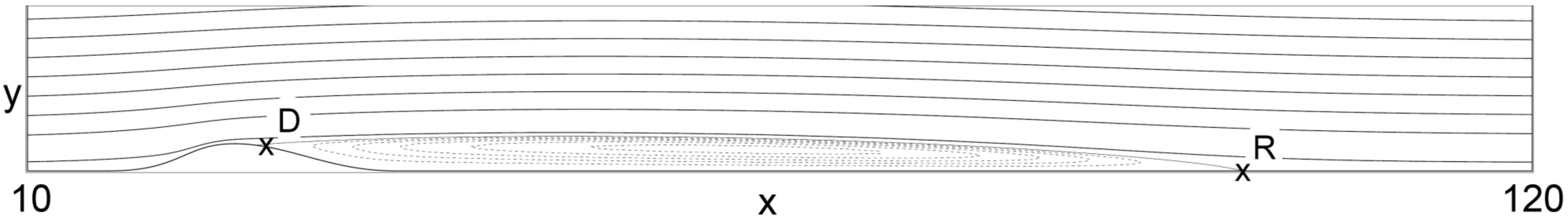
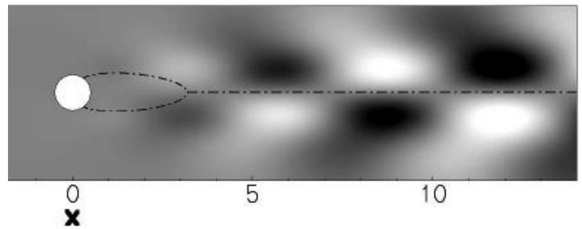
Global linear instability of flows



Global linear instability of flows



Stability of viscous flow past a circular cylinder
A. Zebib

1D — **1971**

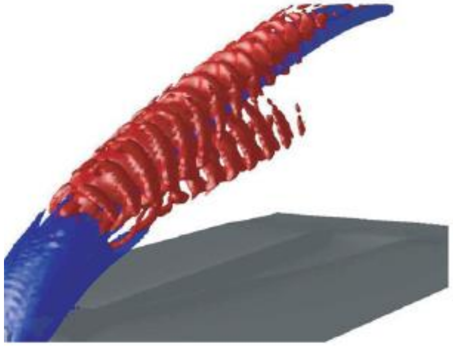
Accurate solution of the Orr–Sommerfeld stability equation
S. A. Orszag

1987….. — **2 D**

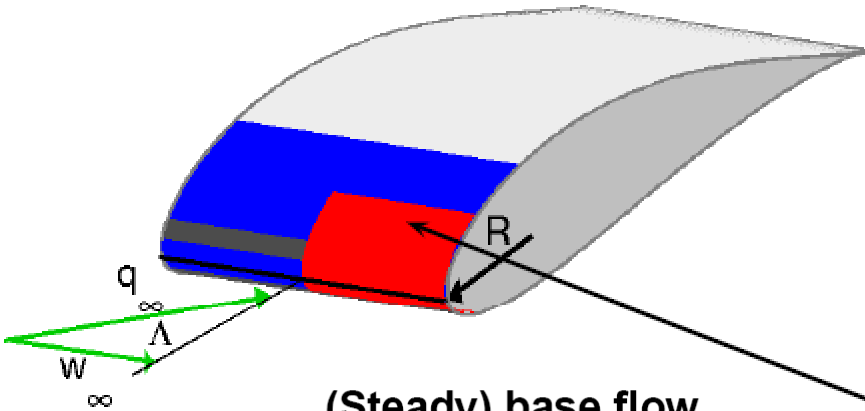
2005

3 D — **2009**

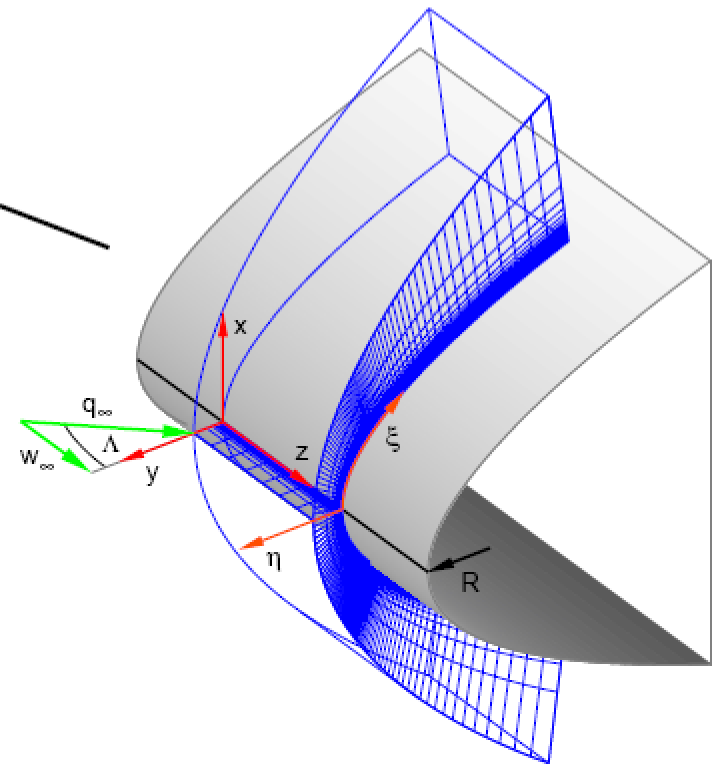
Global stability of a jet in crossflow
S. Bagheri



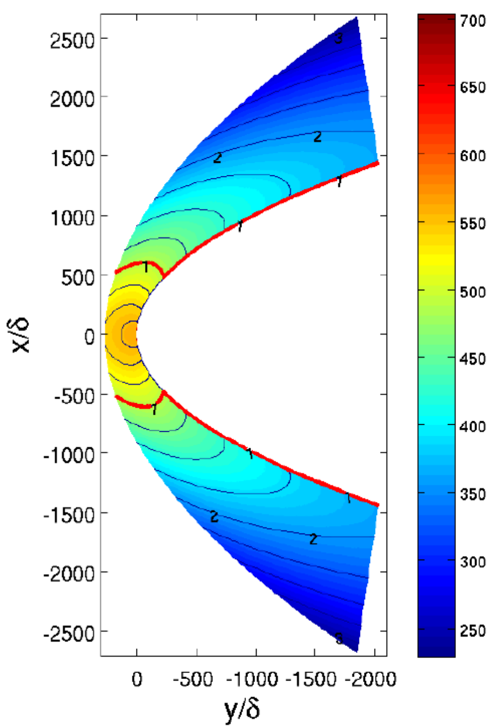
Global linear instability of flows in complex geometry



Leading-edge region
(Parabolic body of infinite span)

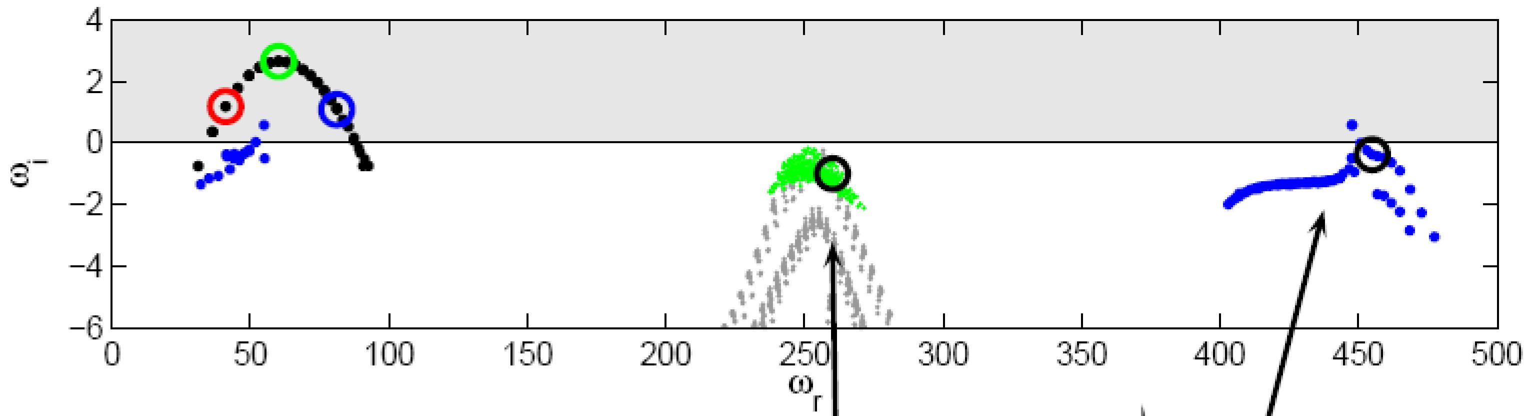


(Steady) base flow
(Temperature & Mach-lines)

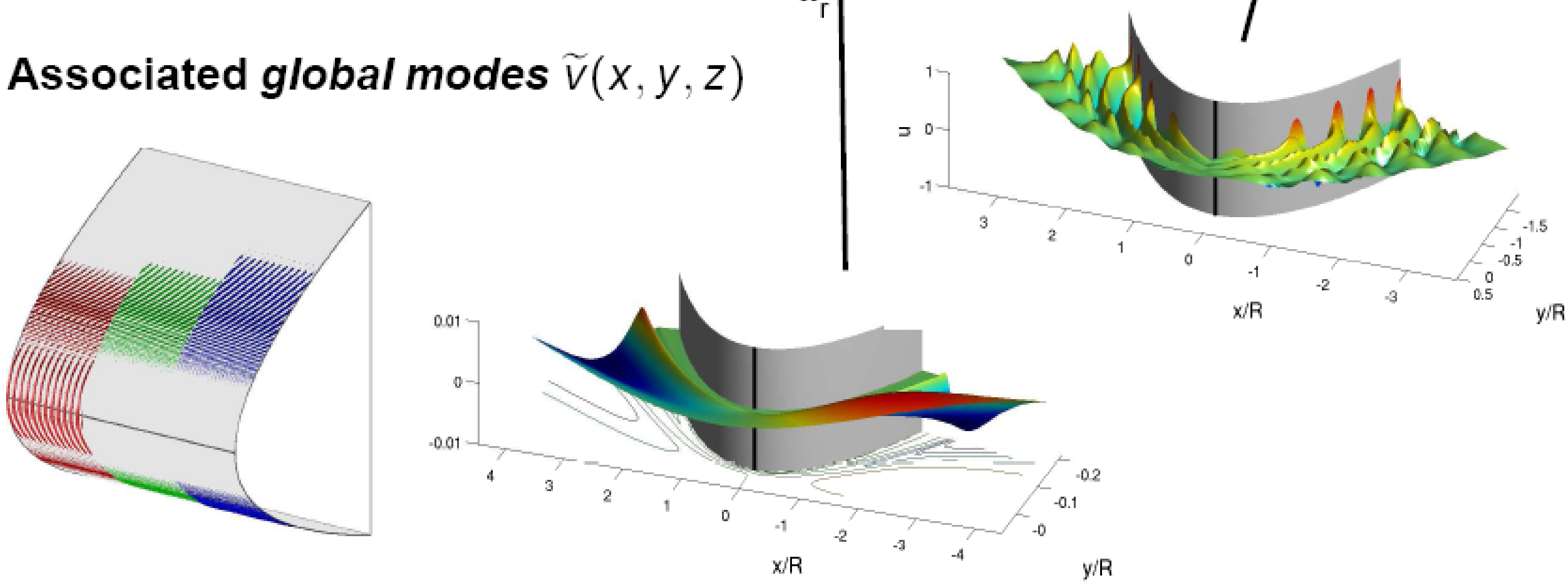


Mack and Schmid 2010

***Global* spectrum:** $\phi'(x,y,z,t) = \tilde{\phi}(x,y)e^{i(\beta z - \omega t)}$ with $\omega = \omega_r + i\omega_i$



Associated *global modes* $\tilde{v}(x,y,z)$



Mack and Schmid 2010

Discretization methods

All standard methods

- Spectral methods (memory saving)
- Finite elements
- Finite volumes
- Finite differences

Sparse and parallelizable

Very few compressible studies

Apparently no use of « non-standard » methods:

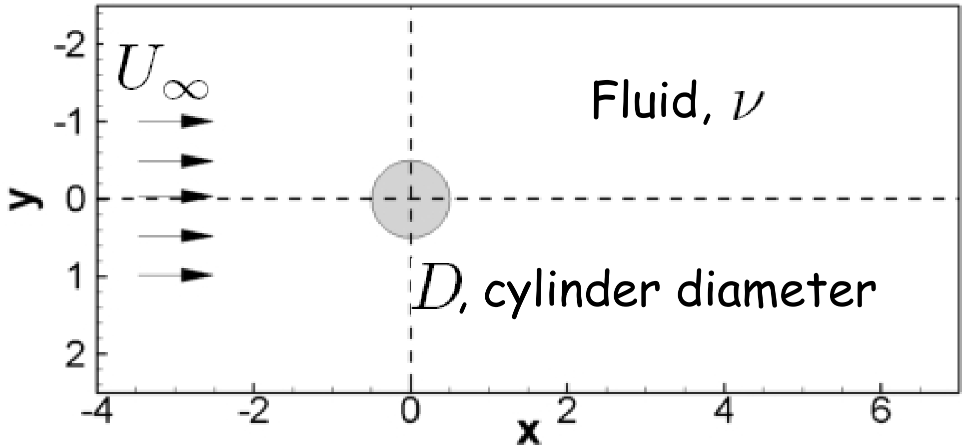
•particle methods, vortex methods,…

•Lattice-Bolzmann

Global linear instability of flows in complex geometry

1. Gobal instabilty of a classical 2D flow
2. Less classical unstable flows
3. Stable flows

Wake behind cylinder flow



$$Re = \frac{U_\infty D}{\nu}$$

Incompressible 2D flow described by

$$(\boldsymbol{u}, p)$$

Velocity   Pressure

Navier-Stokes equations

$$\partial_t \boldsymbol{u} + \boldsymbol{\nabla}\boldsymbol{u} \cdot \boldsymbol{u} = -\boldsymbol{\nabla} p + Re^{-1} \boldsymbol{\nabla}^2 \boldsymbol{u},$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

Stability analysis

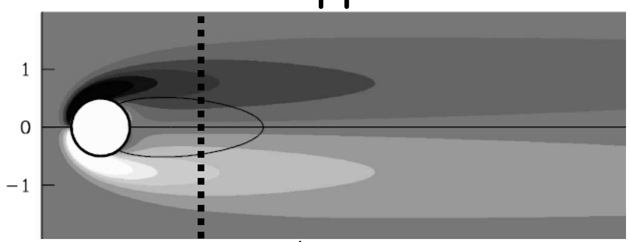
Perturbation expansion

$$(\boldsymbol{u}, p) = (\boldsymbol{U}, P) + (\boldsymbol{u}', p')$$

Perturbations

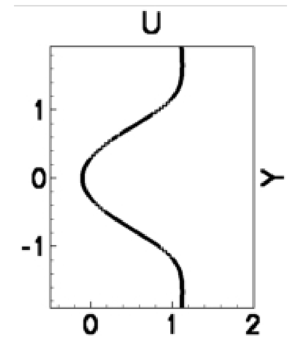
Stationary base flow

Global approach $\longleftarrow$ $\boldsymbol{U}(x, y)$ $\qquad$ $U(y)$ $\longrightarrow$



Local approach

Huerre & Monkewitz (1990)



Zebib, Jackson (1987)

Base flow equations

$$\nabla \boldsymbol{U} \cdot \boldsymbol{U} = -\nabla P + Re^{-1} \nabla^2 \boldsymbol{U},$$
$$\nabla \cdot \boldsymbol{U} = 0$$

Linearized **perturbation equations**

$$\partial_t \boldsymbol{u}' + \nabla \boldsymbol{U} \cdot \boldsymbol{u}' + \nabla \boldsymbol{u}' \cdot \boldsymbol{U} + \cancel{\nabla \boldsymbol{u}' \cdot \boldsymbol{u}'} = -\nabla p' + Re^{-1} \nabla^2 \boldsymbol{u}',$$
$$\nabla \cdot \boldsymbol{u}' = 0$$

Analyse de stabilité globale

$$\boldsymbol{U}(x,y)$$

$$(\boldsymbol{u}', p')(x,y,t) = (\hat{\boldsymbol{u}}, \hat{p})(x,y)\exp[\sigma t]$$

Global mode(complex)

$$\sigma = \lambda + i\omega$$

Growth-rate

frequency

$$St = \frac{\omega}{2\pi}$$

Global stability equations

$$\sigma\hat{\boldsymbol{u}} + \boldsymbol{\nabla}\hat{\boldsymbol{u}}\cdot\boldsymbol{U} + \boldsymbol{\nabla}\boldsymbol{U}\cdot\hat{\boldsymbol{u}} = -\boldsymbol{\nabla}\hat{p} + Re^{-1}\boldsymbol{\nabla}^2\hat{\boldsymbol{u}},$$

$$\boldsymbol{\nabla}\cdot\hat{\boldsymbol{u}} = 0,$$

Global stability analysis solvers

For a given value of Re , numerically solve

- non linear equations,

$$\boldsymbol{\nabla U} \cdot \boldsymbol{U} = -\boldsymbol{\nabla} P + Re^{-1} \boldsymbol{\nabla}^2 \boldsymbol{U},$$

(Newton method)

- Eigenvalue problem

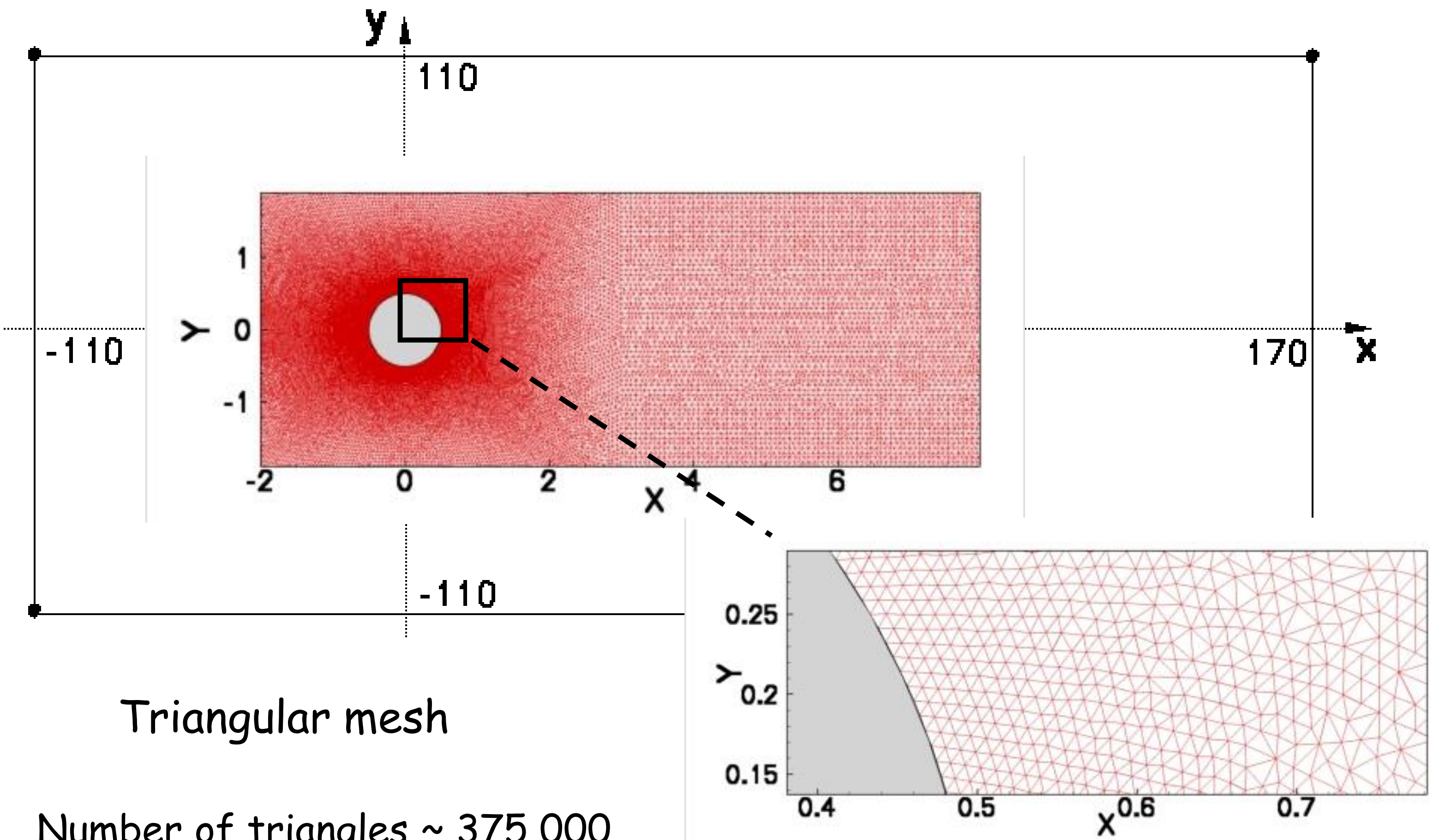
$$\sigma \hat{\boldsymbol{u}} + \boldsymbol{\nabla} \hat{\boldsymbol{u}} \cdot \boldsymbol{U} + \boldsymbol{\nabla U} \cdot \hat{\boldsymbol{u}} = -\boldsymbol{\nabla} \hat{p} + Re^{-1} \boldsymbol{\nabla}^2 \hat{\boldsymbol{u}},$$

(Krylov-Arnoldi method)

Spatial discretization = finite element methods

(FreeFem++ freeware)

Computational model and mesh



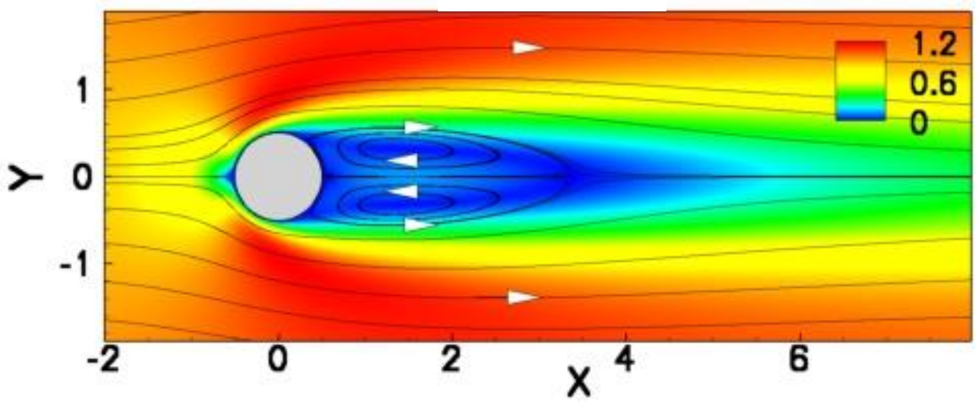
Triangular mesh

Number of triangles ~ 375 000

Number of nodes ~ 190 000

Taylor-Hood finite elements (P2,P2,P1) $\rightarrow$ $\boxed{\text{number of degrees of freedom} \sim 1.6\ 10^6}$

Base flow

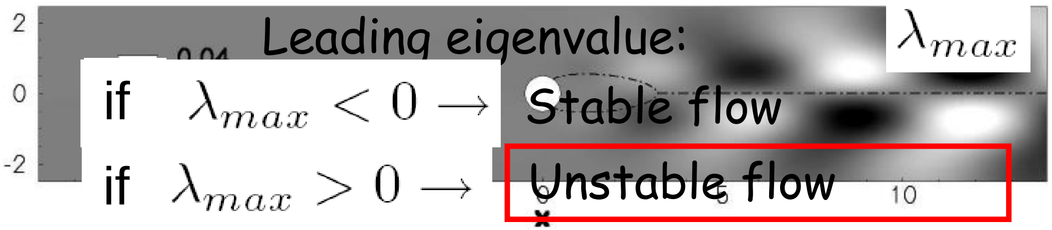


Drag coefficient $$C_D = \frac{F_D}{0.5 D\, U_\infty^2}$$

C_D	$Re = 20$	$Re = 40$
Dennis & Chang (1970)	2.05	1.52
Fornberg (1980)	2.00	1.50
Ye *et al.* (1999)	2.03	1.52
Giannetti & Luchini (2007)	2.05	1.54
Marquet et al. (2008)	2.00	1.50

Valeur propre et mode global dominants

$\Re\{\hat{u}\}$ $\quad (\hat{\boldsymbol{u}}, \hat{p}) \exp[\sigma t] \quad \sigma = \lambda + i\omega$



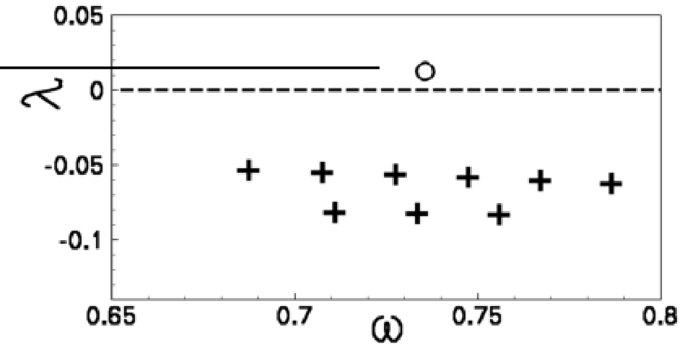
Leading eigenvalue: λ_{max}

if $\lambda_{max} < 0 \rightarrow$ Stable flow

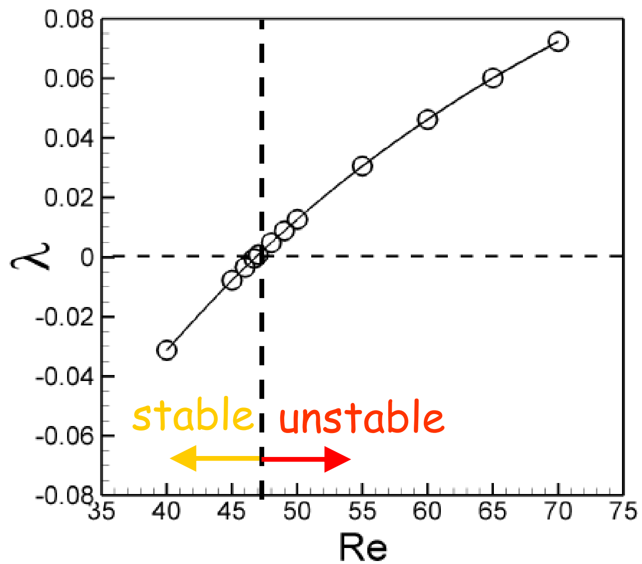
if $\lambda_{max} > 0 \rightarrow$ $\boxed{\text{Unstable flow}}$

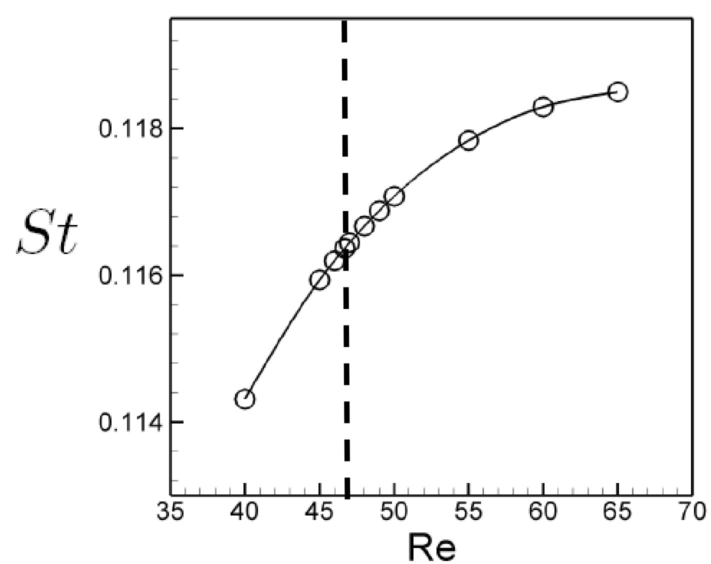
Leading global mode

Spectrum at Re = 50



Evolution as a function of the Reynolds number





$Re_c = 46.7(\pm 0.1)$ $\quad St_c = 0.116$

To keep im mind

Global stability analysis=
Nonlinear problem (base flow) +
Eigenvalue problem

Global linear instability of flows in complex geometry

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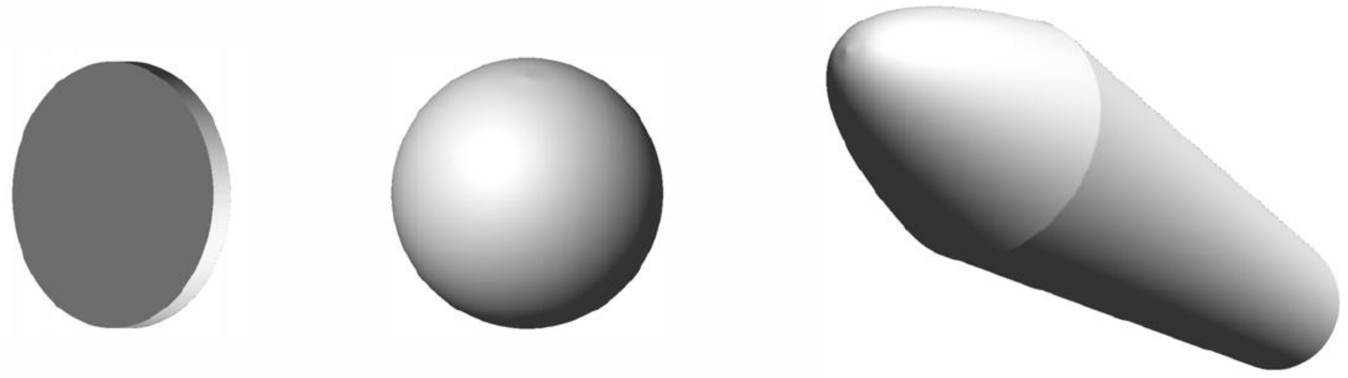
Prototype flows

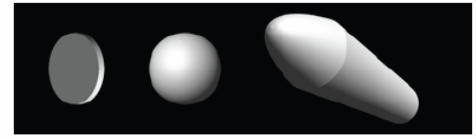
Axisymmetric wakes



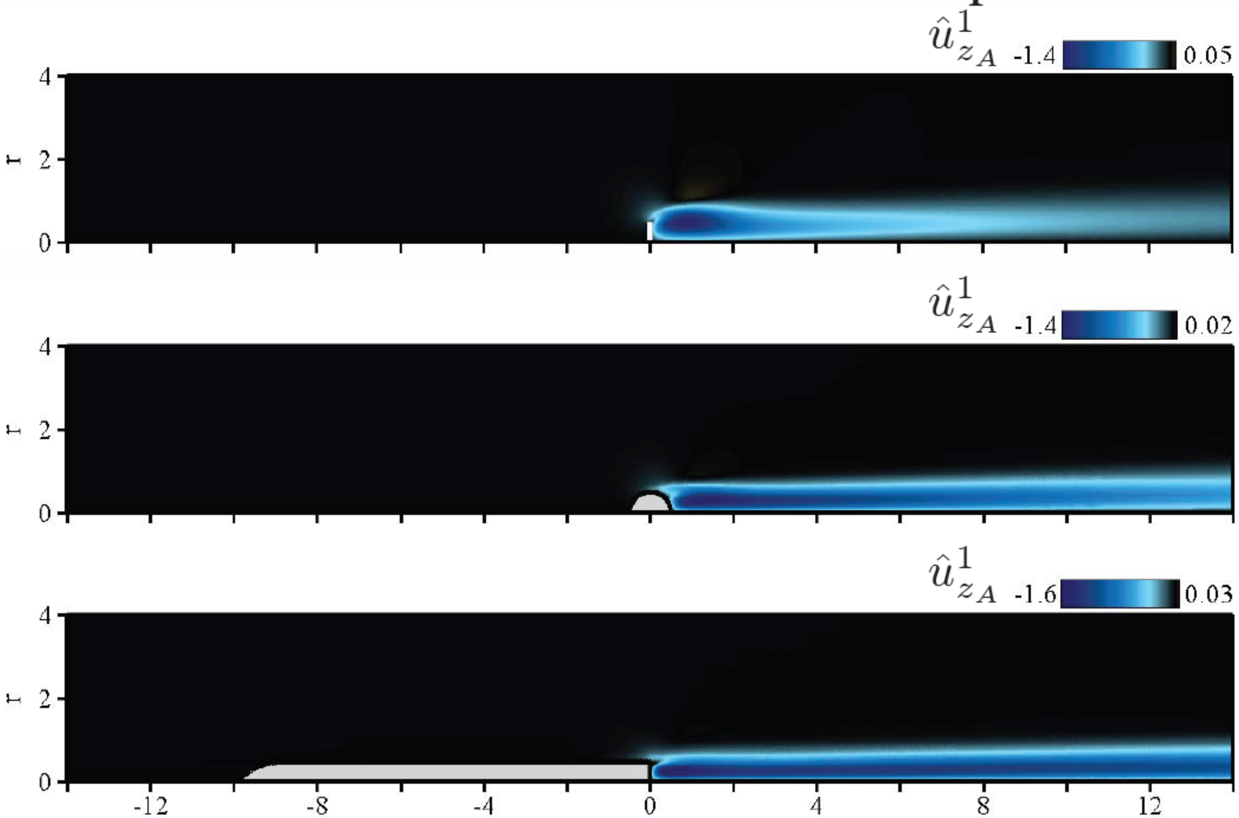
Disk Sphere Blunt base

Meliga et al. 2008,2009,2010

Leading global modes



Same bifurcation sequence for all prototype flows, in the INCOMPRESSIBLE & compressible regimes

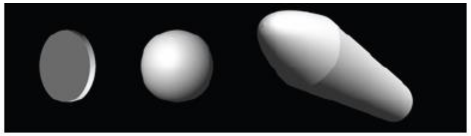


$Re_A = 116.9$

$Re_A = 212.6$

$Re_A = 460.5$

Leading global modes



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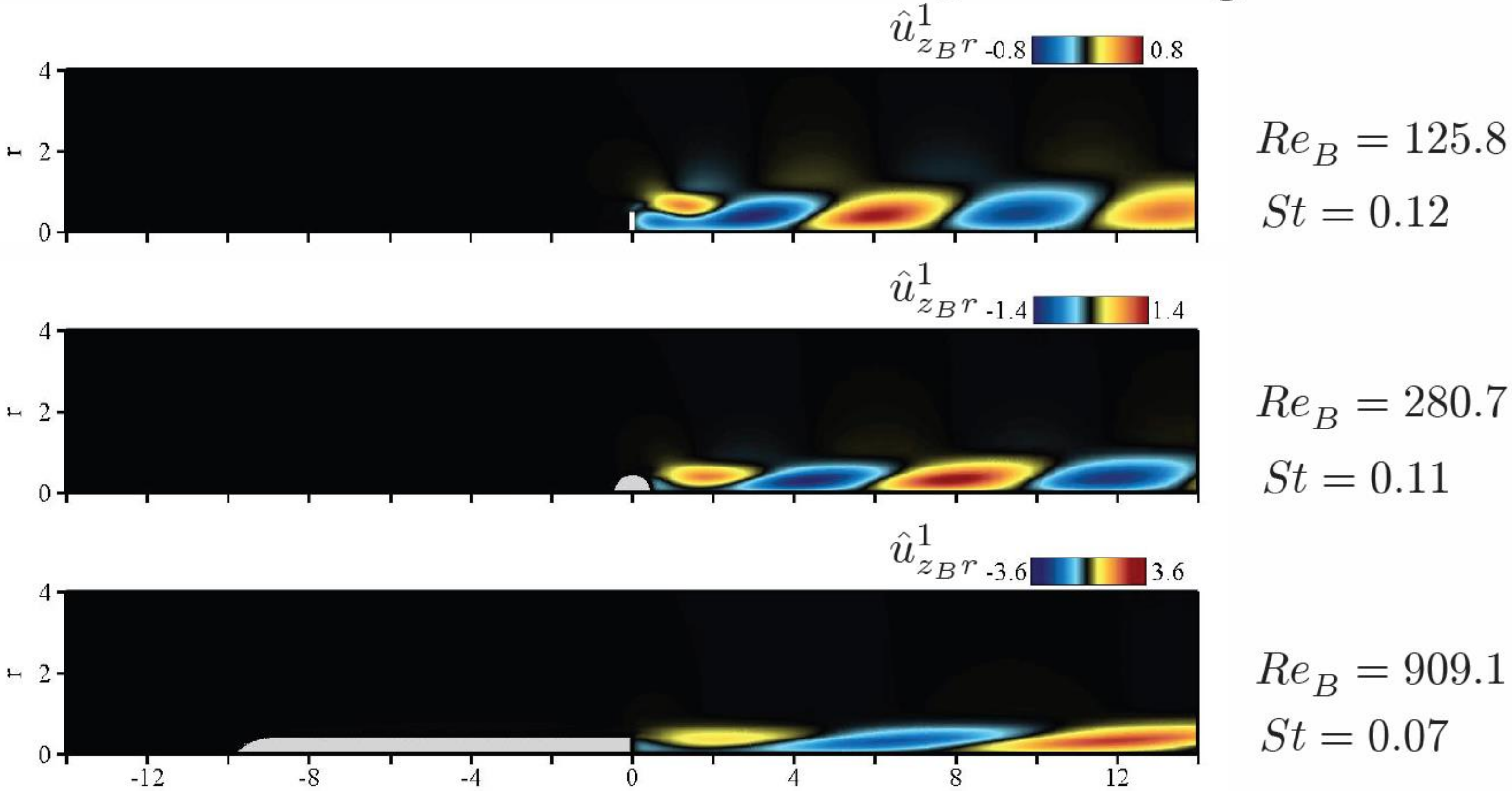
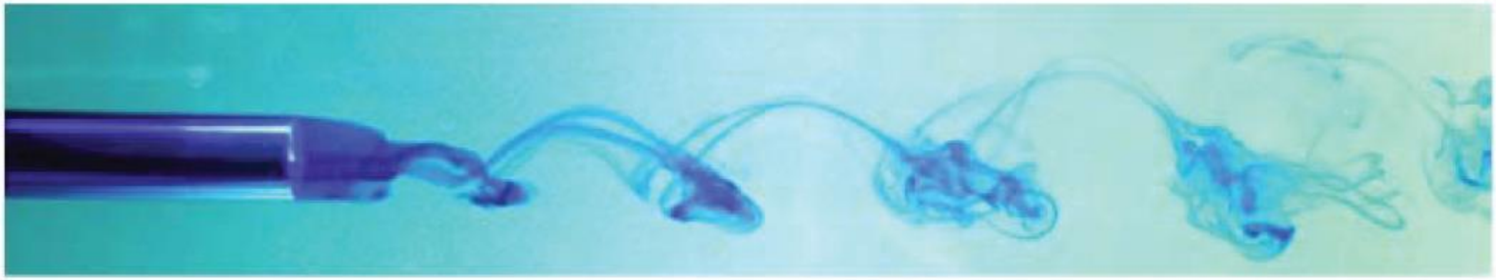


How do these theoretical considerations compare with experiments

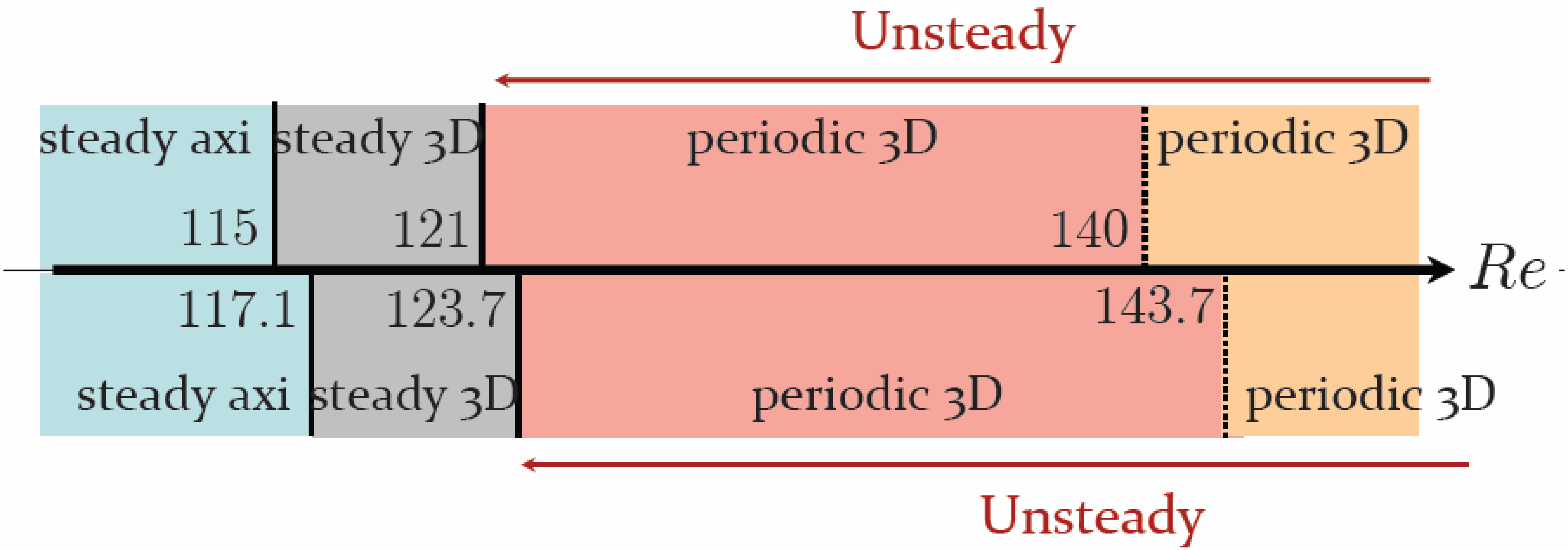
Sphere – Johnson & Patel (1999)



Blunt base – Siegel et al. (2008)

Remarkable predictions of successive symmetry breakings and thresholds

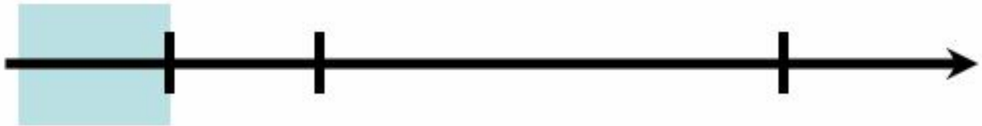
Direct numerical simulation of the 3D flow *Fabre et al. (2008)*



Weakly nonlinear global stability **Meliga et al. 2009**

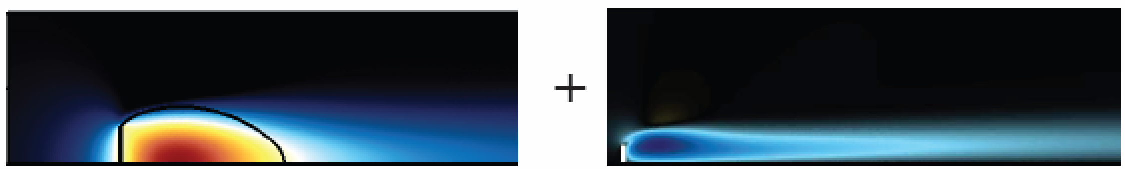
Stable solutions

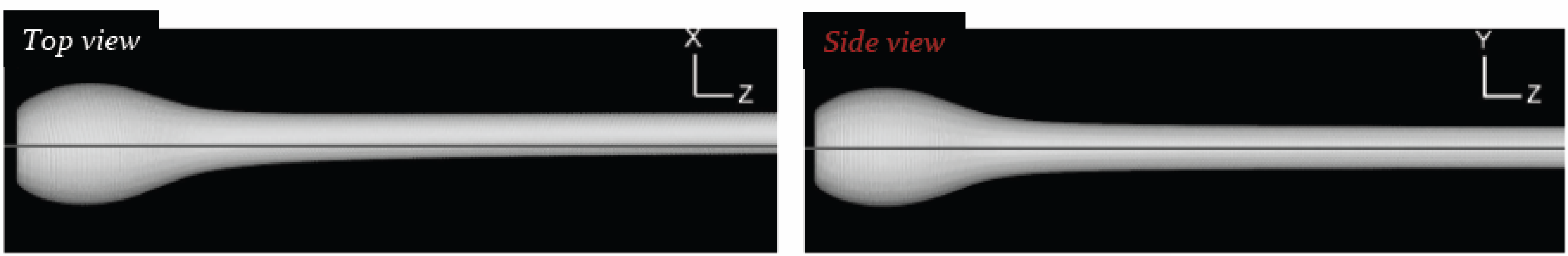
- Steady axi



Stable solutions

- Steady 3D + planar sym.

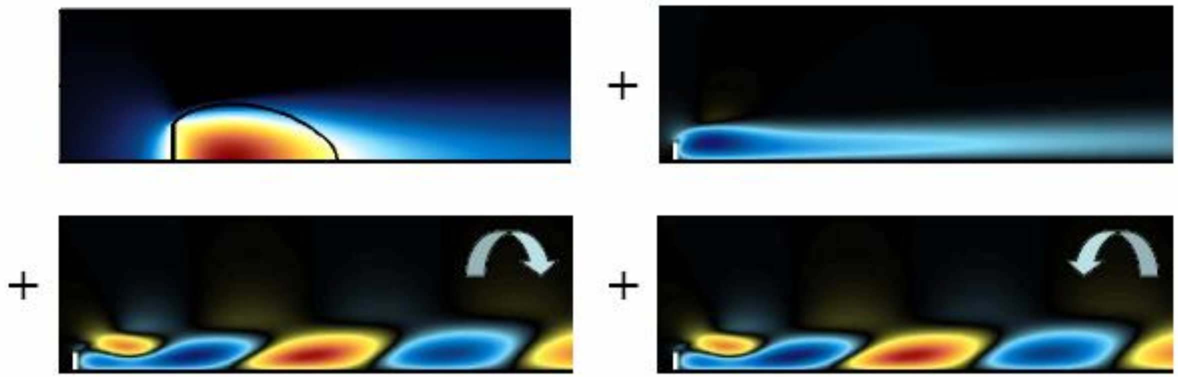


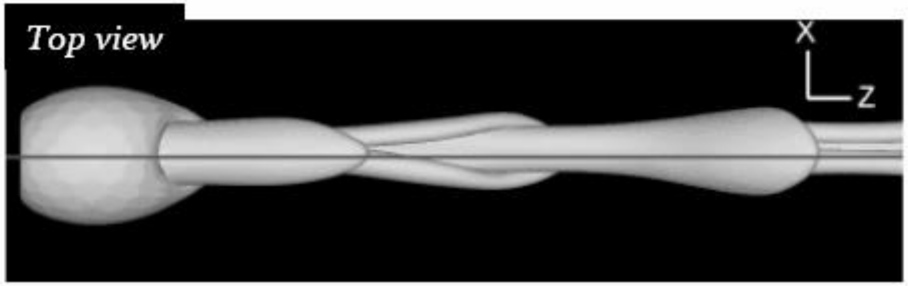


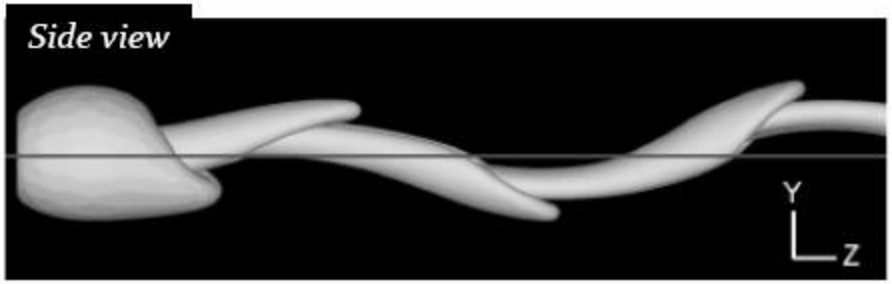
Numerical dye lines

Stable solutions

- Periodic 3D, no sym.

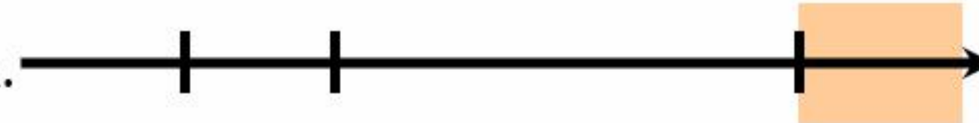


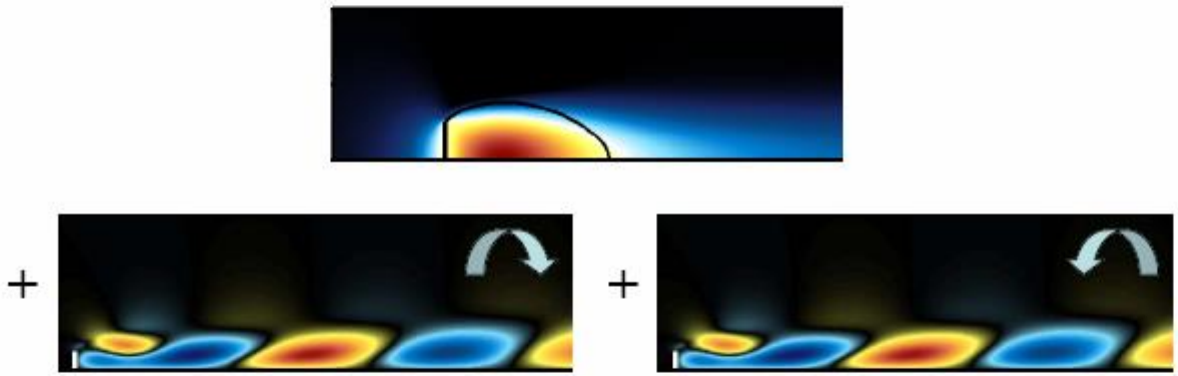


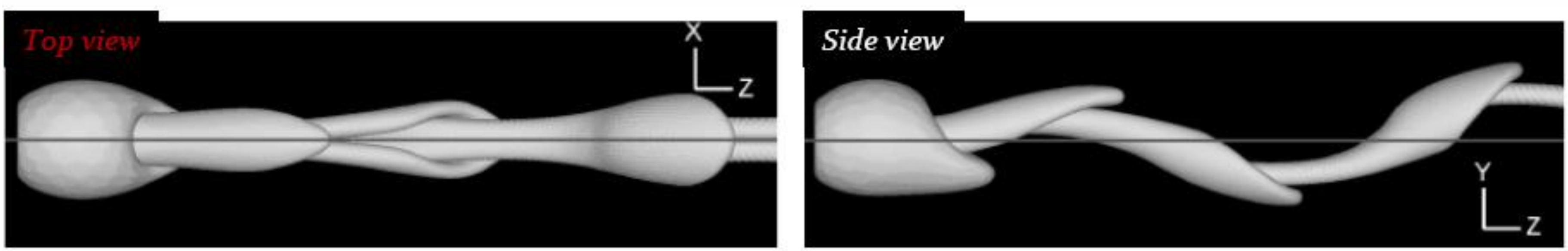


Stable solutions

• Periodic 3D + planar sym.







Global linear instability of flows in complex geometry

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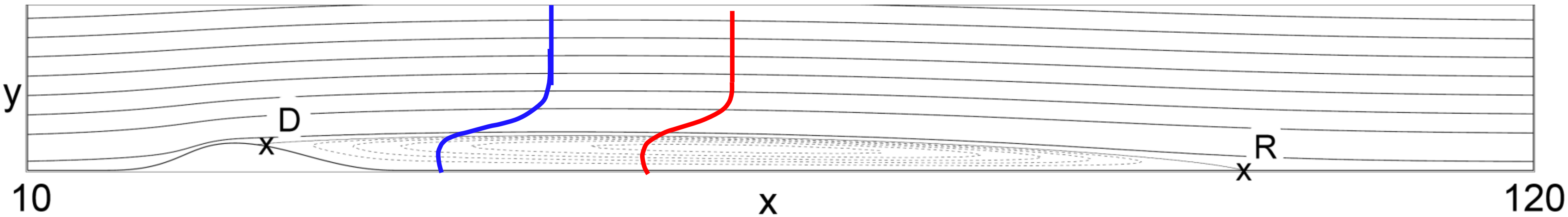
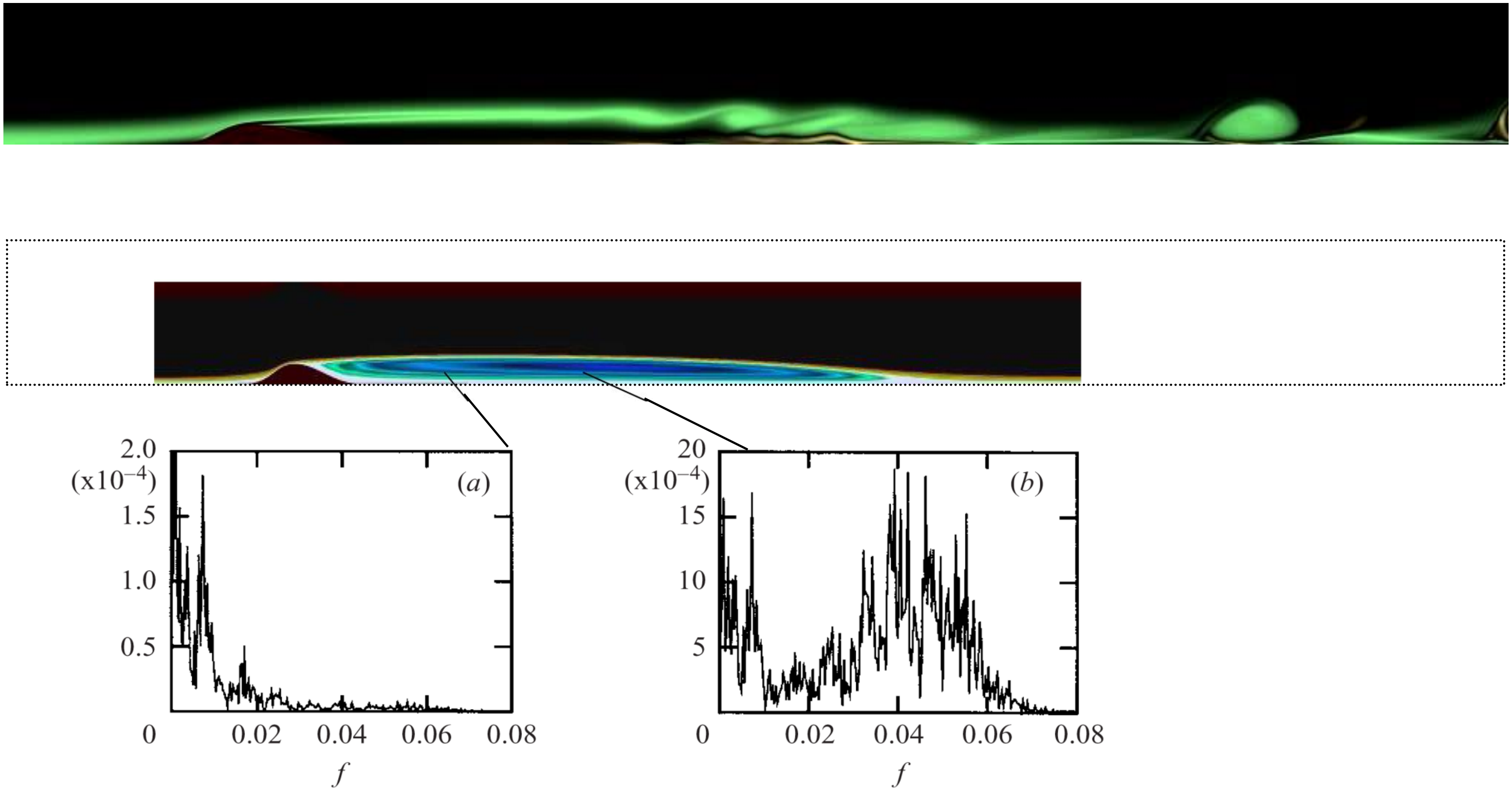
Global low frequency oscillations in a detached boundary layer



Locally parallel 1D instability fails!

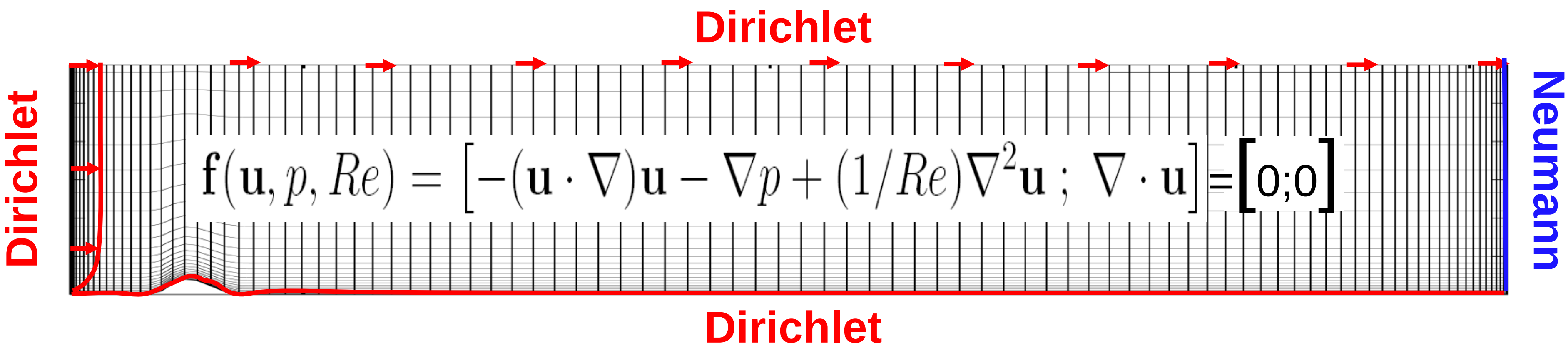
Marquillie and Ehrenstein (2003)

Flapping in the DNS



Marquillie and Ehrenstein (2003)

Navier-Stokes steady states



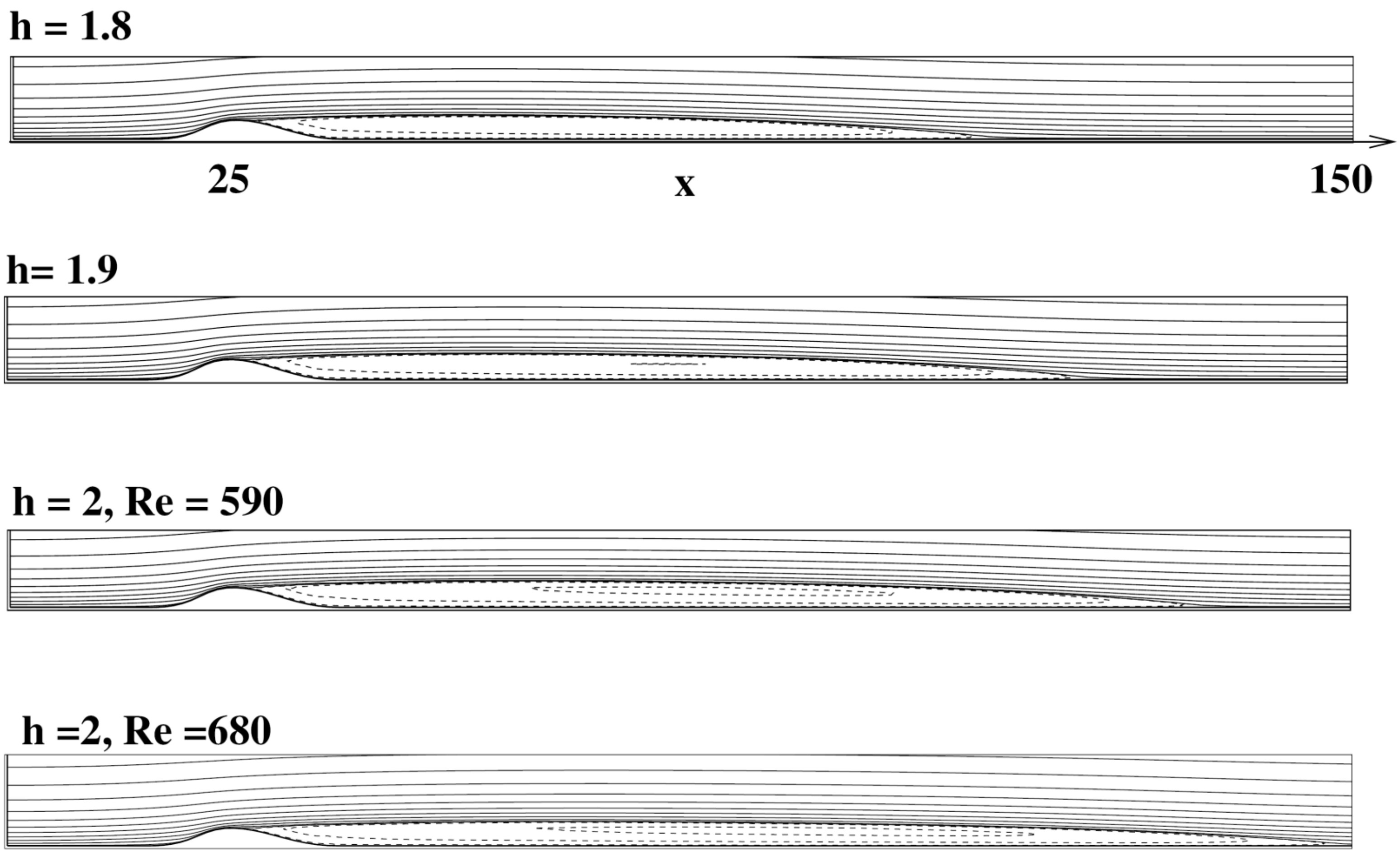
Mapping and chebyshev-collocation in both streamwise x and normal y coordinates

Chebyshev discretization in a square: spurious pressure modes

4 corners, p_i =0 and ∇p_i =0
8 extra conditions: continuity of normal derivative for p at the corners and p_i =0.

Domain length L=300, H=30, 250x40 collocation points

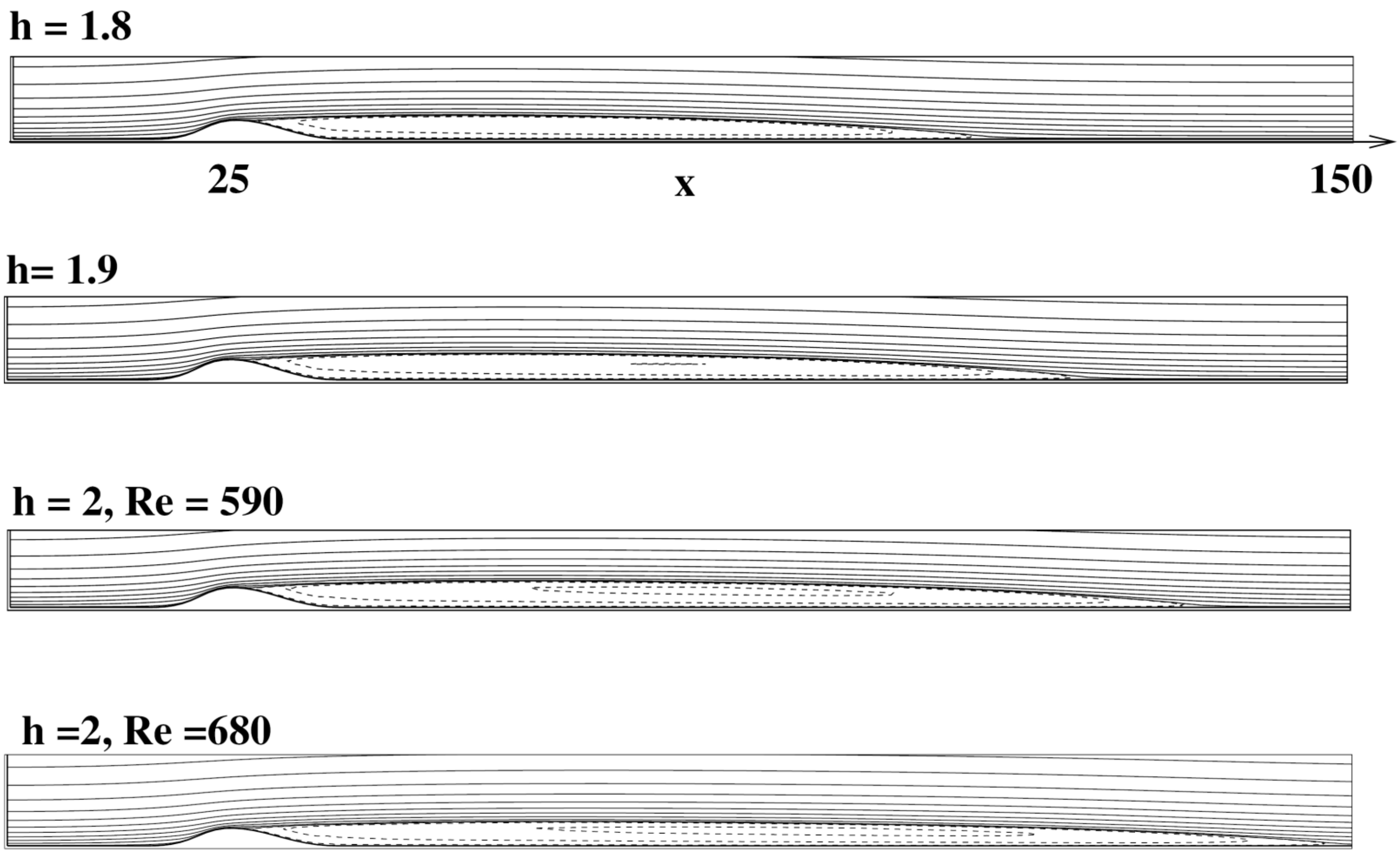
Continuation procedure to obtain unstable steady states



Ehrenstein and Gallaire (2008)

No topology change

Continuation procedure to obtain unstable steady states



Ehrenstein and Gallaire (2008)

Reattachment increasingly difficult to capture

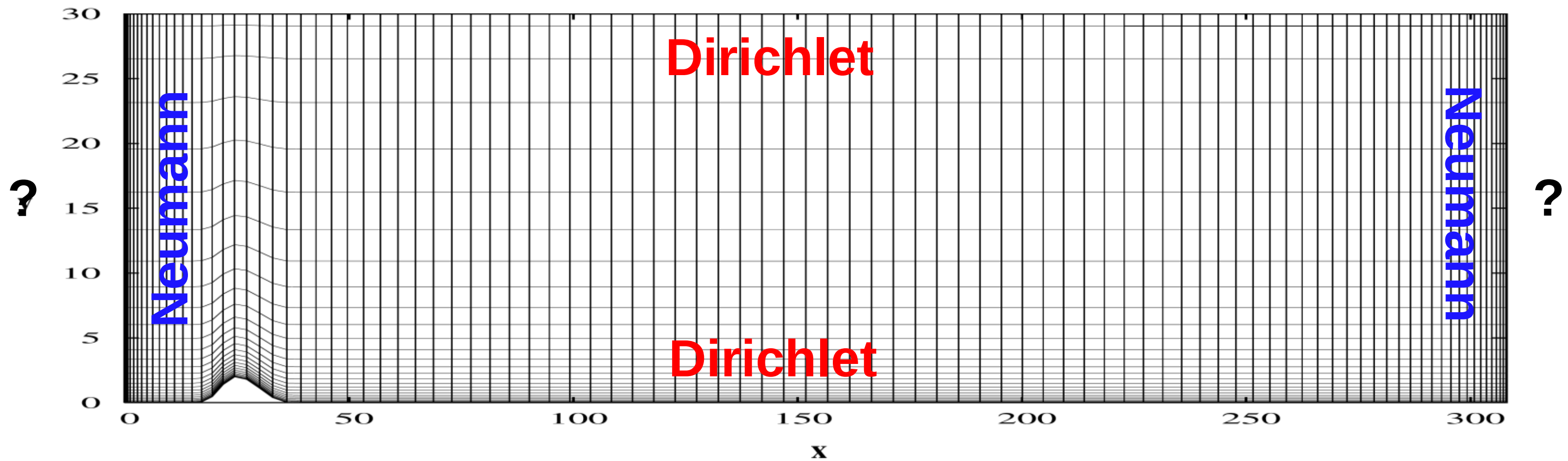
2D temporal global modes

$$\frac{\partial \mathbf{u}}{\partial t} = -\underbrace{(\mathbf{U}.\nabla)}_{\text{Base flow}}\overbrace{\mathbf{u}}^{\text{Perturbation}} - (\mathbf{u}.\nabla)\mathbf{U} - \nabla p + \frac{1}{Re}\nabla^2\mathbf{u},$$

$$\nabla.\mathbf{u} = 0,$$

$$[\mathbf{u}(x,y,z,t), p(x,y,z,t)] = [\hat{\mathbf{u}}(x,y), \hat{p}(x,y)]\, e^{\sigma t}$$

σ : growth rate

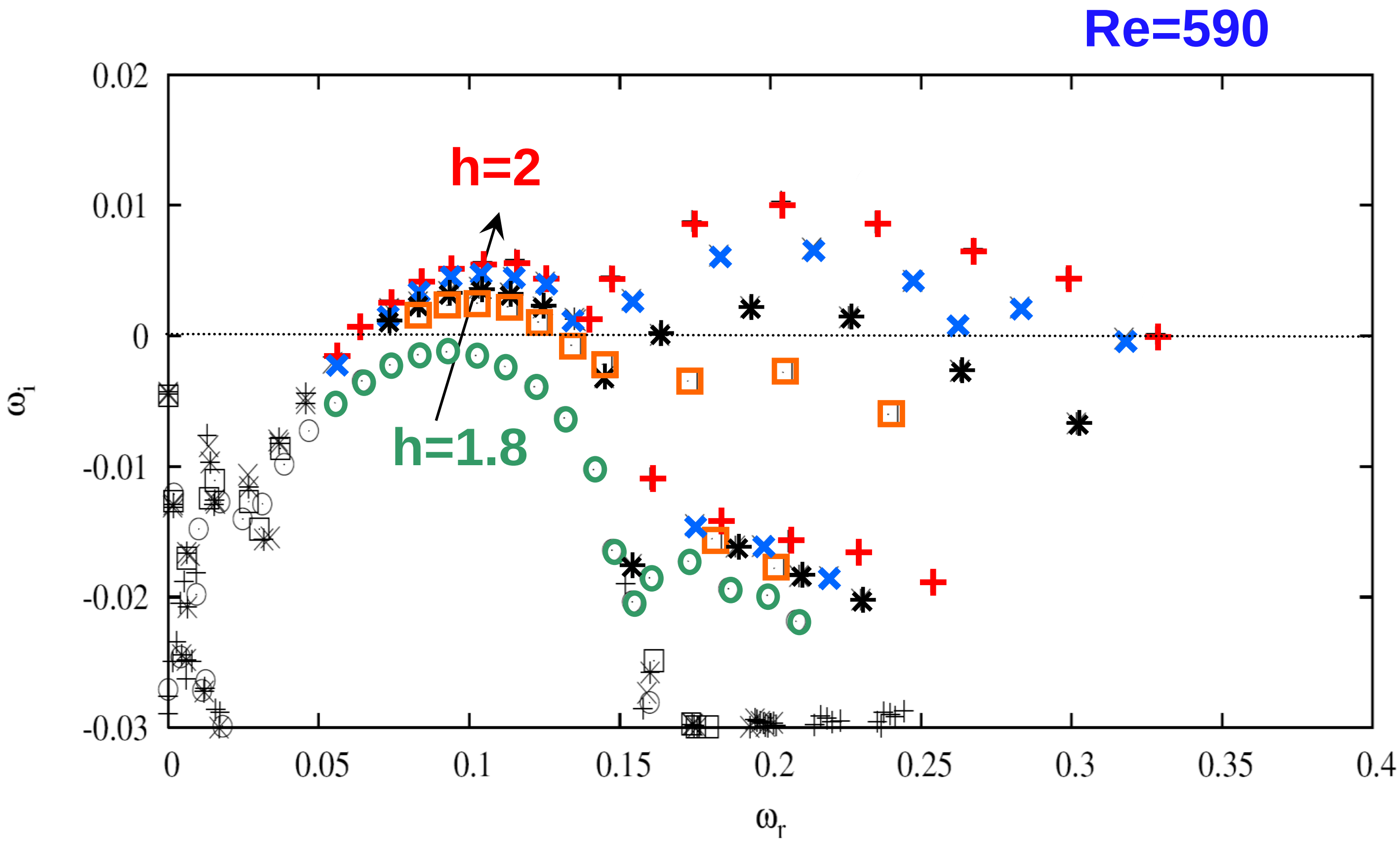


Generalized eigenvalue problem

$$\mathbf{A}\hat{\mathbf{v}} = \sigma \mathbf{B}\hat{\mathbf{v}}$$

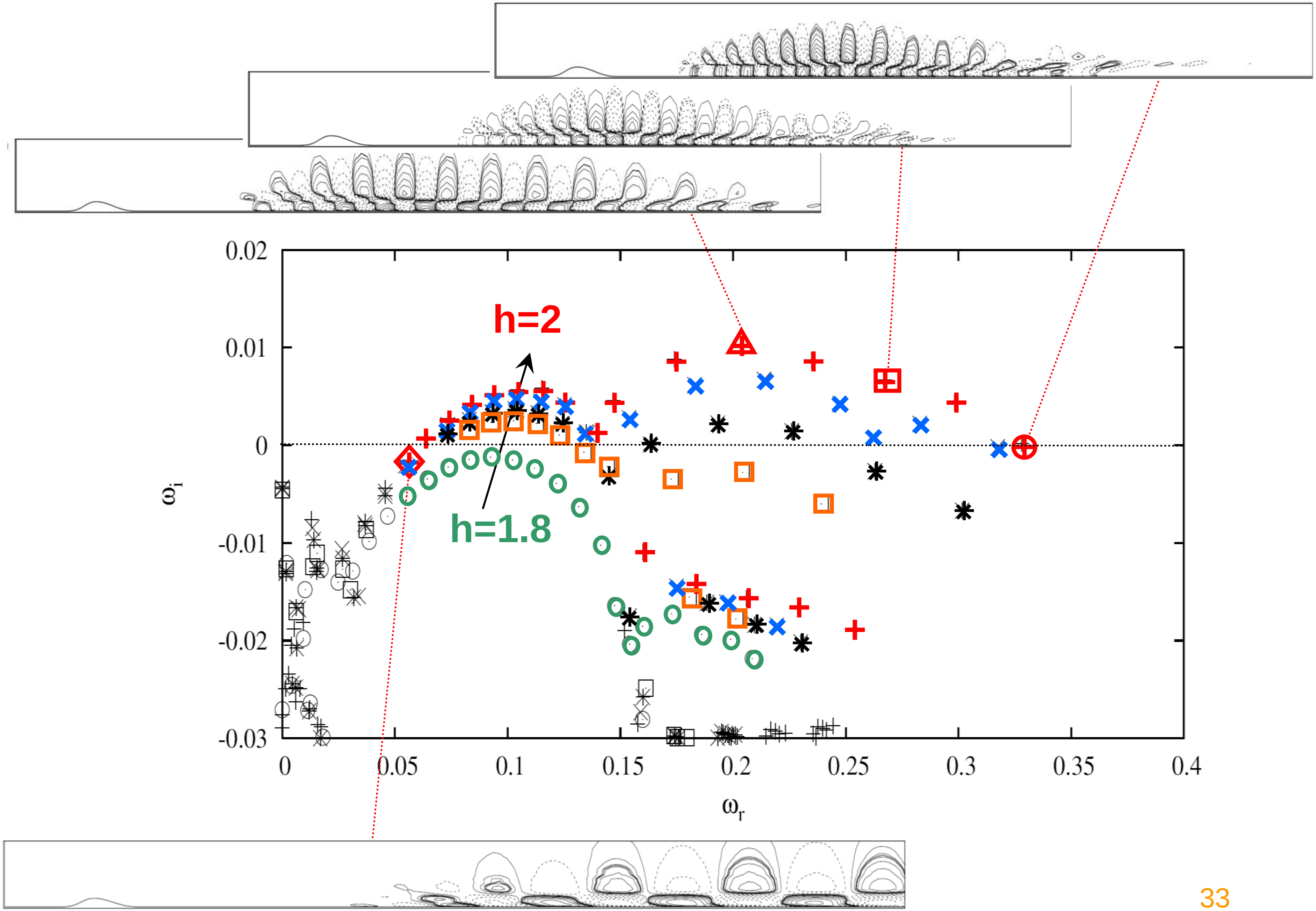
Inverse Krylov-Arnoldi method

Unstable spectrum



Ehrenstein and Gallaire(2007)

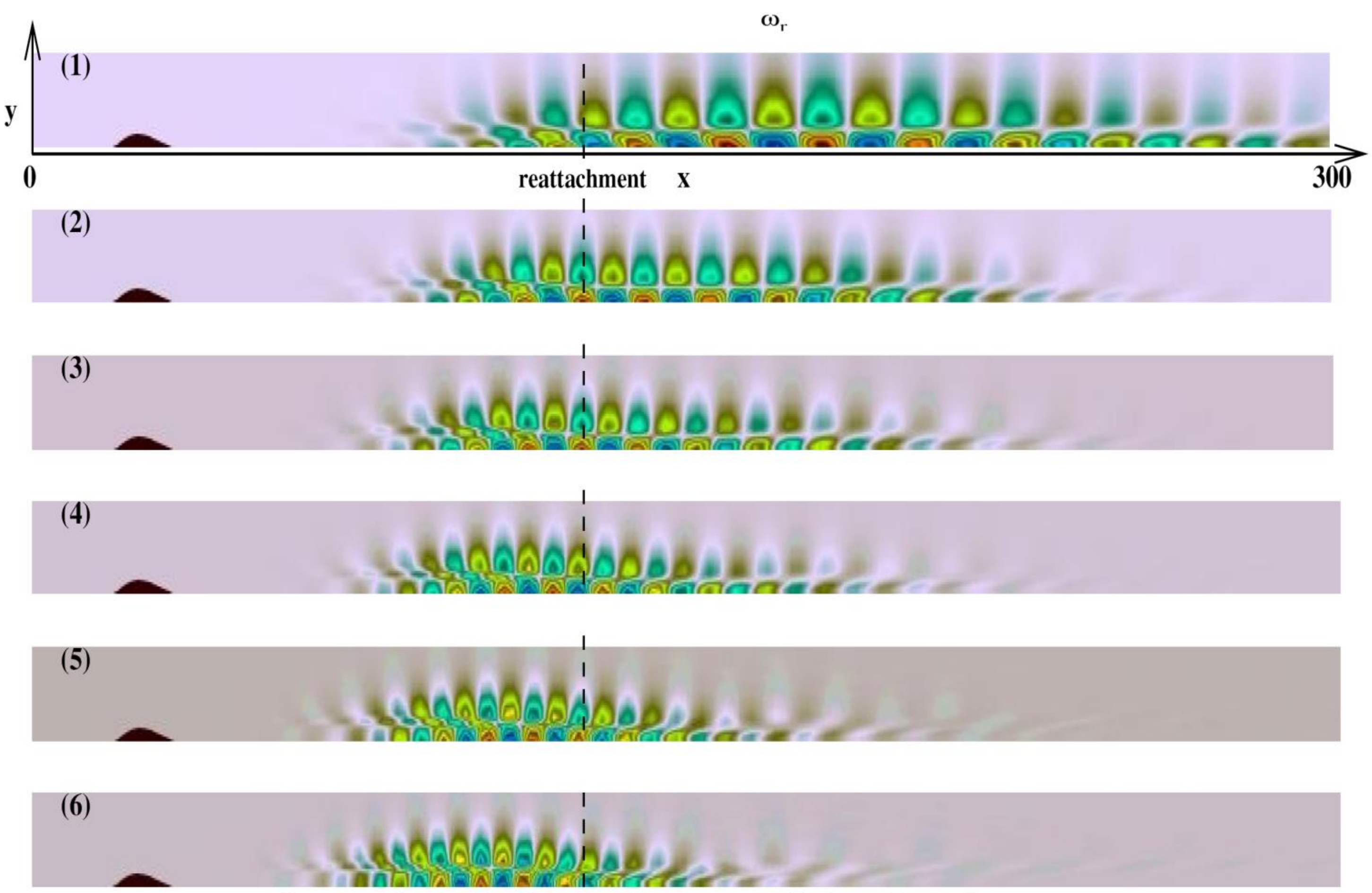
Eigenmodes



h=2
h=1.8
0.02
0.01
0
-0.01
-0.02
-0.03
ω_i
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
ω_r

Localized modes

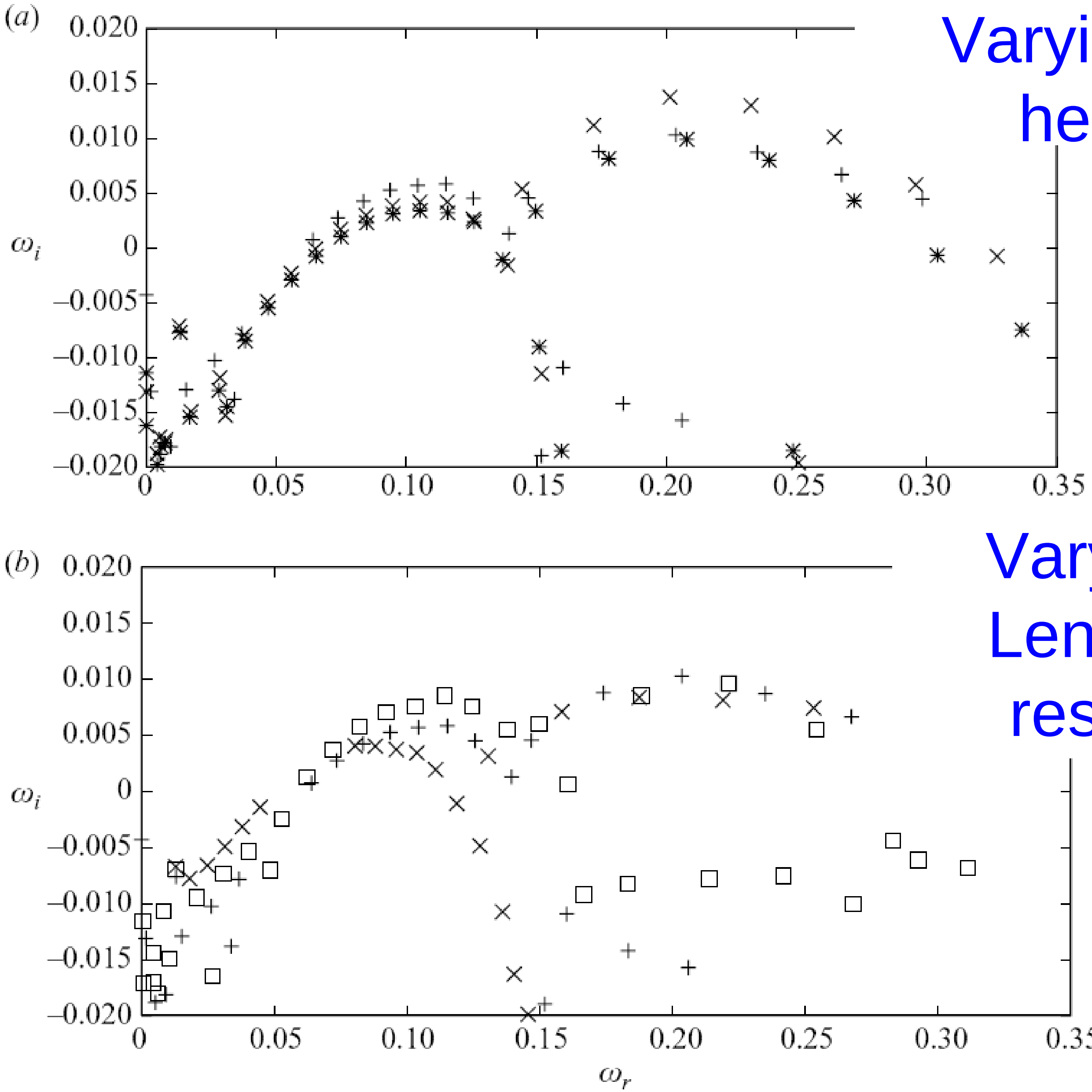
Low ω_r



High ω_r

Steamwise velocity components of eigenfunctions

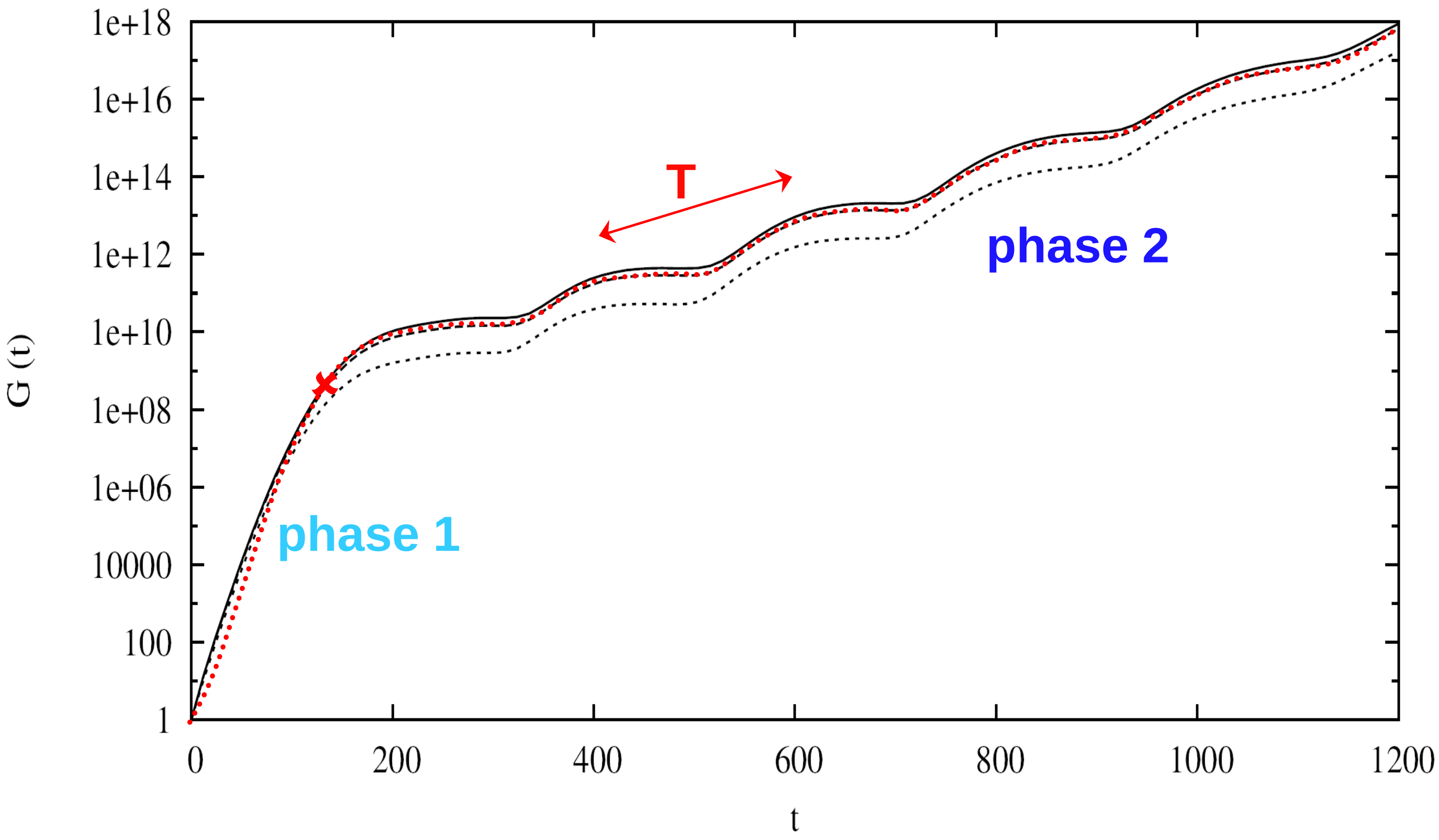
Convergence issues



Varying the height

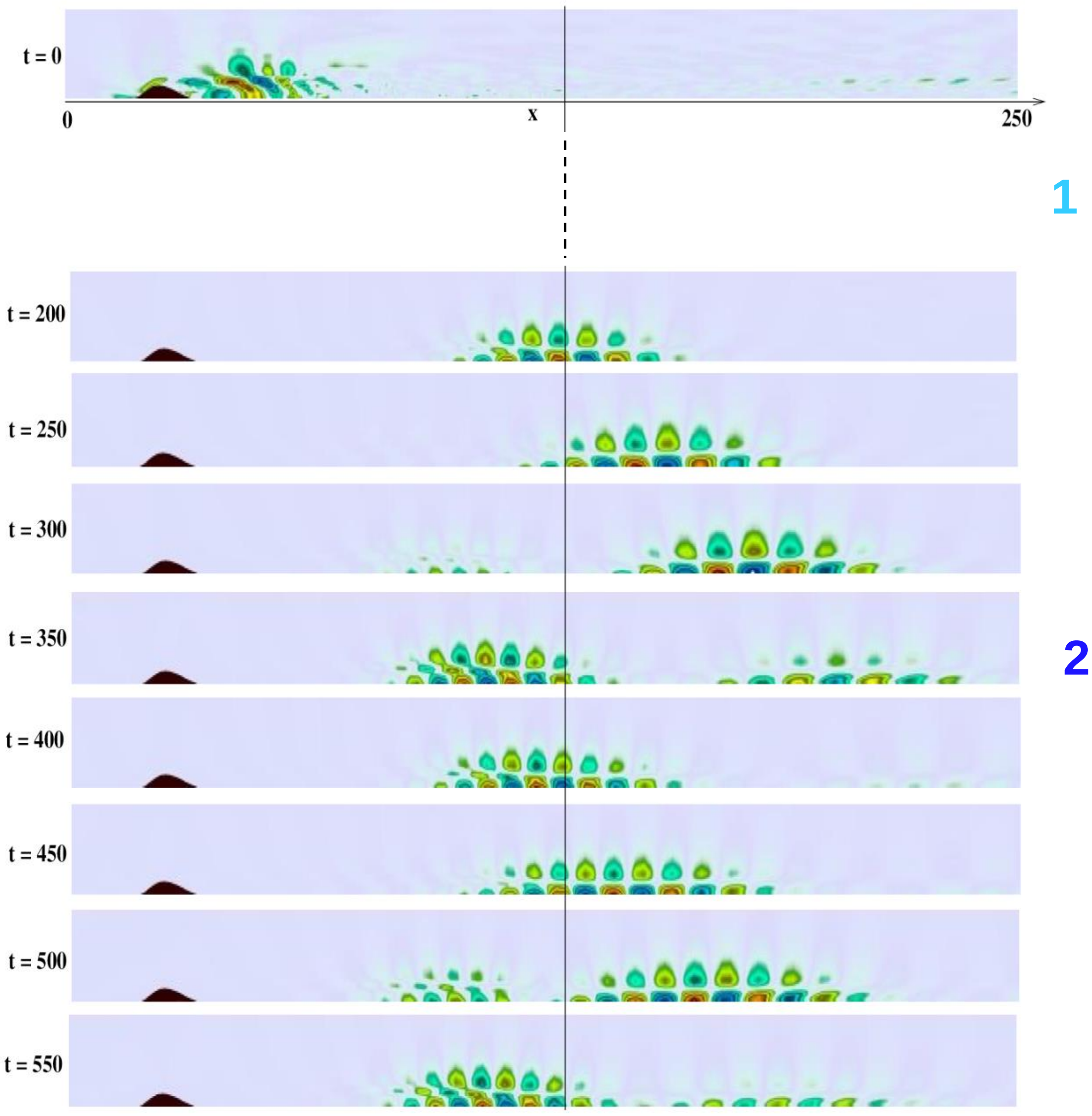
Varying the Length and resolution

Optimal perturbation

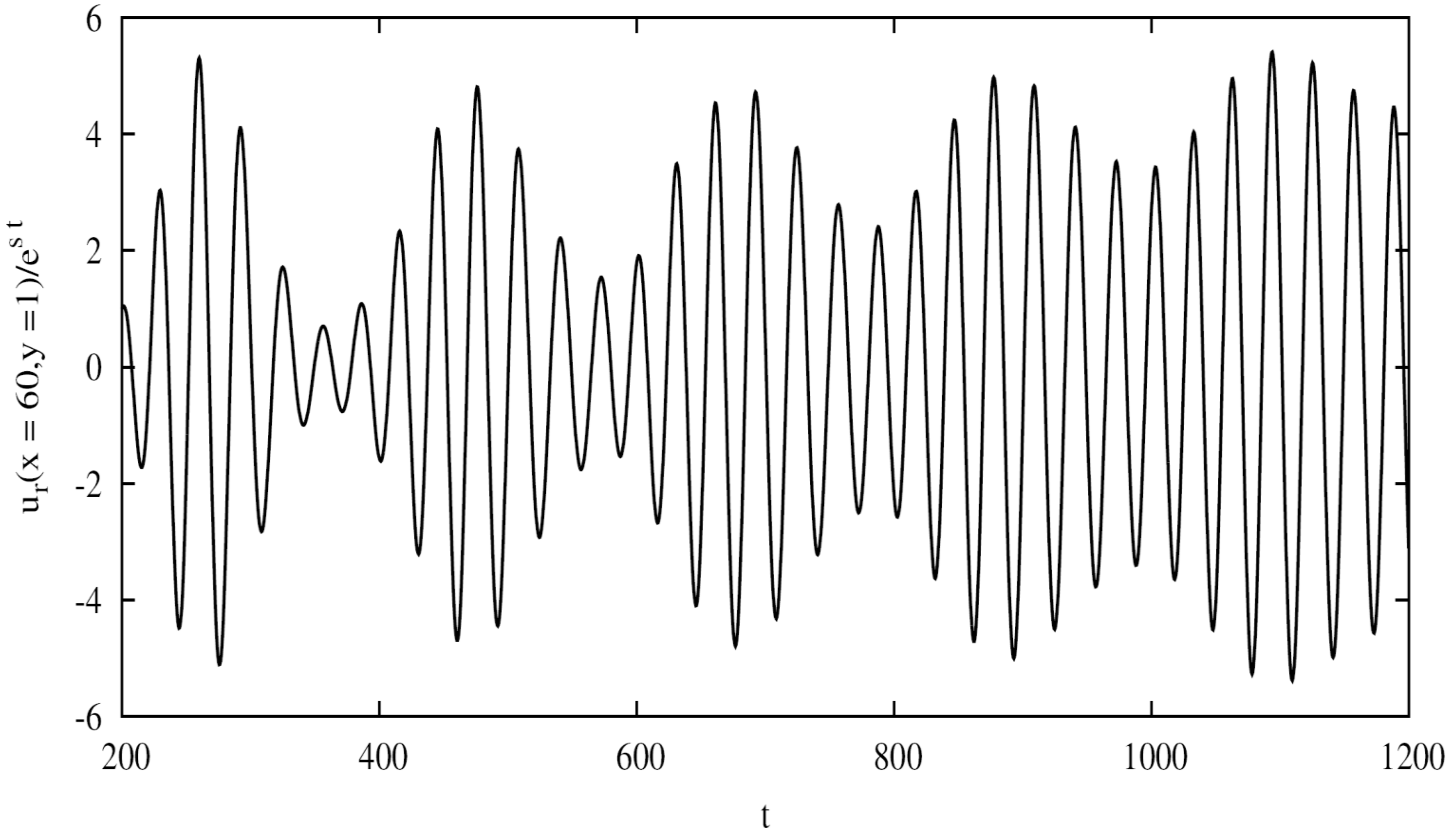


Perturbation evolution

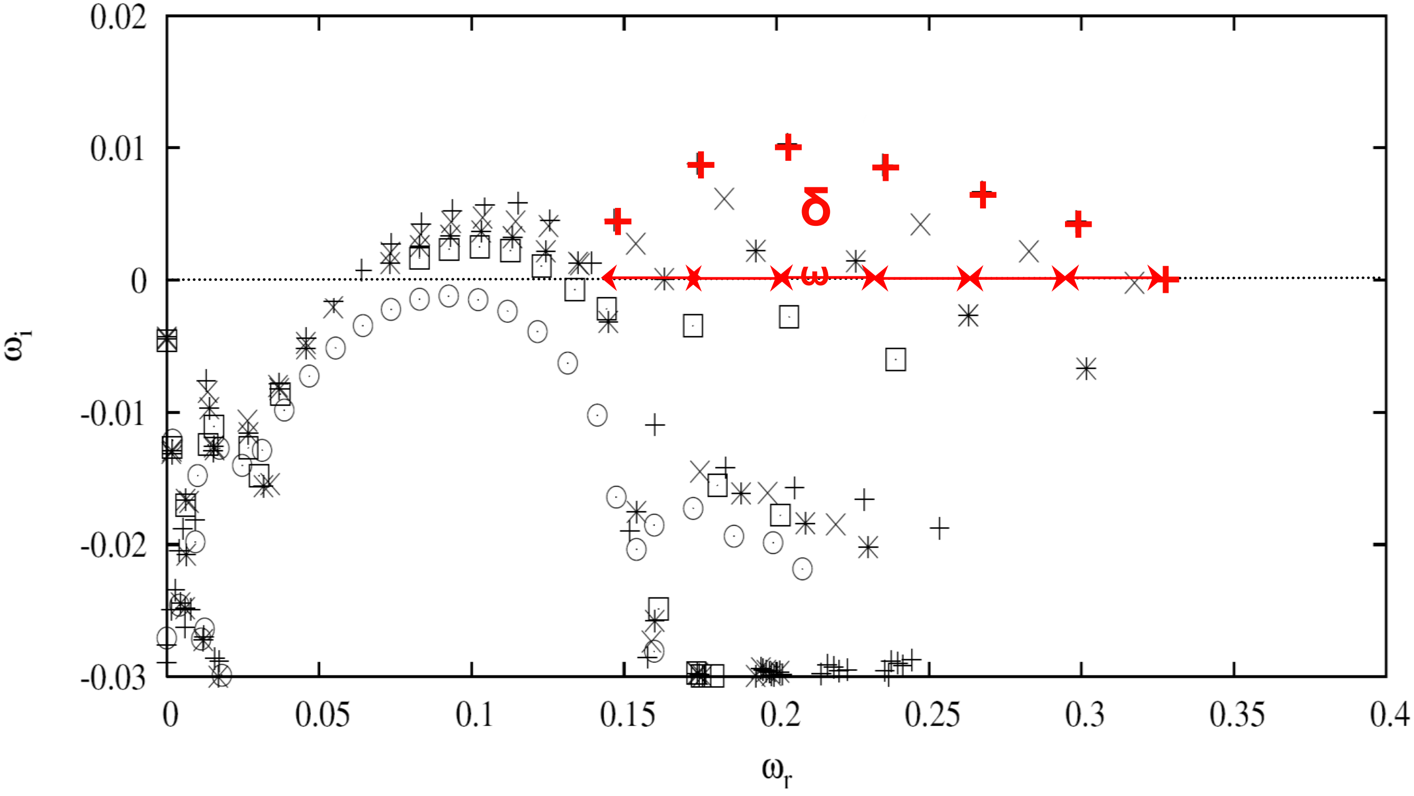
Perturbation evolution



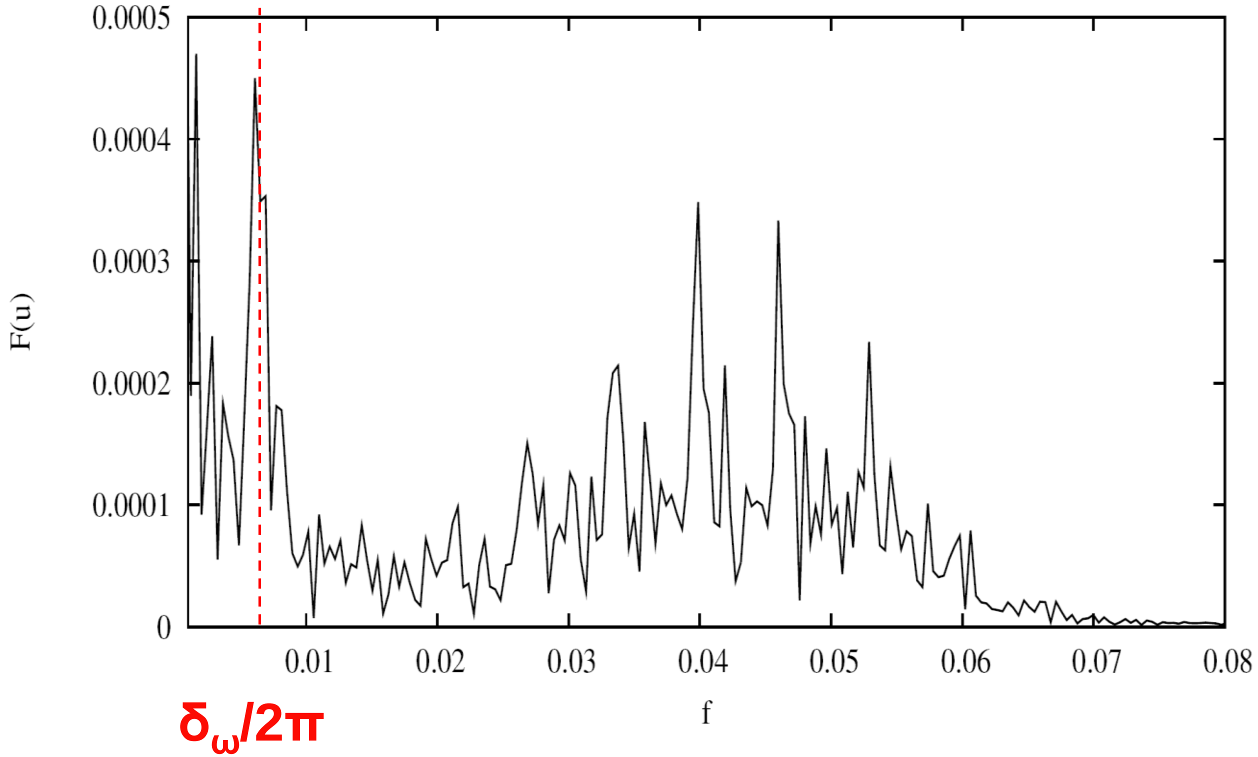
Beating behavior



Similar modes with equidistant frequencies



Power spectrum in DNS



Global linear instability of flows in complex geometry

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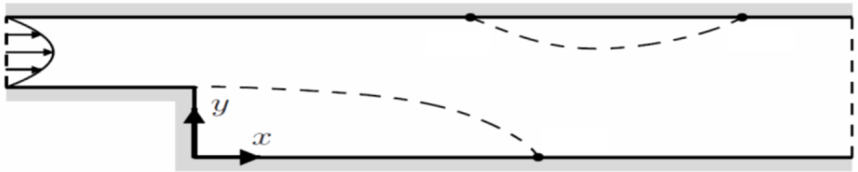
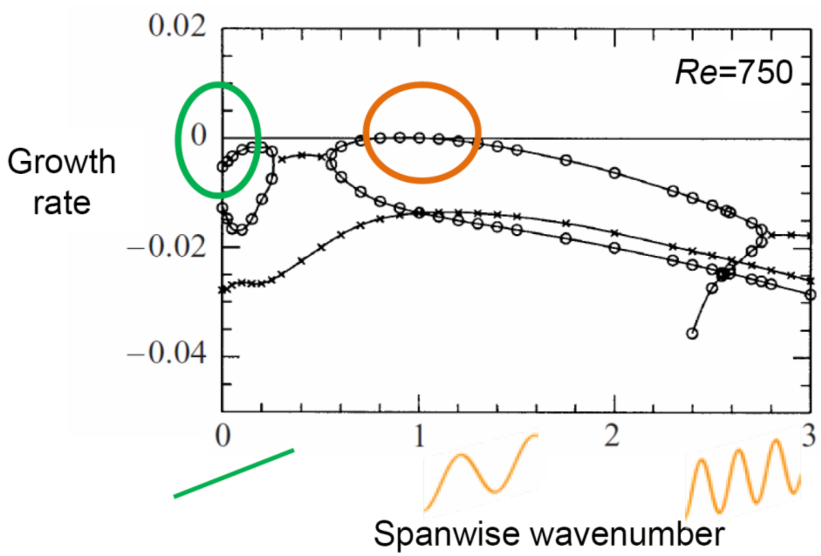
A typical amplifier:
the backward-facing step



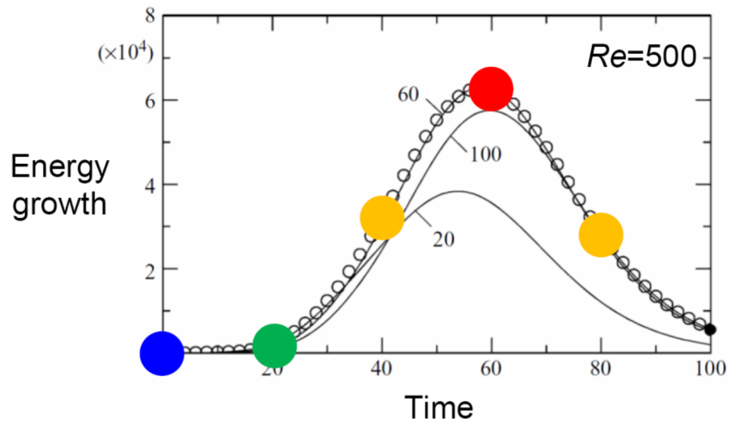
Global linear stability analysis: [Barkley, Gomes & Henderson, 2002; Lanzerstorfer & Kuhlmann, 2012]

- Stable to 2D perturbations for any *Re*
- Stable to 3D perturbations up to *Re*=715-750

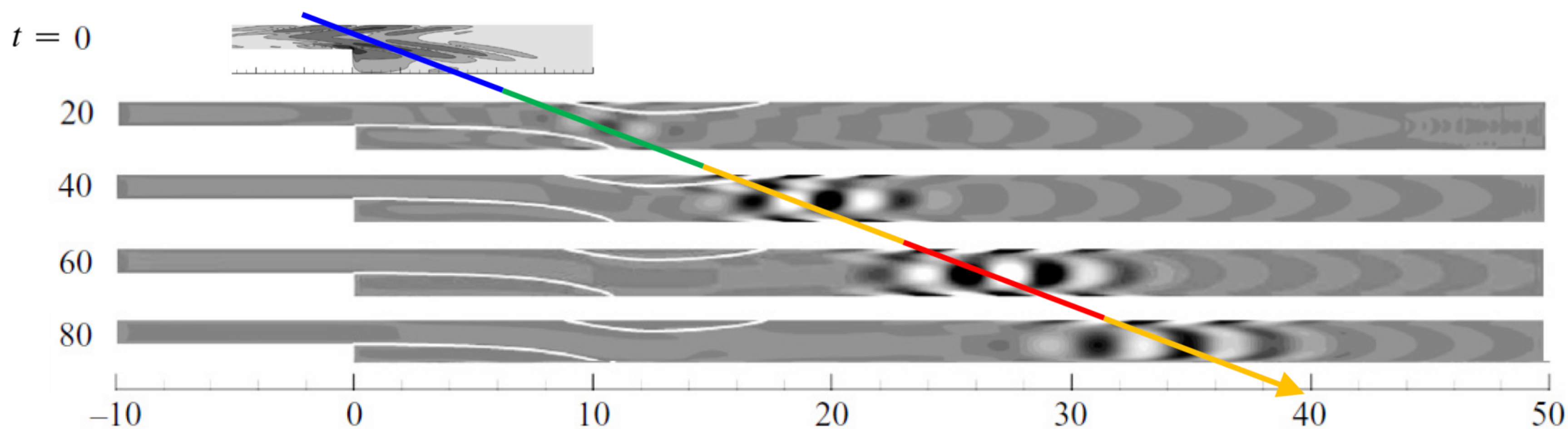


Large transient growth

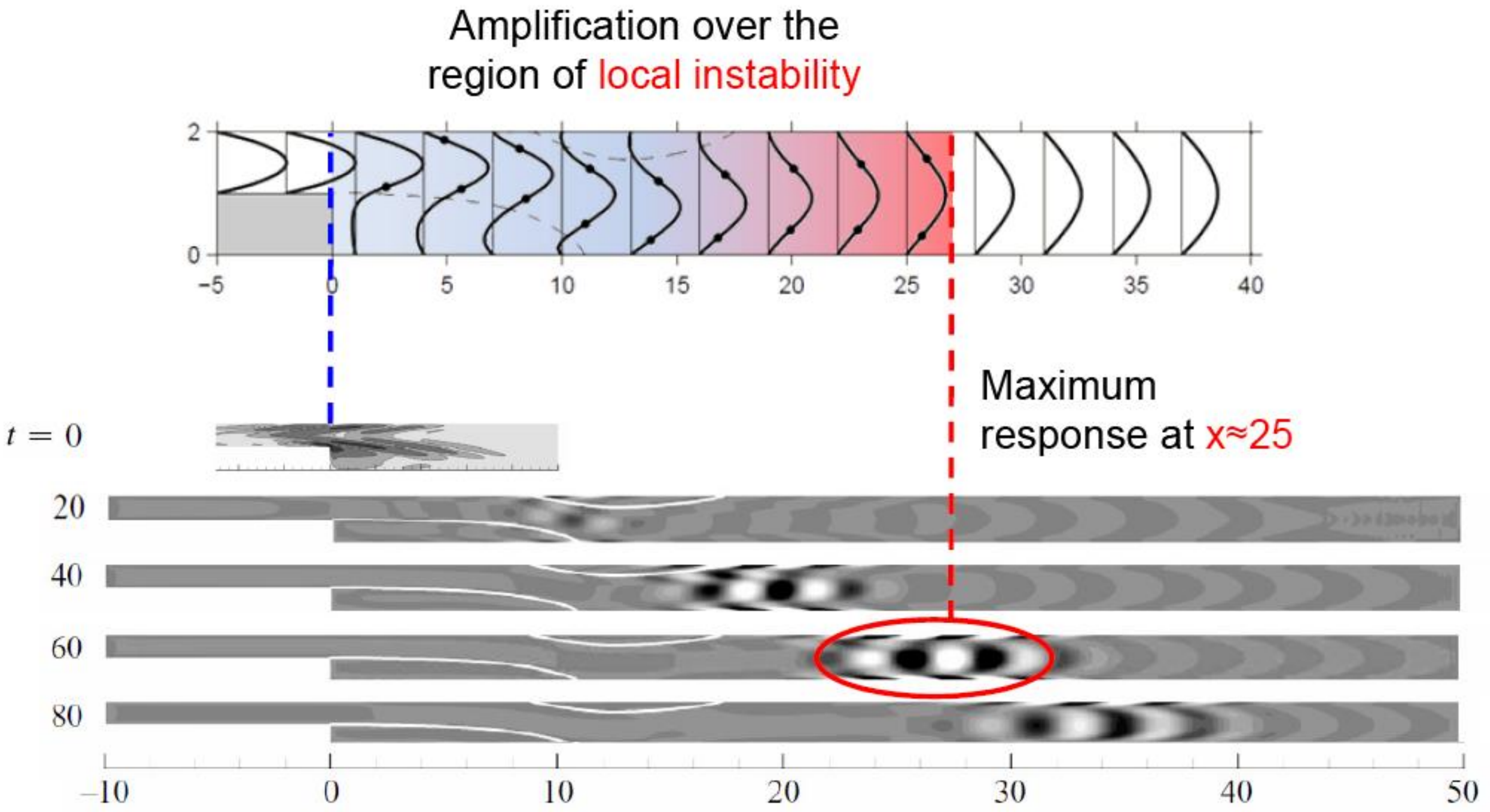
[Blackburn, Barkley & Sherwin, 2008]



Growth in space & time: optimal perturbations amplified while convected downstream

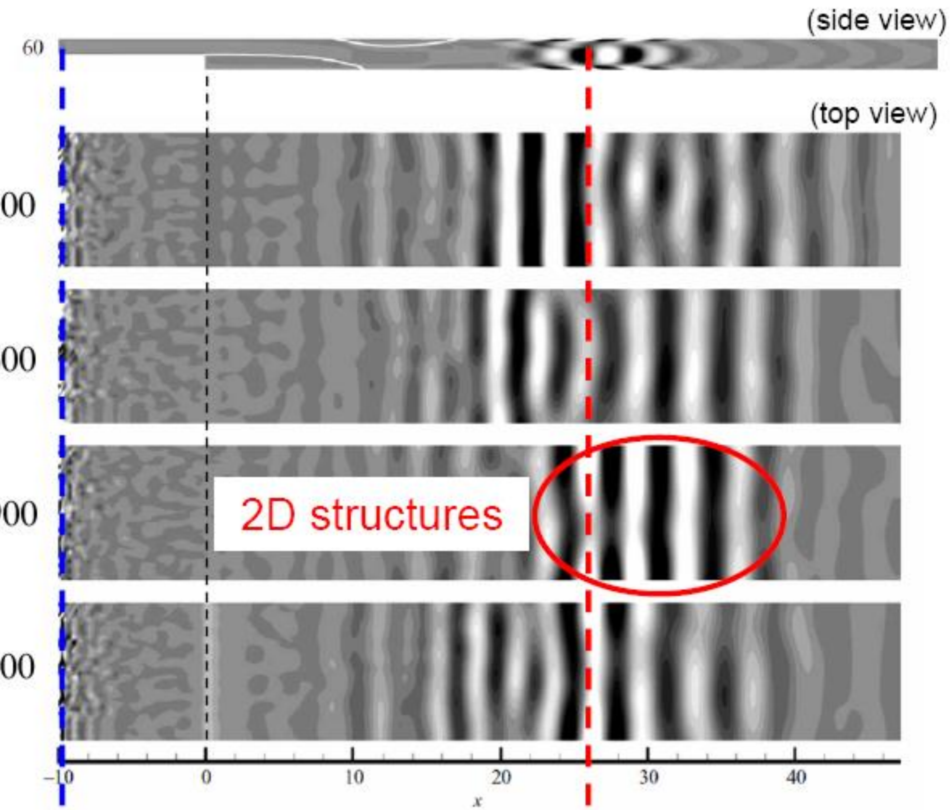


The global transient growth is due to the local spatial growth

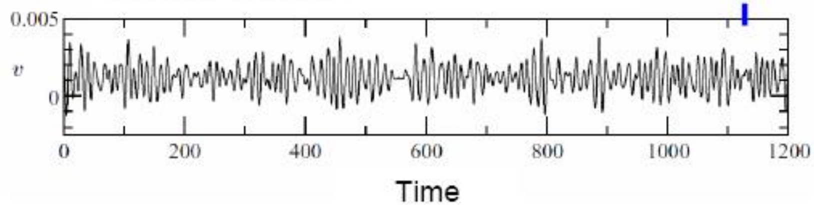


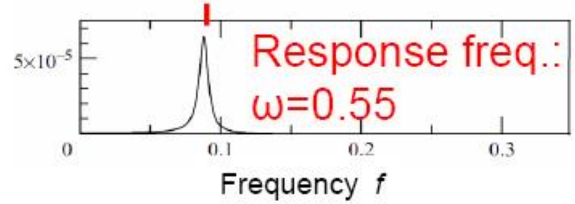
Response to white noise

Response to sustained inlet noise (3D DNS):
[Blackburn *et al.*, 2008]



White noise at the inlet:



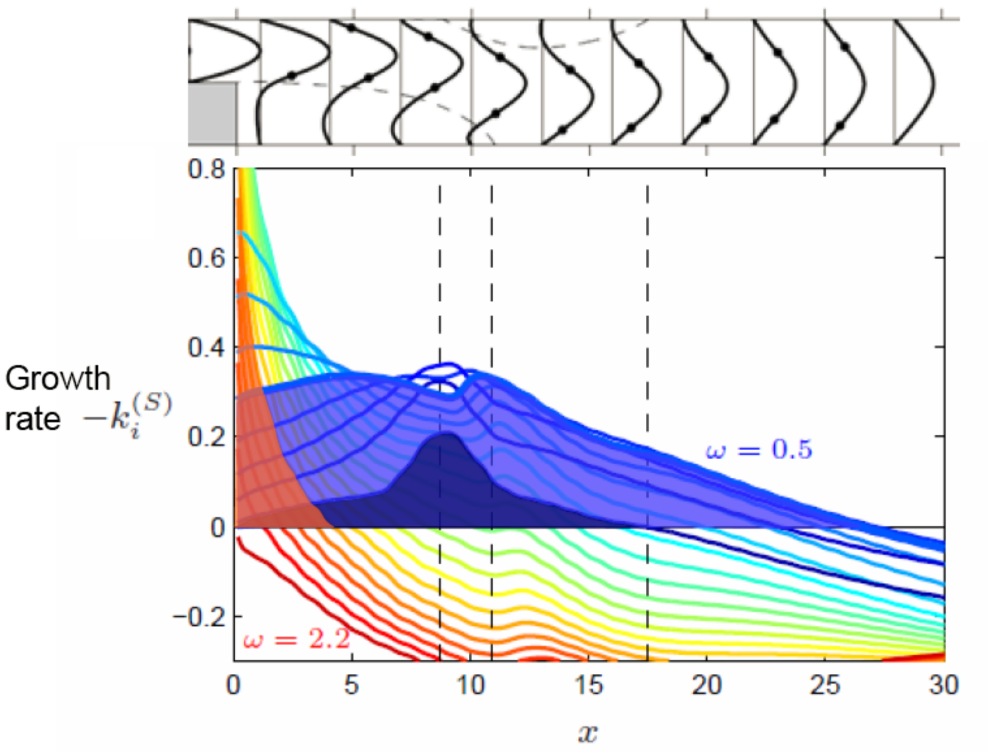


⇨Very selective amplifier

Method 1: weakly non parallel spatial stability analysis

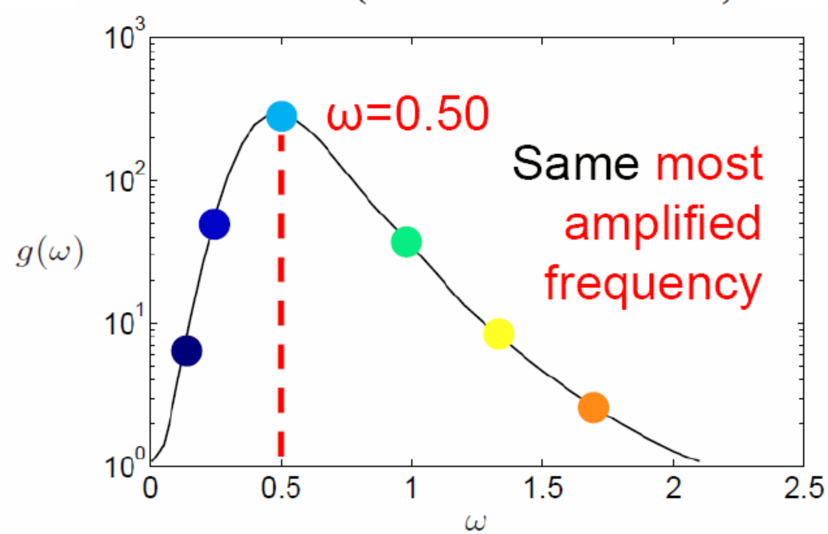
Parallel flow: $\mathbf{u}'(x,y,t) = \mathbf{u}(y)\,\mathrm{e}^{i(kx-\omega t)}$

Spatial analysis $\omega \in \mathbb{R}, k \in \mathbb{C}$



Overall amplification: integral gain

$$g(\omega) = \exp\left(\int -k_i^{(S)}(\omega,x)\,\mathrm{d}x\right)$$



Method 2: transfer function

Nonlinear problem (base flow)

$$\boldsymbol{\nabla} \cdot \mathbf{U}_b = 0,$$

$$\mathbf{U}_b \cdot \boldsymbol{\nabla} \mathbf{U}_b + \boldsymbol{\nabla} P_b - Re^{-1} \boldsymbol{\nabla}^2 \mathbf{U}_b = \mathbf{0},$$

+ linear perturbation

$$\boldsymbol{\nabla} \cdot \mathbf{u}' = 0,$$

$$\partial_t \mathbf{u}' + \boldsymbol{\nabla} \mathbf{u}' \cdot \mathbf{U}_b + \boldsymbol{\nabla} \mathbf{U}_b \cdot \mathbf{u}' + \boldsymbol{\nabla} p' - Re^{-1} \boldsymbol{\nabla}^2 \mathbf{u}' = \mathbf{F}$$

Method 2: transfer function

Nonlinear problem (base flow)

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for harmonic forcing $\mathbf{F}(t) = \mathbf{f} e^{i\omega t}$.

look for solution $(\mathbf{u}', p')(\mathbf{x}, t) = (\mathbf{u}, p)(\mathbf{x})\, \mathrm{e}^{i\omega t}$

Method 2: transfer function

harmonic forcing

$$\nabla \cdot \mathbf{u} = 0,$$

$$i\omega \mathbf{u} + \nabla \mathbf{u} \cdot \mathbf{U}_b + \nabla \mathbf{U}_b \cdot \mathbf{u} + \nabla p - Re^{-1} \nabla^2 \mathbf{u} = \mathbf{f}$$

Resolvent operator (transfer function)

$$\mathbf{u} = \mathcal{R}(\omega)\mathbf{f}$$

Method 2: transfer function

harmonic forcing

$$\boldsymbol{\nabla} \cdot \mathbf{u} = 0,$$

$$i\omega\mathbf{u} + \boldsymbol{\nabla}\mathbf{u} \cdot \mathbf{U}_b + \boldsymbol{\nabla}\mathbf{U}_b \cdot \mathbf{u} + \boldsymbol{\nabla} p - Re^{-1}\boldsymbol{\nabla}^2\mathbf{u} = \mathbf{f}$$

Resolvent operator (transfer function)

$$\mathbf{u} = \mathcal{R}(\omega)\mathbf{f}$$

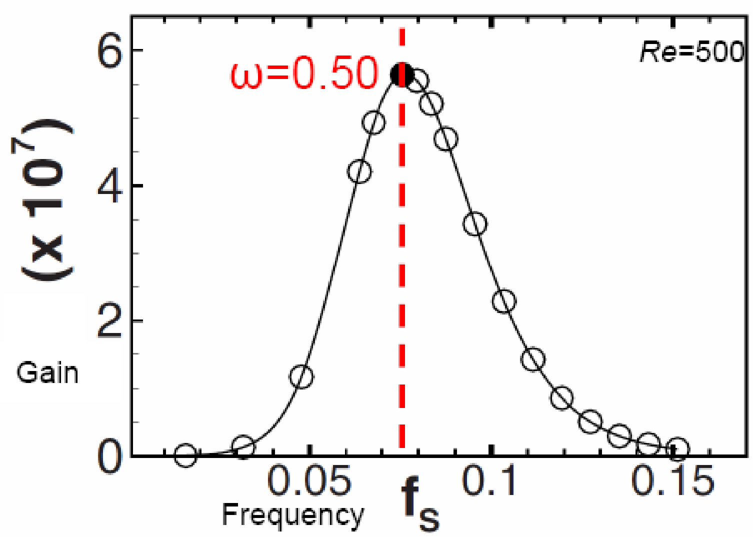
Harmonic gain

$$G^2(\omega) = \frac{||\mathbf{u}||^2}{||\mathbf{f}||^2} = \frac{(\mathcal{R}\mathbf{f} \mid \mathcal{R}\mathbf{f})}{(\mathbf{f} \mid \mathbf{f})} = \frac{(\mathcal{R}^\dagger\mathcal{R}\mathbf{f} \mid \mathbf{f})}{(\mathbf{f} \mid \mathbf{f})}$$

Method 2: transfer function

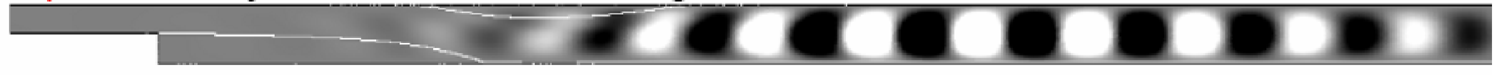
Optimal harmonic gain [Marquet &Sipp, 2010; Marquet *et al.*, 2010, priv. comm.]:

same preferred frequency,
large gain

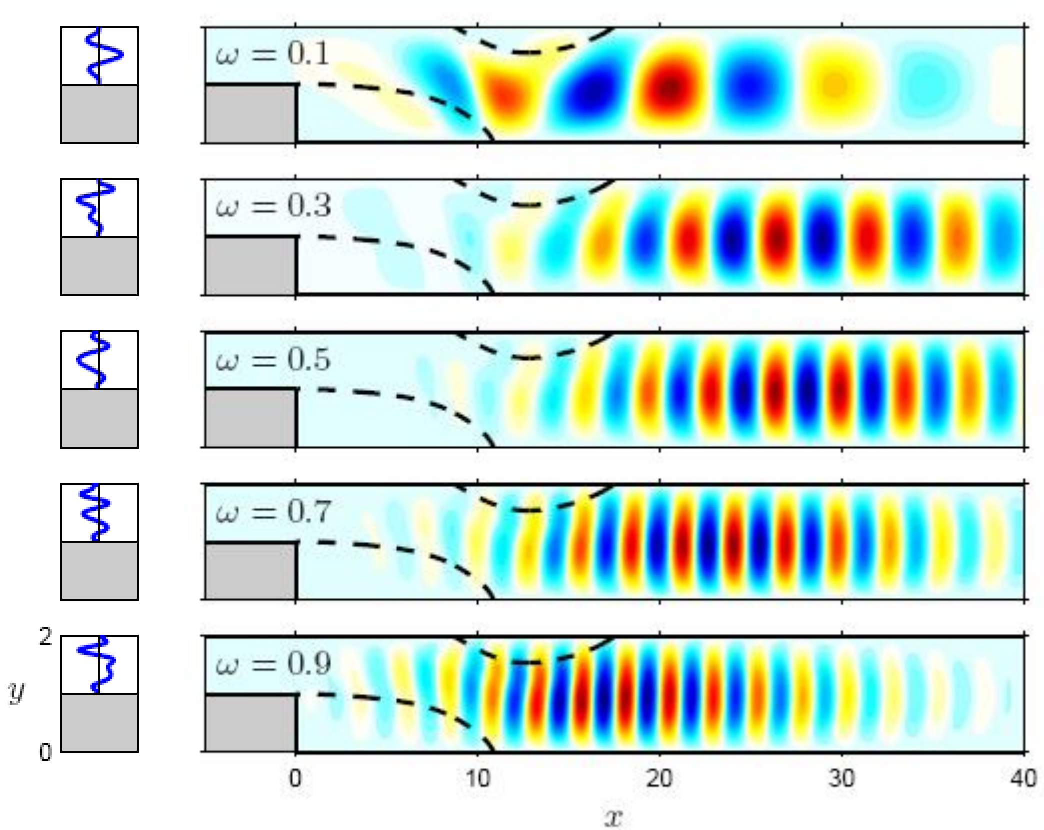


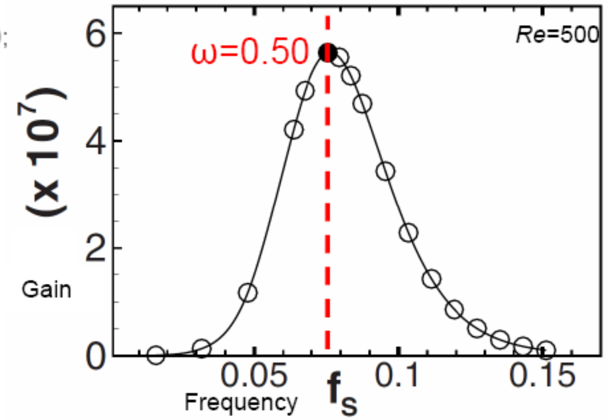
Optimal forcing

and optimal response very similar to transient growth:



Method 2: transfer function

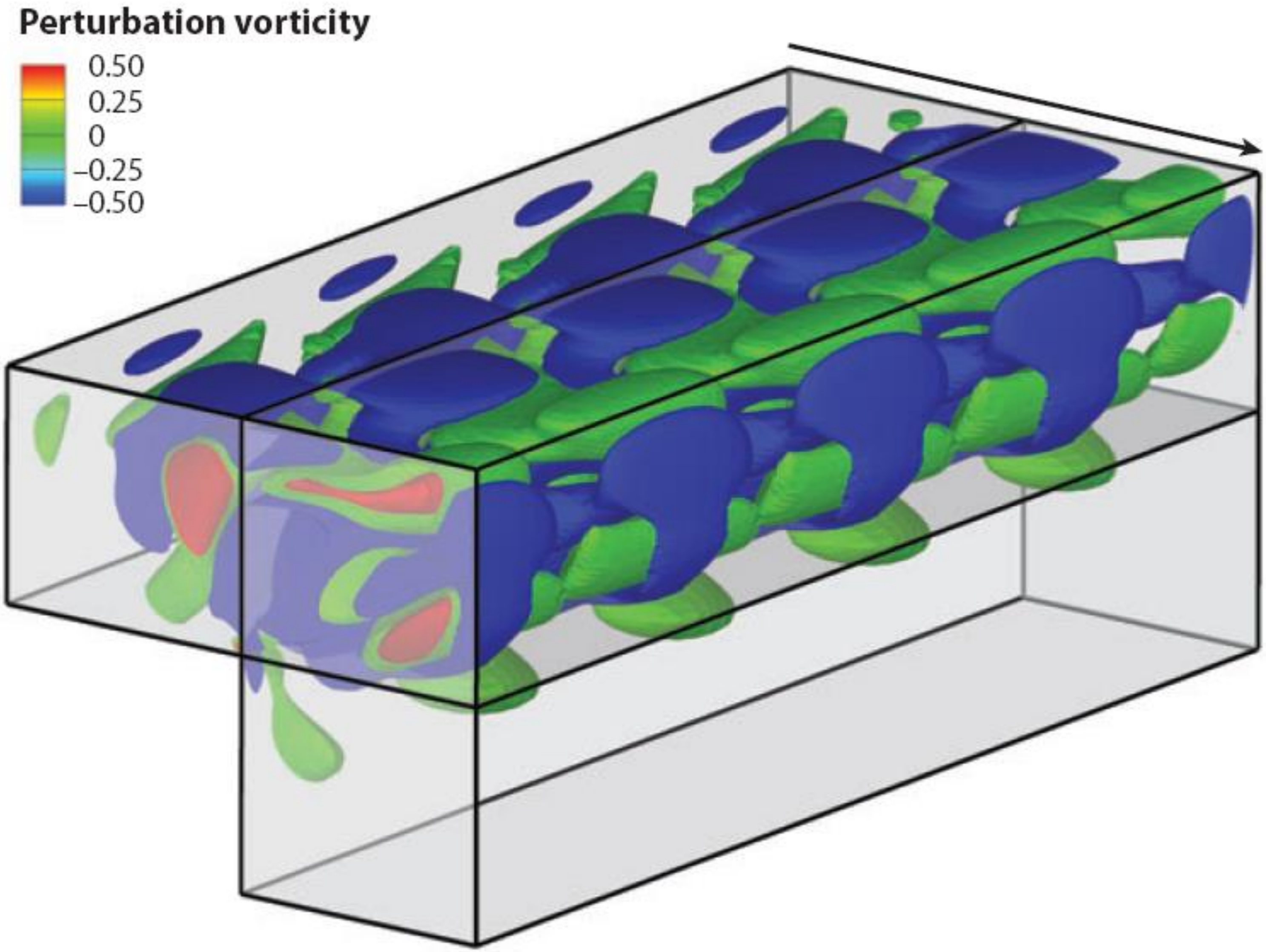




Global linear instability of flows in complex geometry

1. Gobal instabilty of a classical 2D flow
2. Less classical unstable flows
3. Stable flows

Global linear instability of flows in complex geometry



Da Vincente 2010