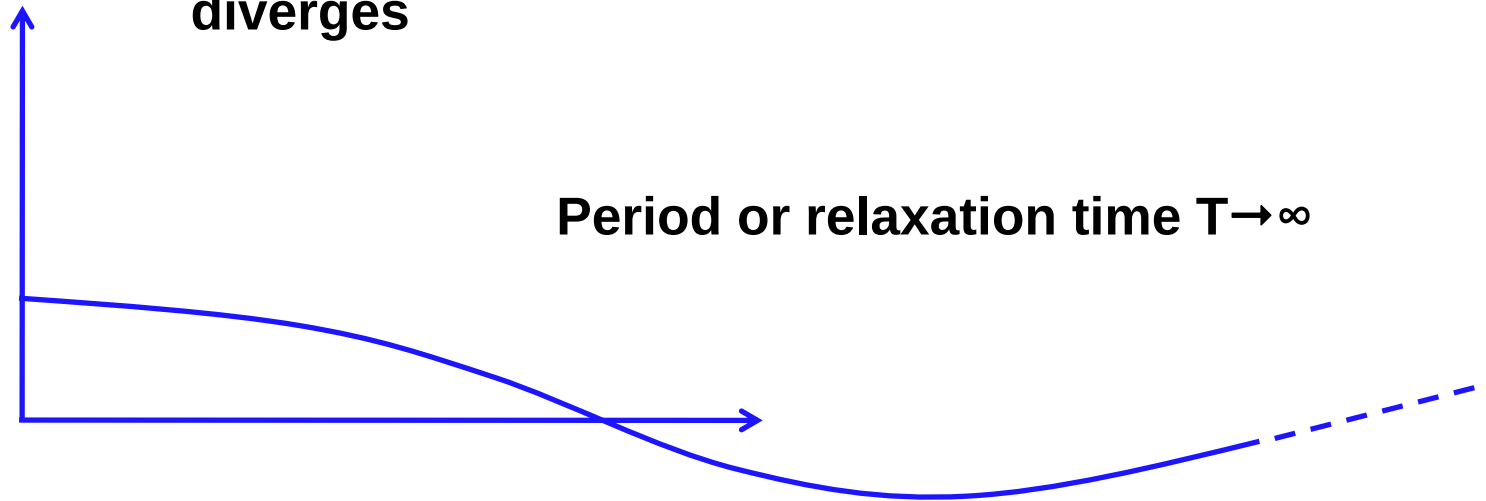
STABLE

$$s = \pm (\omega^2 - \omega_0^2)^{1/2}$$

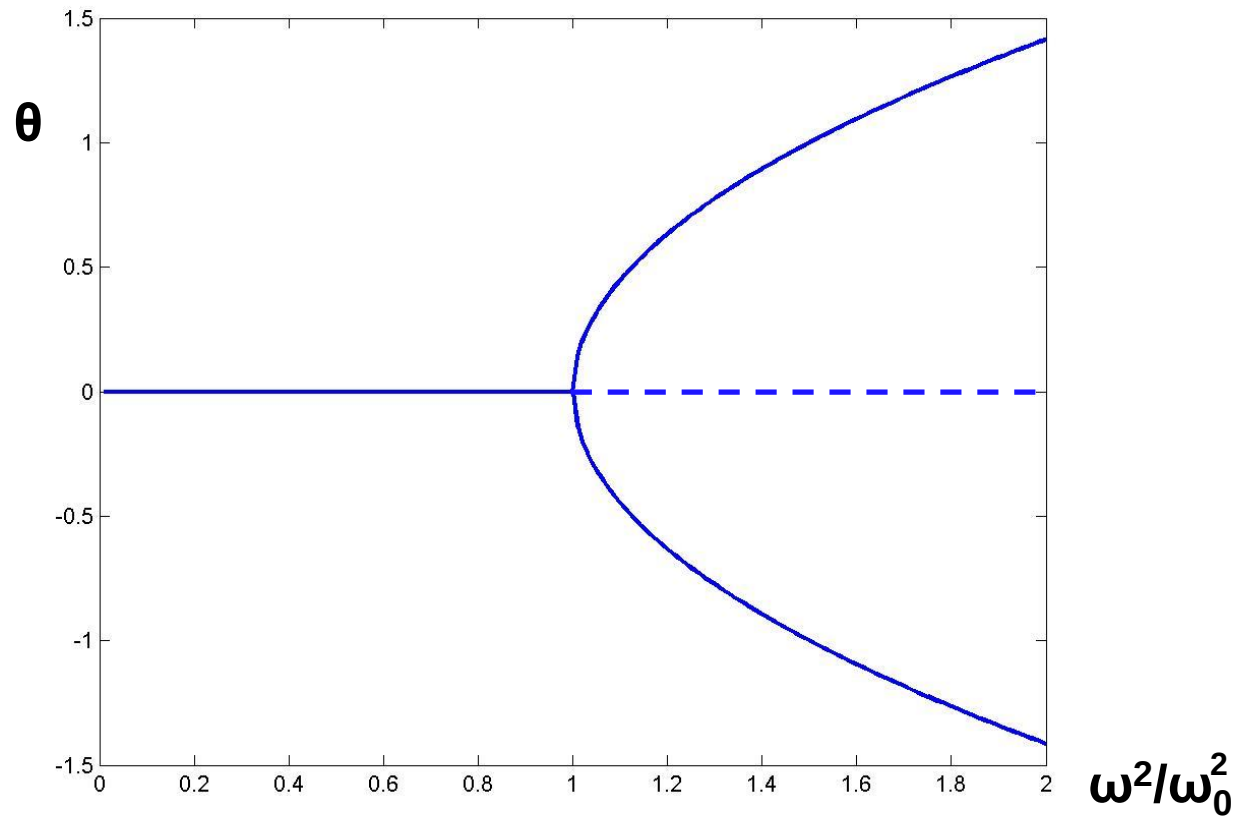
UNSTABLE

Important concept: critical slowing

When $\omega^2 \sim \omega_0^2$, the characteristic time $\tau = 1/|s|$ diverges



What about nonlinearities?



What about nonlinearities?

But recall the full nonlinear equation

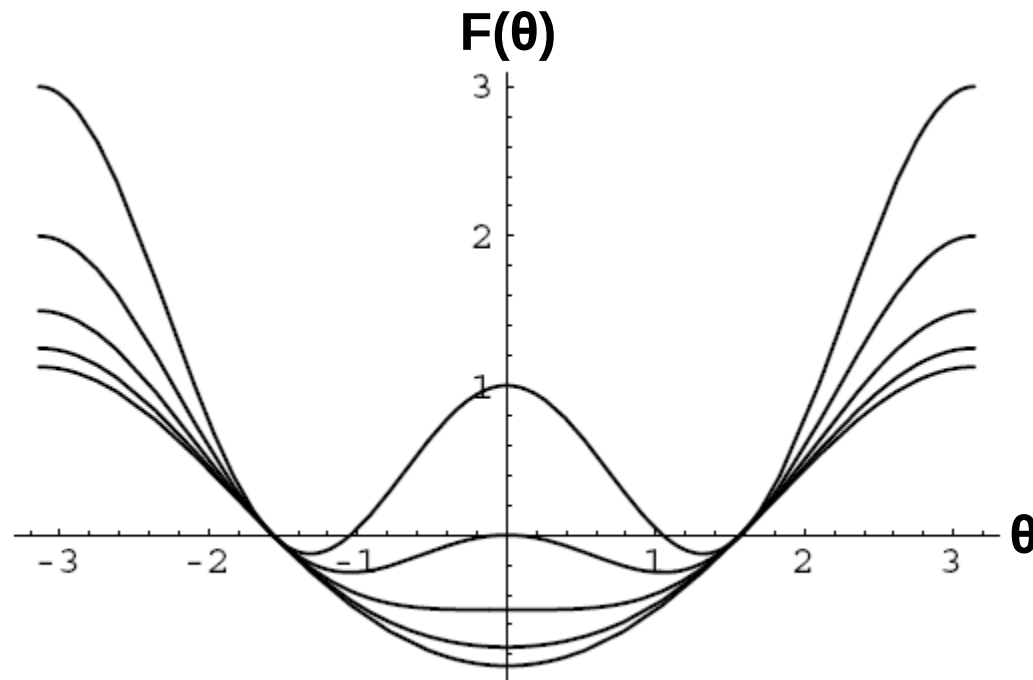
$$\ddot{\theta} = -\omega_0^2 \sin(\theta) + \omega^2 \sin(\theta) \cos(\theta)$$

It has another 2 steady solutions

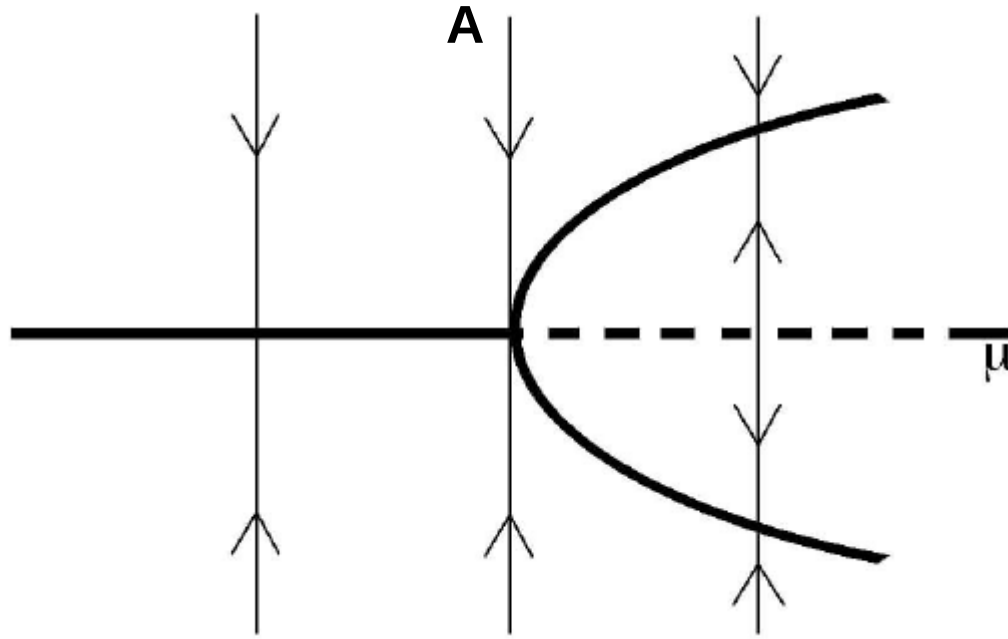
$$\theta_s = \arccos((\omega_0/\omega)^2)$$

Physical interpretation (Potential)

$$m\ddot{\theta} = F'(\theta)$$



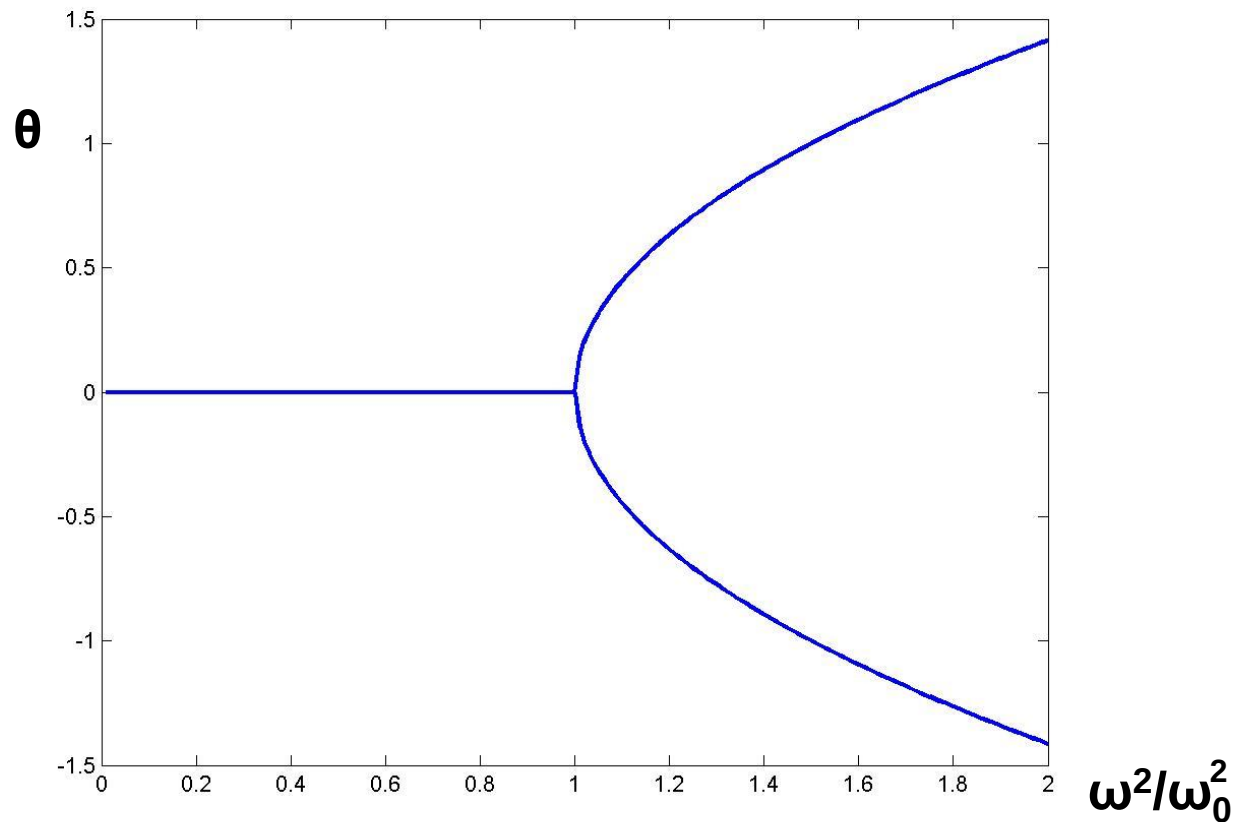
What about nonlinearities?



Supercritical fork bifurcation

- stable
- unstable

Simple exercise for next week



Demonstrate the dynamic stability of the bifurcated states