

Spatial analysis of Kelvin-Helmholtz

We have found for the dispersion relation of the Kelvin-Helmholtz instability of two inviscid, equally dense and immiscible streams of velocities U_1 and U_2

$$(\omega - k\bar{U})^2 = -\left(\frac{\Delta U k}{2}\right)^2 + \frac{\gamma}{2\rho}k^3 \quad (1)$$

where $\bar{U} = (U_1 + U_2)/2$ and $\Delta U = U_1 - U_2$.

1. Show that this dispersion relation can be put, once suitable made dimensionless, in the form

$$(\omega - k)^2 = R(-k^2 + k^3). \quad (2)$$

What length scale has been used? What time scale? What is the expression of R ?

2. What is the cut-off wavelength? the maximum growth-rate? its associated wavenumber?
3. Write a simple matlab code to find the spatial branches of this dispersion relation. Use a small value of R to test it.
4. Compare with the results from Gaster's transform.
5. Determine the value of R where the transition from absolute to convective takes place.