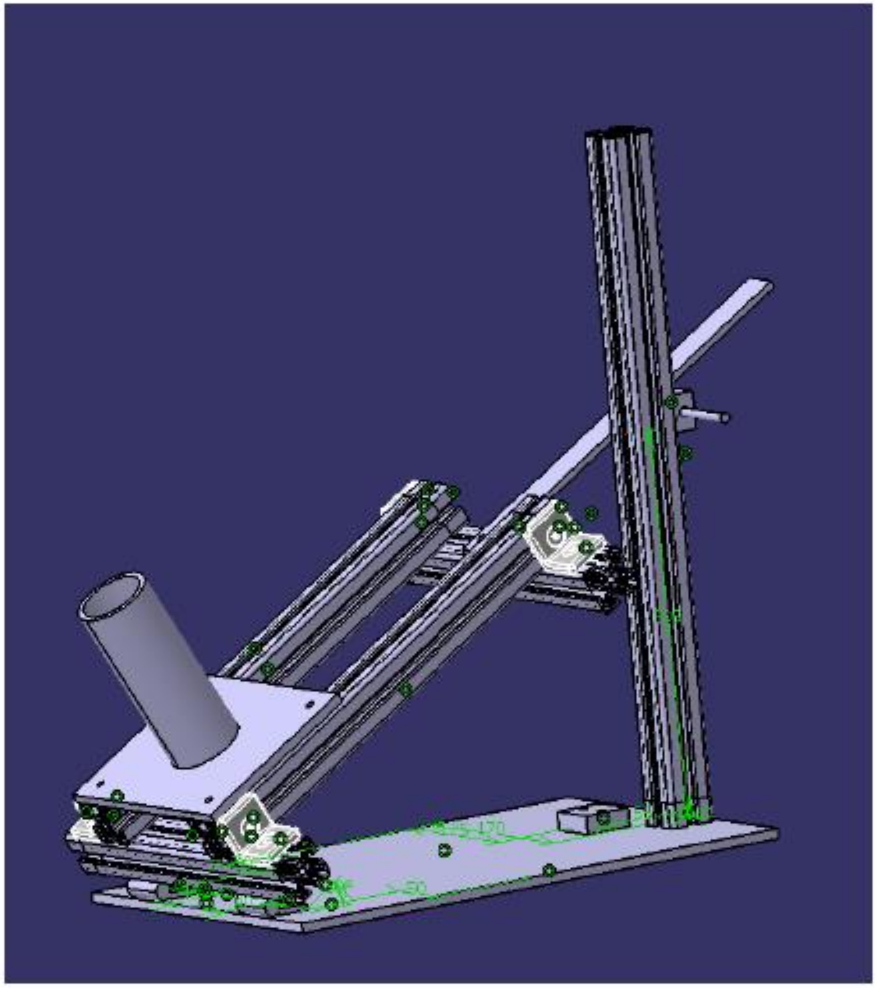
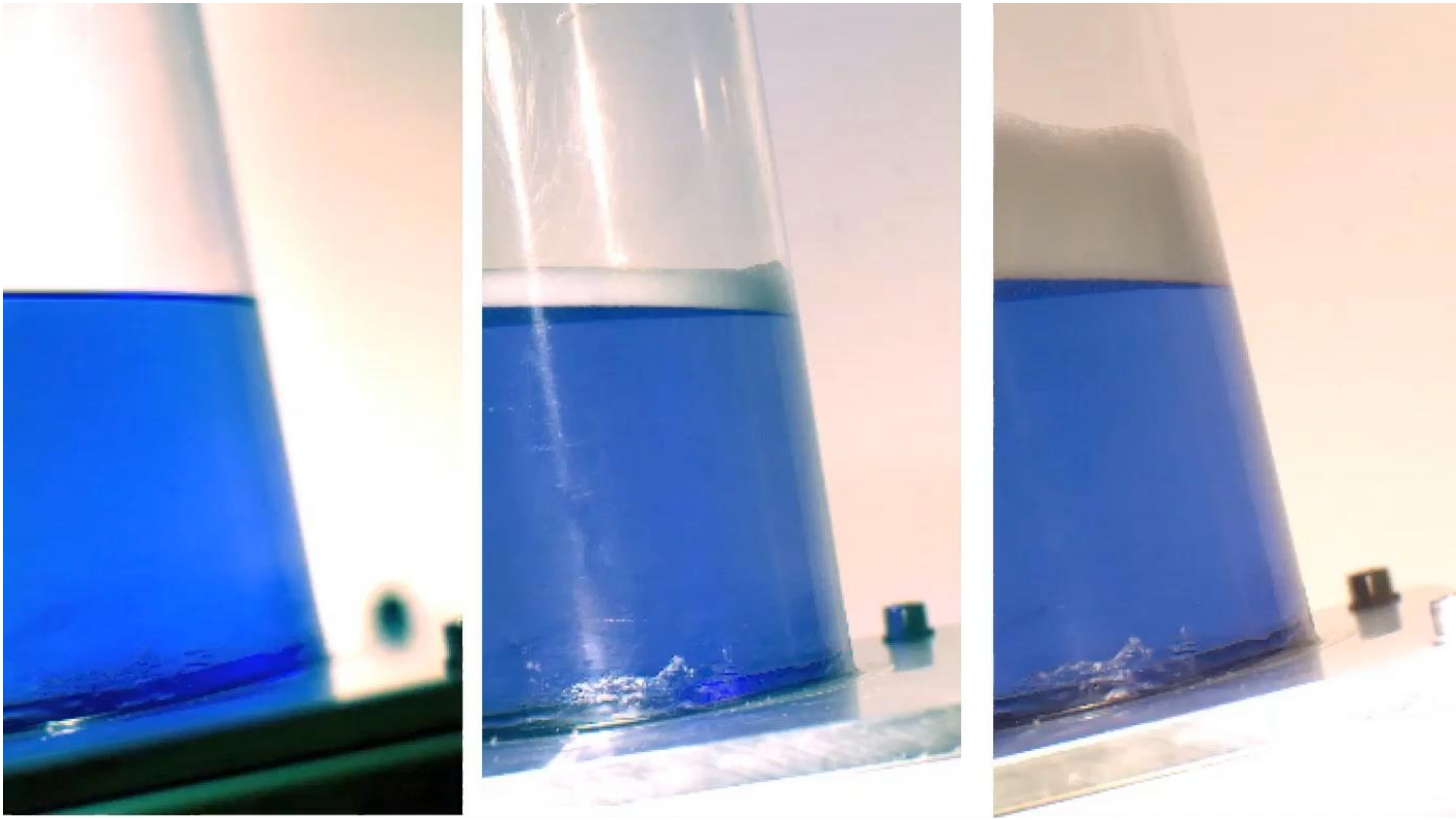
Impulse response

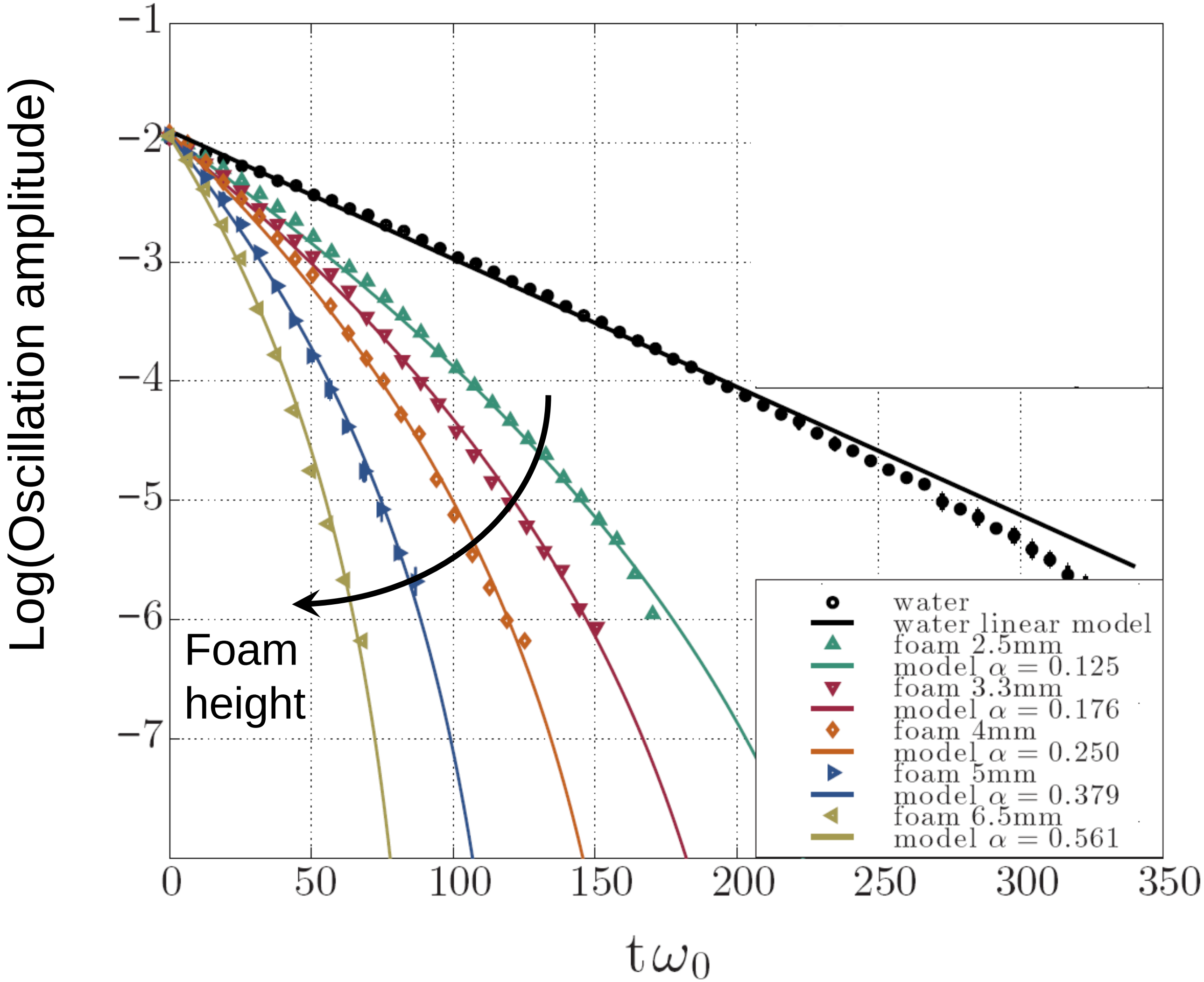


Damping by foam



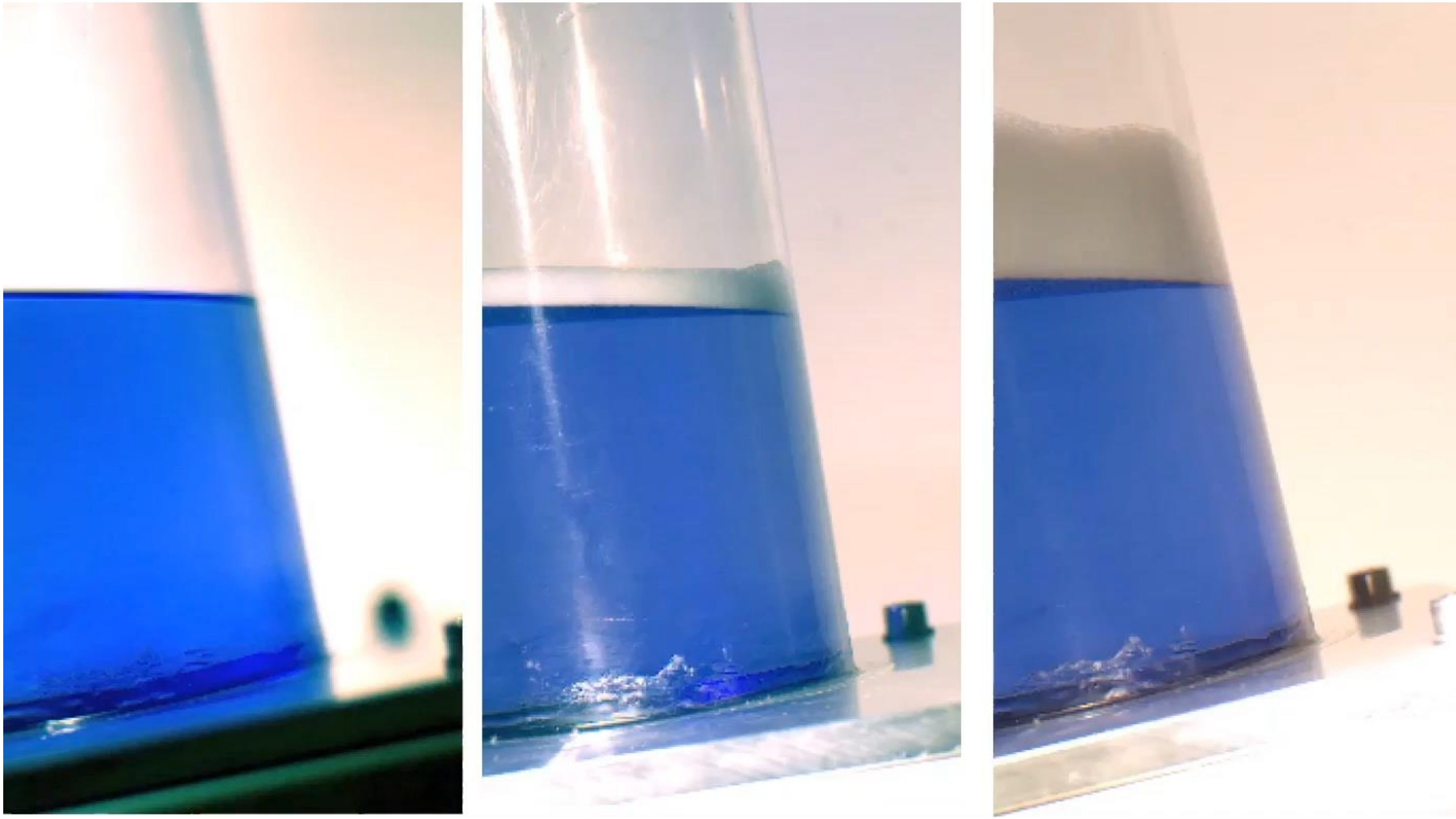
see also Sauret et al. 2015

Damping by foam



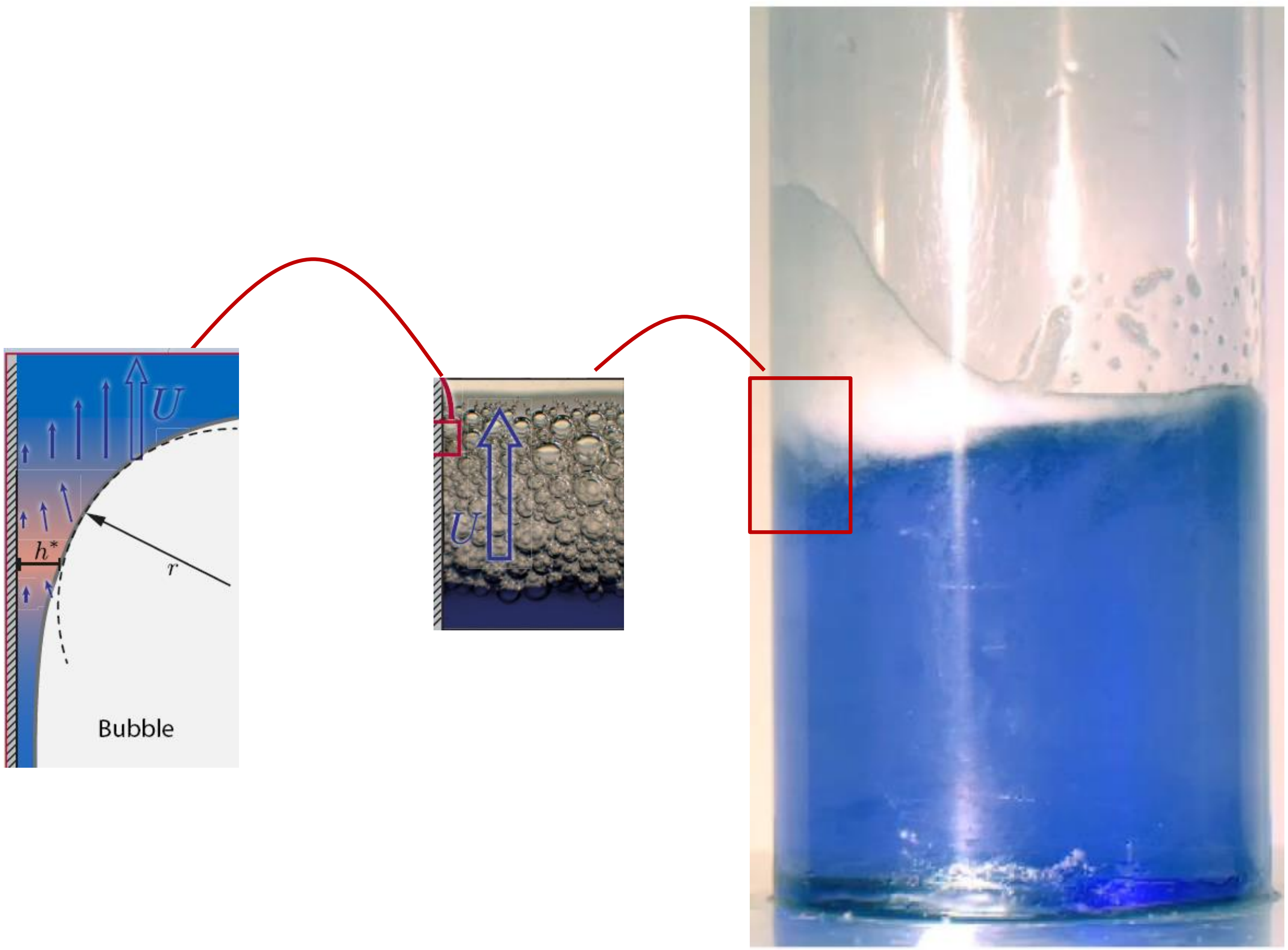
There must be something nonlinear!

Damping by foam to exacerbate nonlinear effects



see also Sauret et al. 2015

Damping by foam



A nonlinearly damped oscillator

$$\pi R^2(\rho\lambda + \rho_f h)\ddot{X} + F_g = F_w + F_b + F_s + F_f$$

- Wall friction $F_w \sim \dot{X}$
- Bulk dissipation $F_b \sim \dot{X}$
- Free surface dissipation $F_s \sim \dot{X}$
- Contact line friction $F_f \sim \dot{X}|\dot{X}|^{-1/3}$

$$\boxed{\ddot{X} + \omega_0^2 X = -2\sigma\omega_0\dot{X} - \alpha\dot{X}|\dot{X}|^{-1/3}}$$

$$\ddot{x} + \omega_0^2 x = -\epsilon \dot{x} |\dot{x}|^{-1/3}$$

Asymptotic expansion:

$$x = x_0 + \epsilon x_1 + \epsilon^2 x_2 + \ldots \qquad t = t + \epsilon t_1$$

$\boxed{\epsilon^0}$

$$\ddot{x}_0 + \omega_0^2 x_0 = 0$$

$$x_0 = A(t_1) e^{i\omega_0 t} + \overline{A}(t_1) e^{-i\omega_0 t}$$

$\boxed{\epsilon^1}$

$$\ddot{x}_1 + \omega_0^2 x_1 = -2 \frac{\partial^2 x_0}{\partial t \partial t_1} - \left(\frac{\partial x_0}{\partial t} \right) \left| \frac{\partial x_0}{\partial t} \right|^{-1/3}$$

Let examine the terms on the r.h.s: The first as before:

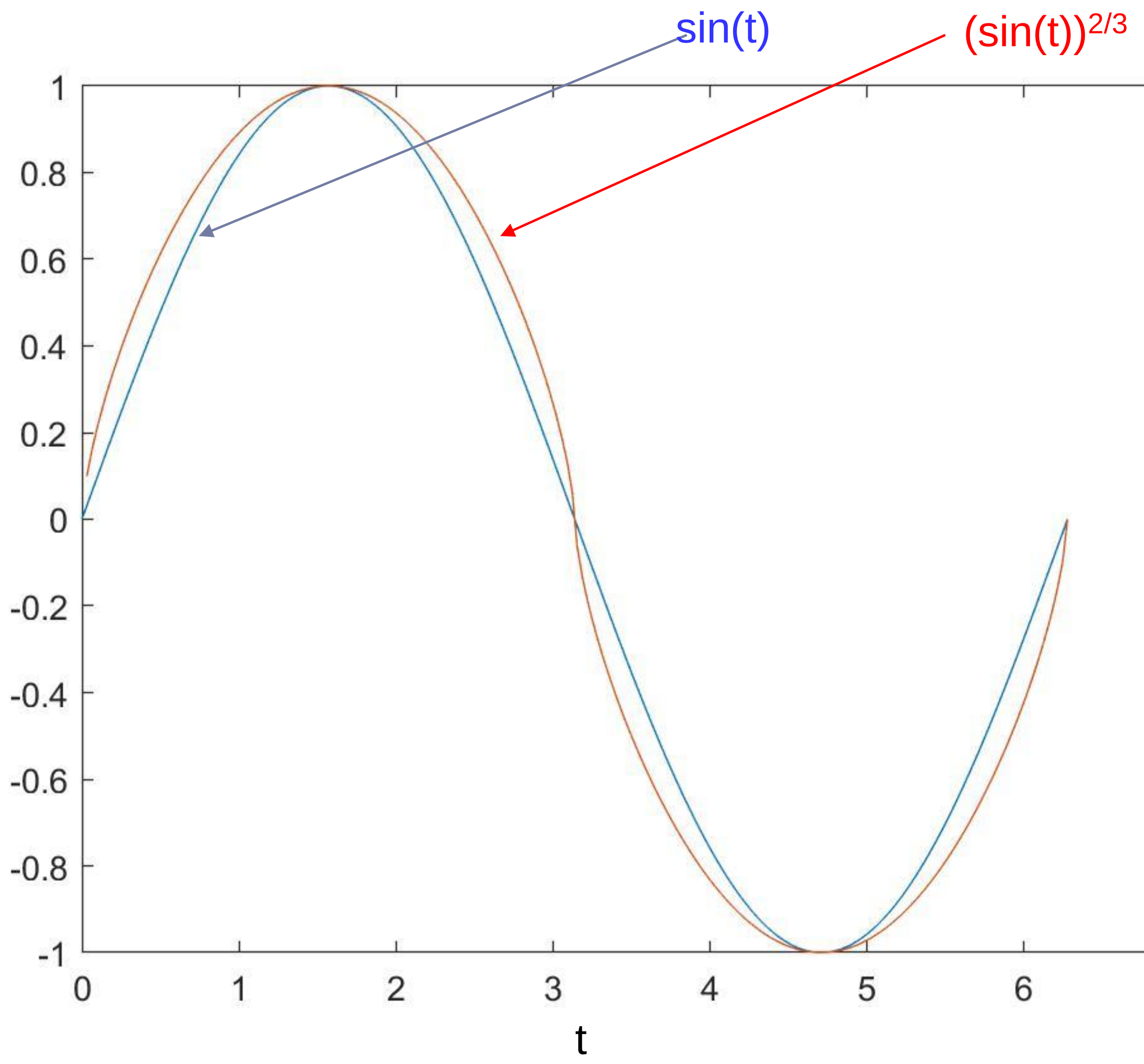
$$2\frac{\partial^2 x_0}{\partial t \partial t_1} = 2i\omega_0 A' e^{i\omega_0 t} + c.c. = 2i\omega_0(a' + i\beta' a)e^{i\beta}e^{i\omega_0 t} + c.c.$$

The second is non-linear, by using $A = ae^{i\beta}$:

$$x_0|x_0|^{-1/3} = (i\omega_0 A e^{i\omega_0 t} - i\omega_0 \overline{A} e^{-i\omega_0 t})|i\omega_0 A e^{i\omega_0 t} - i\omega_0 \overline{A} e^{-i\omega_0 t}|^{-1/3} =$$
$$(i\omega_0 a e^{i\omega_0 t + i\beta} - i\omega_0 a e^{-i\omega_0 t - i\beta})|i\omega_0 a e^{i\omega_0 t + i\beta} - i\omega_0 a e^{-i\omega_0 t - i\beta}|^{-1/3} =$$
$$\omega_0^{2/3} a^{2/3} \underbrace{(i(e^{i\phi} - e^{-i\phi}))|i(e^{i\phi} - e^{-i\phi})|^{-1/3}}_{f(\phi)}$$

where $\phi = \omega_0 t + \beta$, the last term can be transformed with Fourier series

$$f(\phi) = (-2\sin(\phi))| - 2\sin(\phi)|^{-1/3} =$$



sin(t)
(sin(t))^{2/3}
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6
7
t

$$f(\phi) = (-2\sin(\phi))|-2\sin(\phi)|^{-1/3} = \sum_{-\infty}^{\infty} c_n e^{in\phi} = c_1 e^{i\phi} + c_{-1} e^{-i\phi} + NST$$

where $c_{\pm 1}$ are the Fourier series coefficient given by $c_{\pm 1} = \frac{1}{2\pi}\int_0^{2\pi} f(\phi) e^{\mp i\phi} \mathrm{d}\phi$, with $c_{-1} = \overline{c}_1$. $c_1 \approx 0.85i = c_{1i} i$.

$$\dot{x}_0 |\dot{x}_0|^{-1/3} = \omega_0^{2/3} a^{2/3} e^{i\beta} c_{1i} i e^{i\omega_0 t} + c.c. + NST$$

The amplitude equation is obtained:

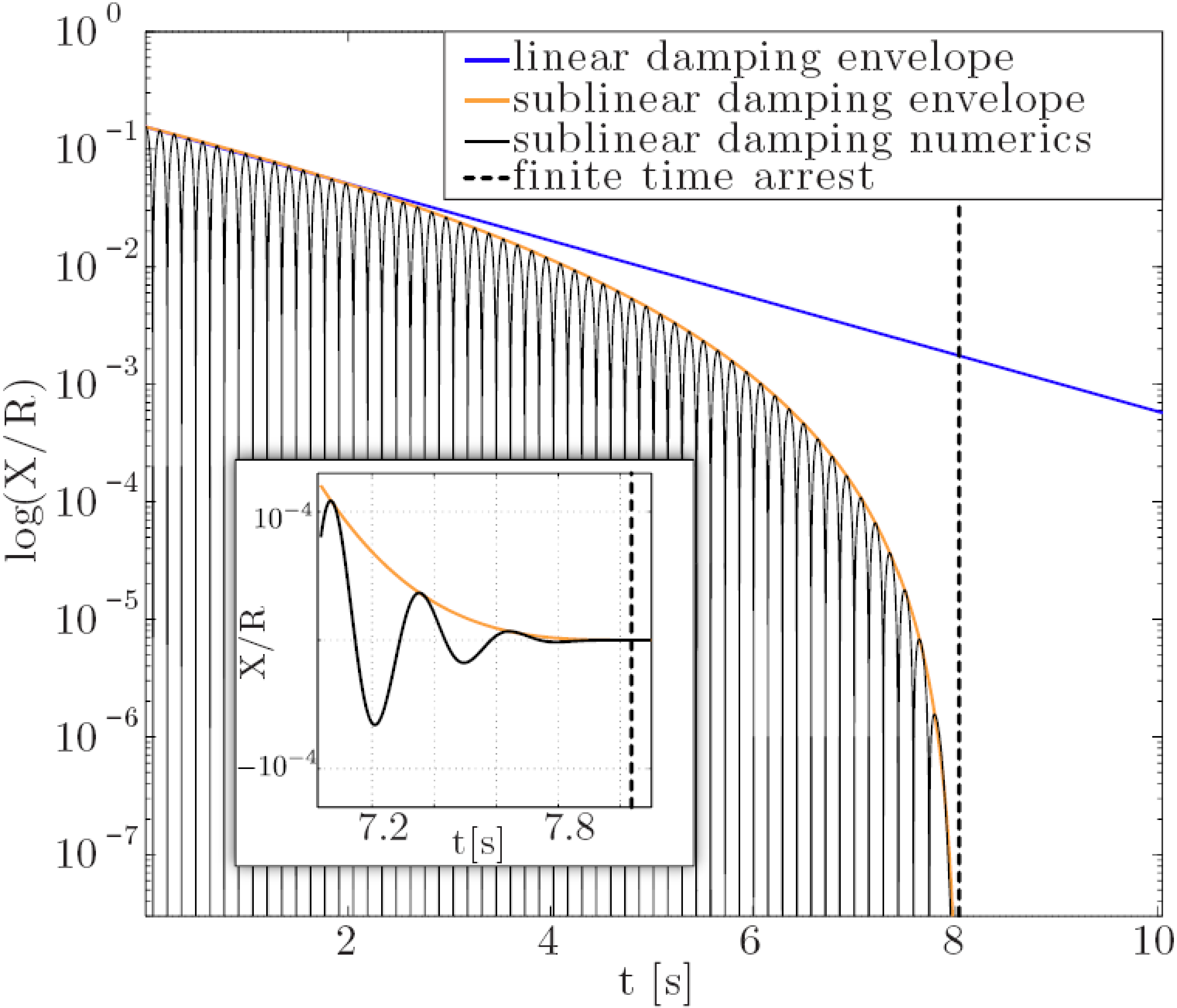
$$2i\omega_0 (a' + ia\beta') e^{i\beta} e^{i\omega_0 t} + \omega_0^{2/3} a^{2/3} e^{i\beta} c_{1i} i e^{i\omega_0 t} + c.c = 0$$

By splitting real and imaginary parts:

$$\begin{cases} 2\omega_0 a' + c_{1i}\omega_0^{2/3} a^{2/3} = 0 \Rightarrow \int_{a_0}^{a} a^{-2/3}\mathrm{d}a = -\frac{c_{1i}}{2\omega_0^{2/3}} \int_0^{t_1} \mathrm{d}t_1 \Rightarrow a(t_1) = (-\frac{c_{1i}t_1}{6\omega_0^{1/3}} + a_0^{1/3})^3 \\ 2\omega_0 a\beta' = 0 \Rightarrow \beta = \beta_0 \end{cases}$$

With the finite time singularity:

$$t^*_{inv} = \frac{6\omega_0^{1/3}}{\epsilon c_{1i}} a_0^{1/3}$$


linear damping envelope
sublinear damping envelope
sublinear damping numerics
finite time arrest
log(X/R)
t [s]
X/R
t[s]
7.2
7.8
10^-4
−10^-4
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
2
4
6
8
10

Damping by foam