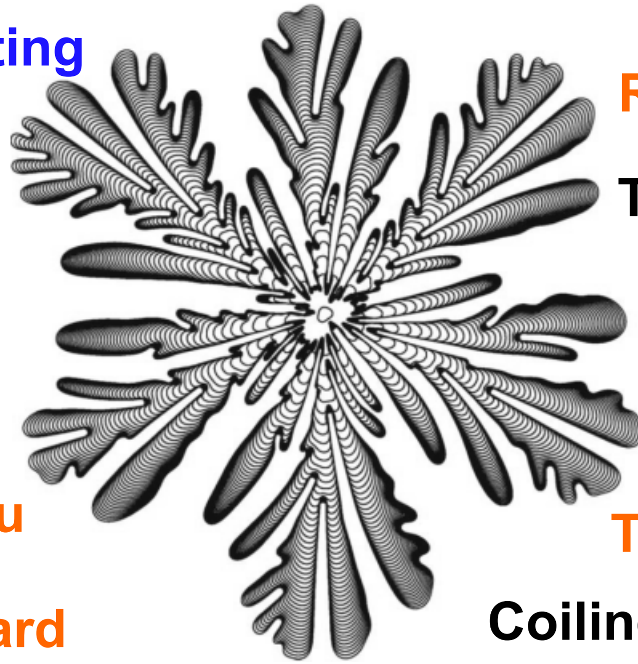
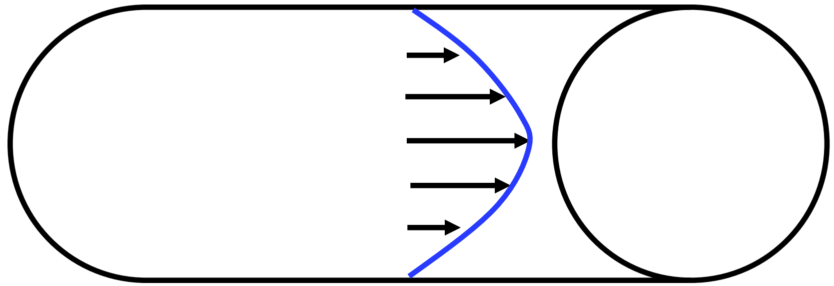
Most flows are unstable...

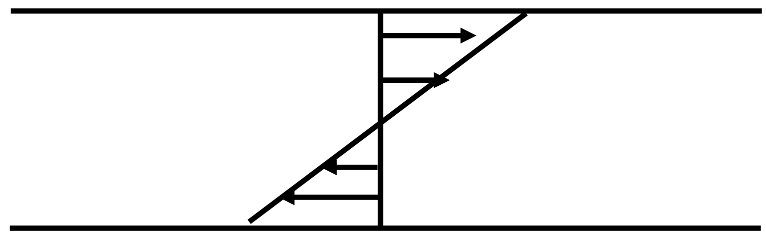
Vortex shedding
Saffman-Taylor
Flow separation
Tollmien-Schlichting
Rayleigh-Taylor
Lift-up and Streaks
Traffic waves
Meandering instability
Gravito-capillary waves
Taylor-Couette
Rayleigh-Plateau
Tearing instability
Rayleigh-Benard
Coiling instability
Benard-Marangoni
Kelvin-Helmholtz



Hagen Poiseuille



Couette



Flow type	$\cancel{Re_{ener}}$	Re_{exp}	Re_{lin}
Pipe flow	~~81.5~~	≈ 2000	∞
Plane Poiseuille flow	~~49.6~~	≈ 1000	5772
Plane Couette flow	~~20.7~~	≈ 360	∞

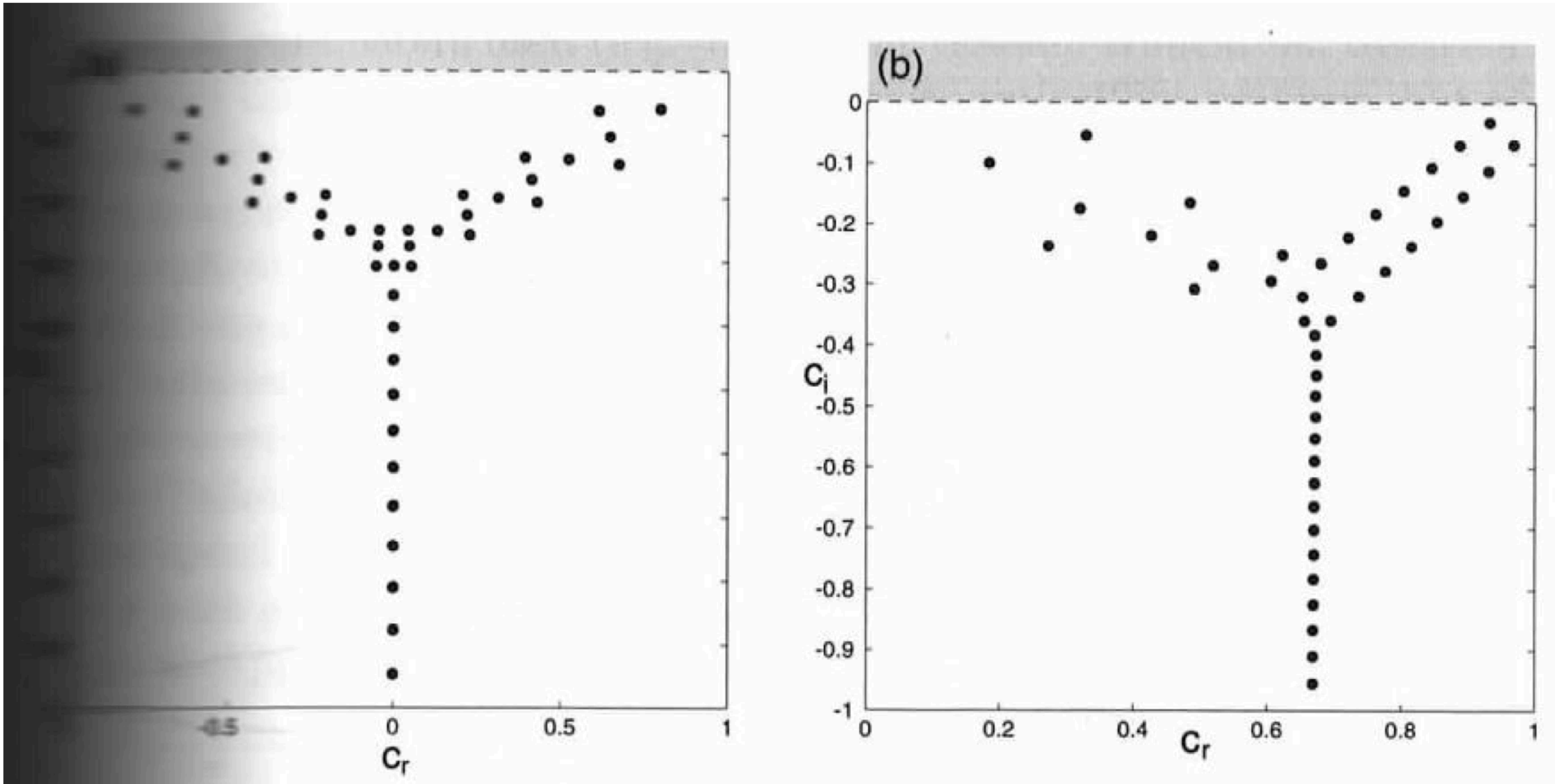
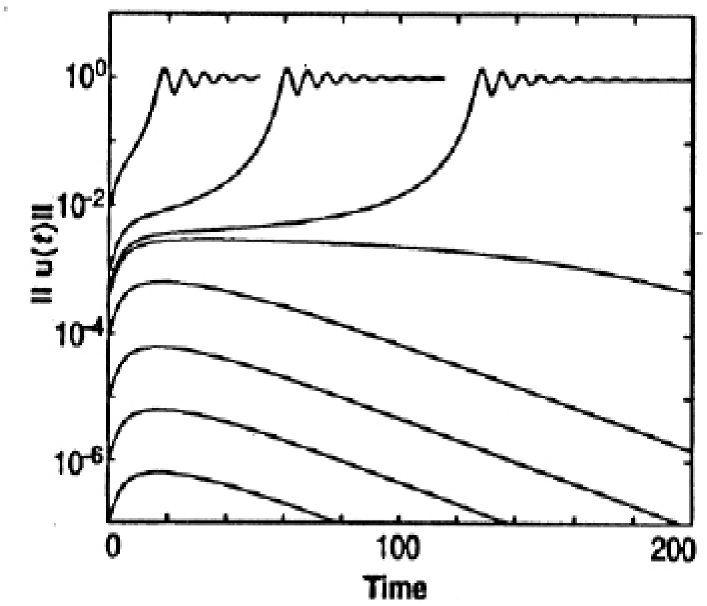
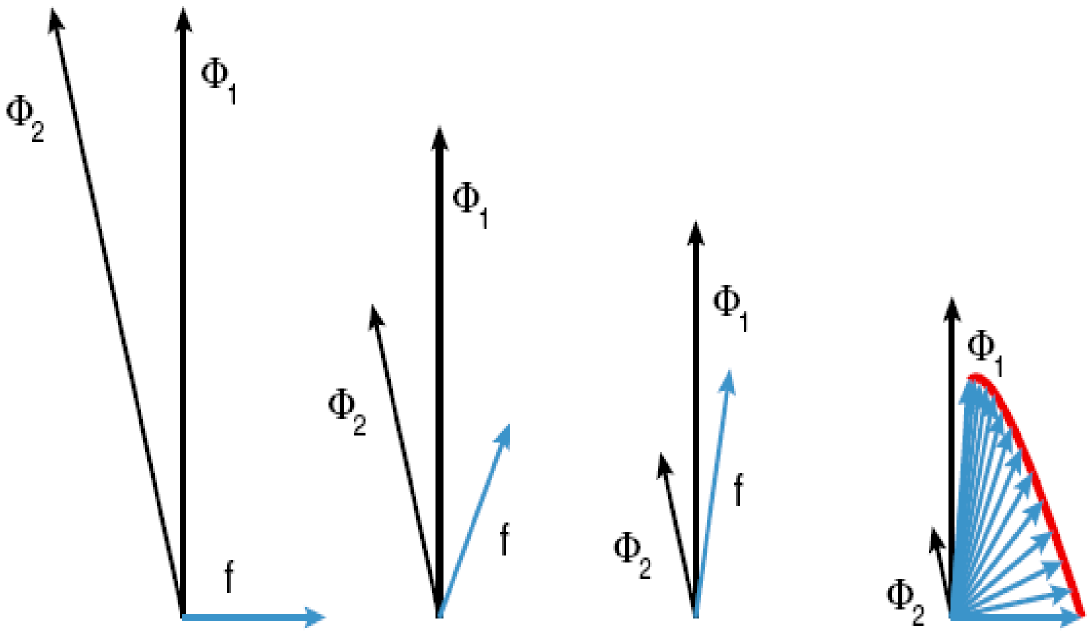


FIGURE 3.3. Spectrum of plane Couette and pipe flow. (a) Plane Couette flow for $\alpha = 1, \beta = 1, \mathrm{Re} = 1000$; (b) Pipe flow for $\alpha = 1, n = 1, \mathrm{Re} = 5000$.

Croissance transitoire et bye pass transition



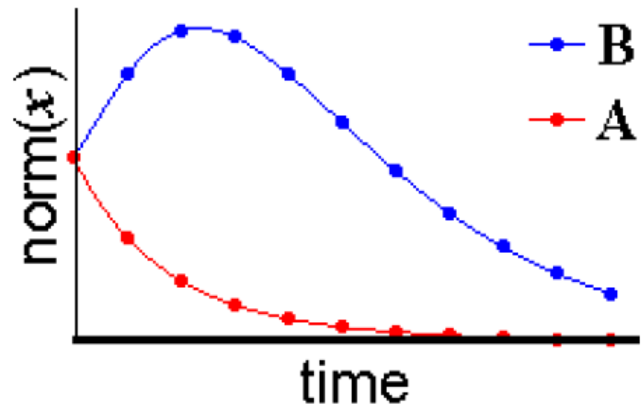
Stabilité conditionnelle; transition sous-critique
Trefethen 1993

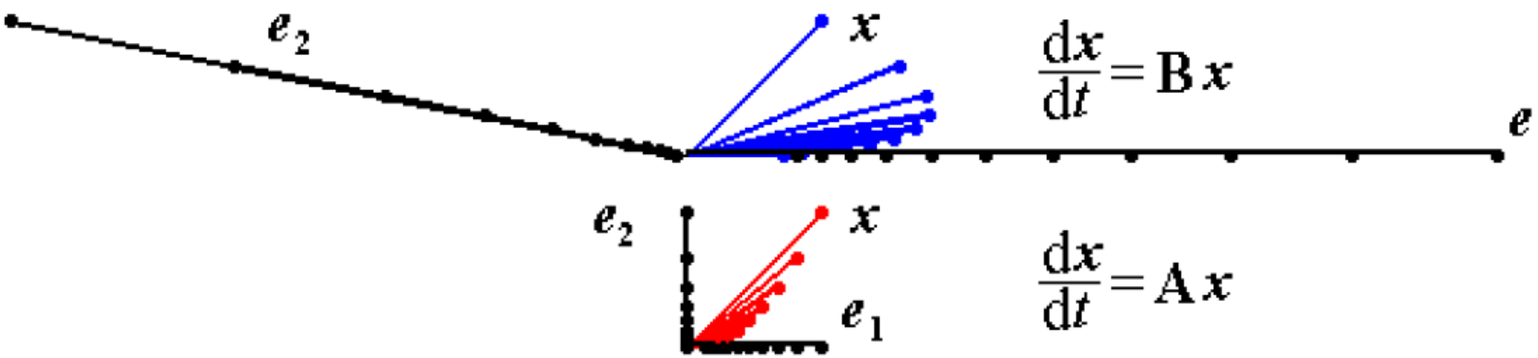


Φ_2
Φ_1
f
Φ_2
Φ_1
f
Φ_2
Φ_1
f
Φ_1
Φ_2

$$\mathbf{B} = \begin{bmatrix} -1 & 5 \\ 0 & -2 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$





Definition of optimal gain of a linear system

$$\frac{dq}{dt} = \mathbf{L}q$$

$$q(t = 0) = q_0$$

Matrix exponential

$$q(t) = \exp(\mathbf{L}t)q_0$$

Definition of optimal gain of a linear system

Linear system $$\frac{dq}{dt} = \mathbf{L}q$$

Initial condition $$q(t=0) = q_0$$

Optimal gain $$G(t) = \max_{q_0} \frac{||q||}{||q_0||} \text{ ?}$$

norm

Definition of optimal gain of a linear system

Optimal gain $G(t)$

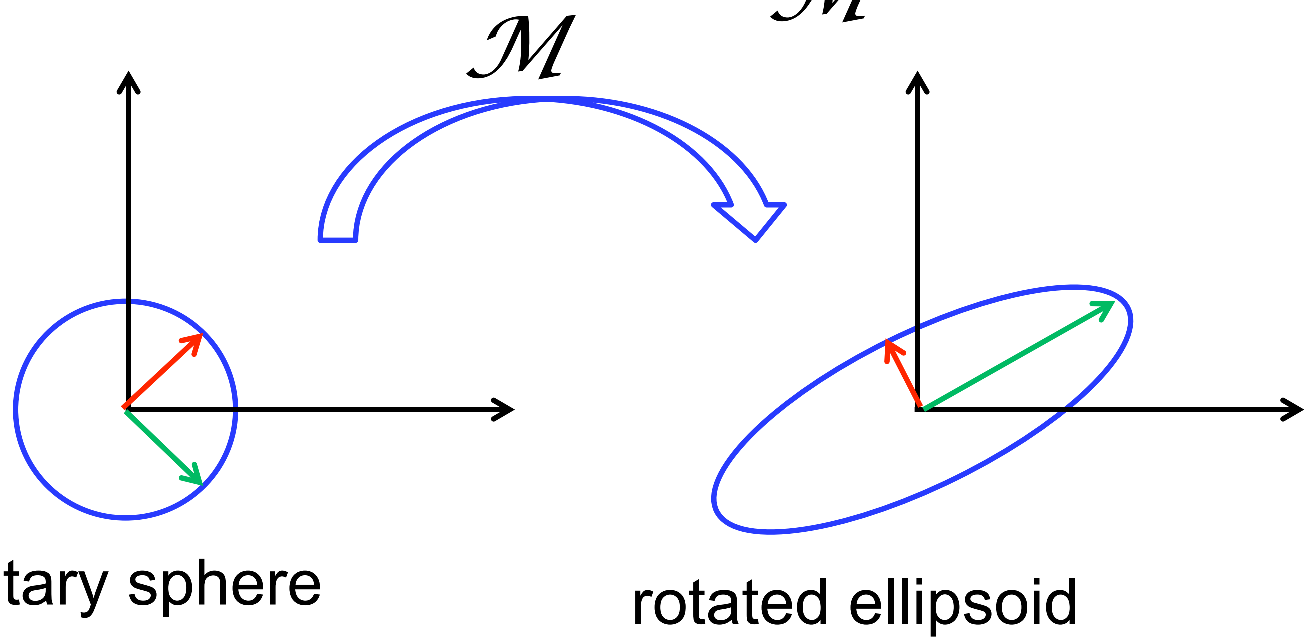
$$G(t) = \max_{q_0} \frac{||q||}{||q_0||}$$

$$= \max_{q_0} \frac{||\exp(t\mathcal{L})q_0||}{||q_0||}$$

$$= ||\exp(t\mathcal{L})||$$

How to compute a matrix 2-norm?

Optimal gain $$G(t) = \underbrace{\|\exp(t\mathcal{L})\|}_{\mathcal{M}}$$



Optimal gain is associated to a norm

Kinetic energy written in v,η form

iαu=Dv-iβw and η=iβu-iαw

$$E(t) = \frac{1}{2k^2}\int_{\Omega}\left[|\mathcal{D}v|^2 + k^2|v|^2 + |\eta|^2\right] \, d\Omega$$

Optimal gain is associated to a norm

Kinetic energy written in v,η form

iαu=Dv-iβw and η=iβu-iαw

$$
\begin{aligned}
E(t) &= \frac{1}{2k^2}\int_{\Omega}\left[|\mathcal{D}v|^2 + k^2|v|^2 + |\eta|^2\right]\, d\Omega \\
&= \|q\|^2 = \frac{1}{2k^2}\int_{\Omega}\begin{pmatrix} v \\ \eta \end{pmatrix}^H \begin{pmatrix} -\mathcal{D}^2 + k^2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} v \\ \eta \end{pmatrix}\, d\Omega \\
&= \frac{1}{2k^2}\int_{\Omega} q^H M q\, d\Omega
\end{aligned}
$$

energy matrix

Definition of optimal gain of a linear system

Cholevski dec. $M = F^H F$

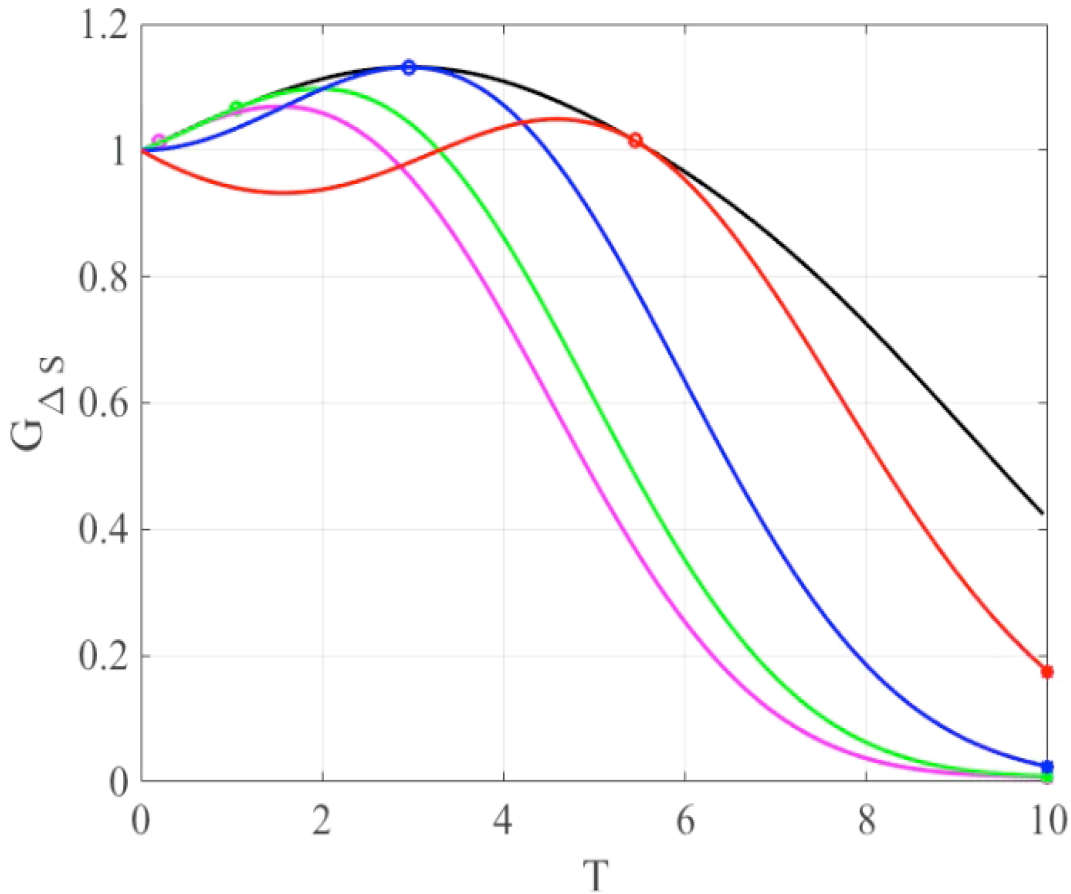
$$||q||^2 = \frac{1}{2k^2}\int_\Omega q^H F^H F q \, d\Omega = \frac{1}{2k^2}\int_\Omega (Fq)^H F q \, d\Omega$$

Optimal gain

$$\begin{aligned} G(t) &= \max_{q_0} \frac{||q||_E}{||q_0||_E} = \max_{q_0} \frac{||Fq||_2}{||Fq_0||_2} = \max_{q_0} \frac{||F\exp(tL)q_0||_2}{||Fq_0||_2} \\ &= \max_{q_0} \frac{||F\exp(tL)F^{-1}Fq_0||_2}{||Fq_0||_2} = ||F\exp(tL)F^{-1}||_2 \end{aligned}$$

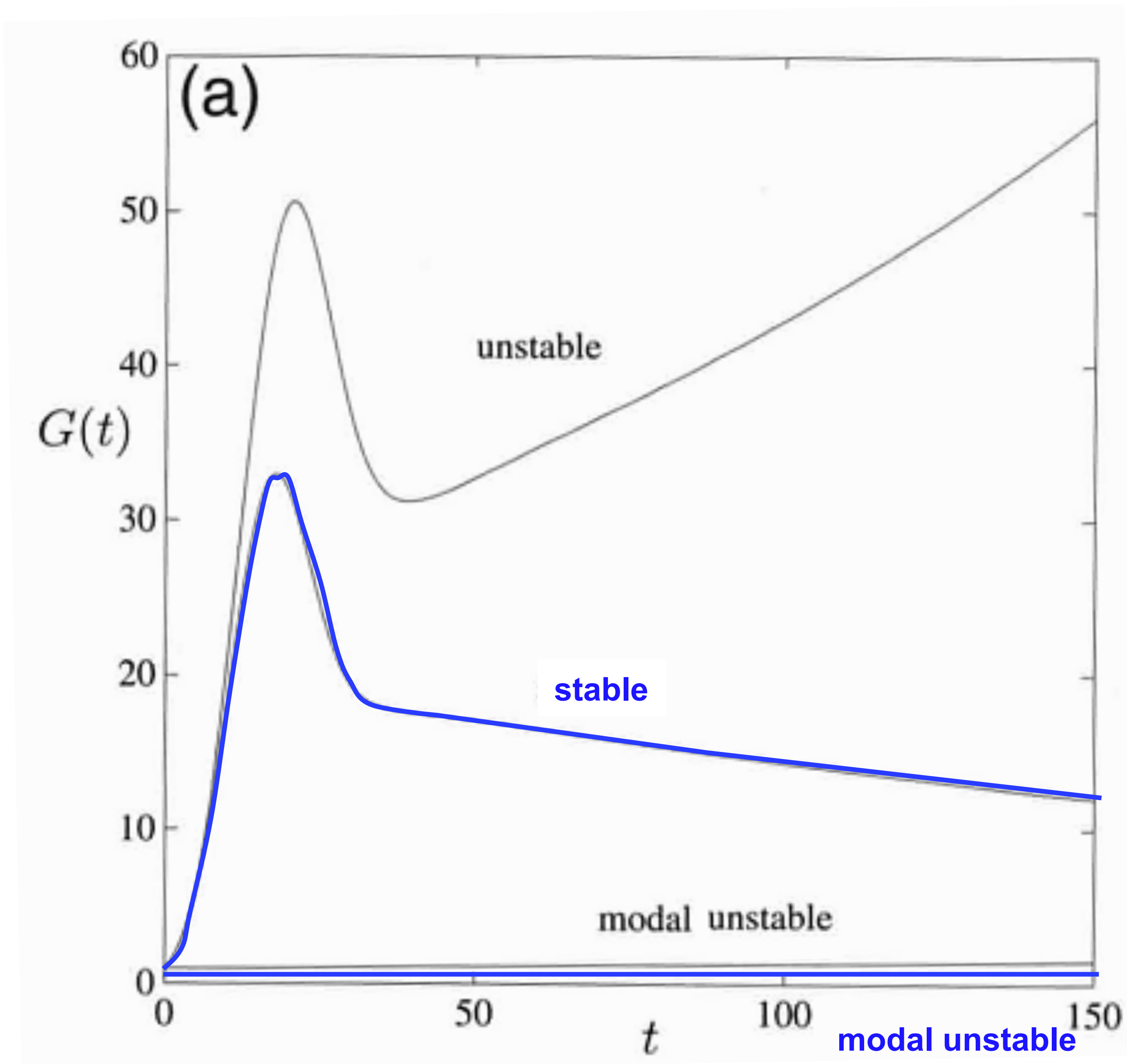
$=q'_0$

Optimal Gain (enveloppe)

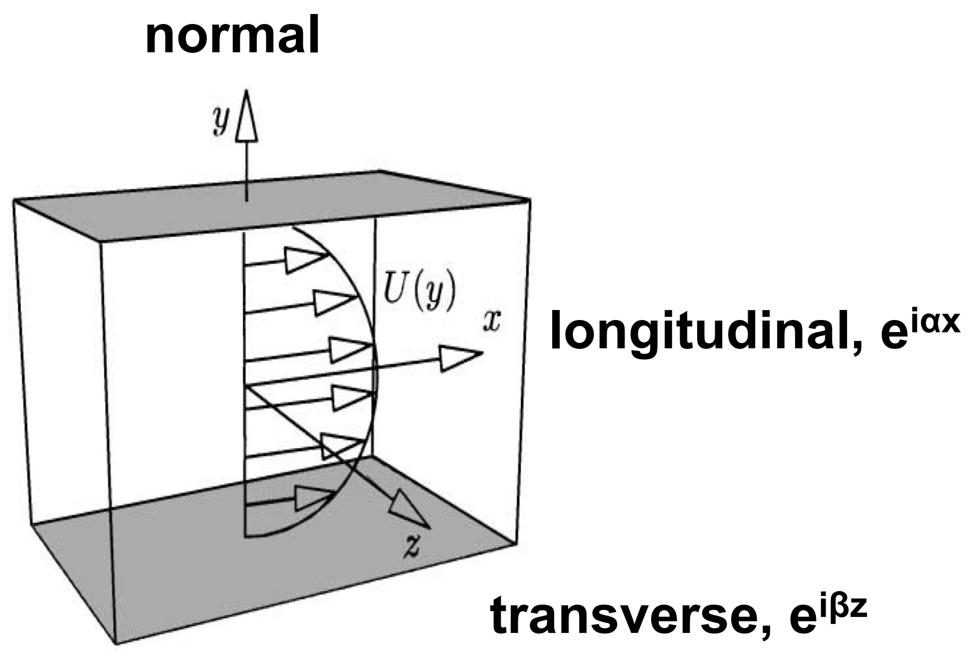


$$G(t) = \max_{q_0} \frac{||q||_E}{||q_0||_E} = \max_{q_0} \frac{||Fq||_2}{||Fq_0||_2} = \max_{q_0} \frac{||F\exp(tL)q_0||_2}{||Fq_0||_2}$$

$$= \max_{q_0} \frac{||F\exp(tL)F^{-1}Fq_0||_2}{||Fq_0||_2} = ||F\exp(tL)F^{-1}||_2$$

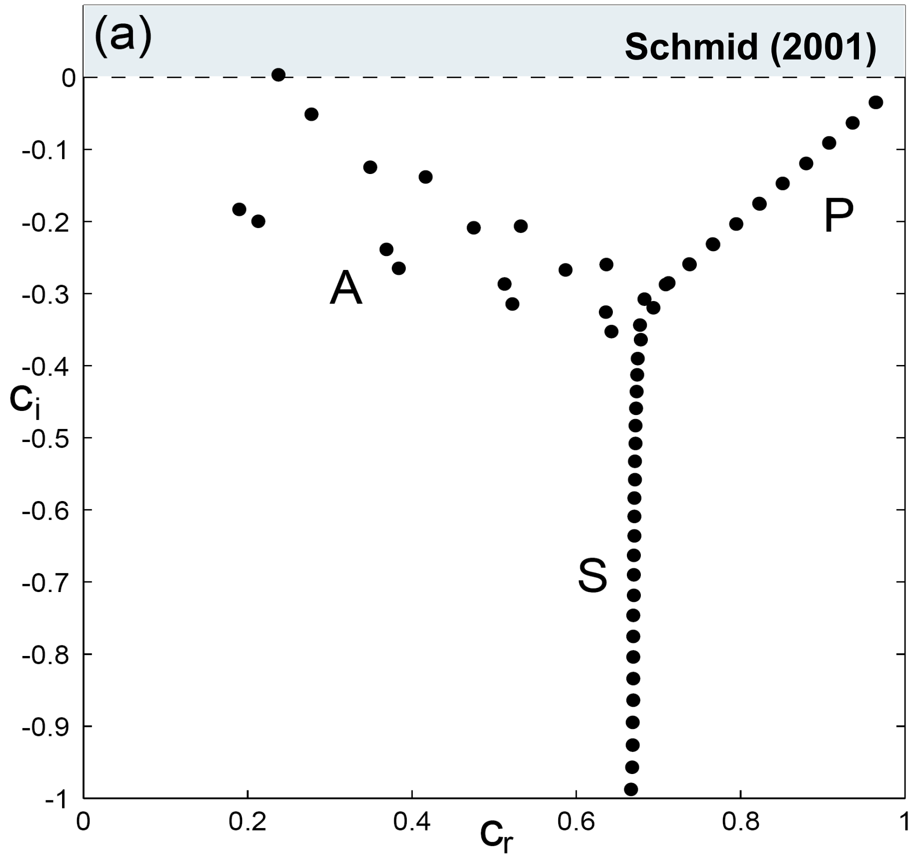

(a)
60
50
40
30
20
10
0
$G(t)$
unstable
stable
modal unstable
0
50
100
150
t
modal unstable

Plane Poiseuille flow



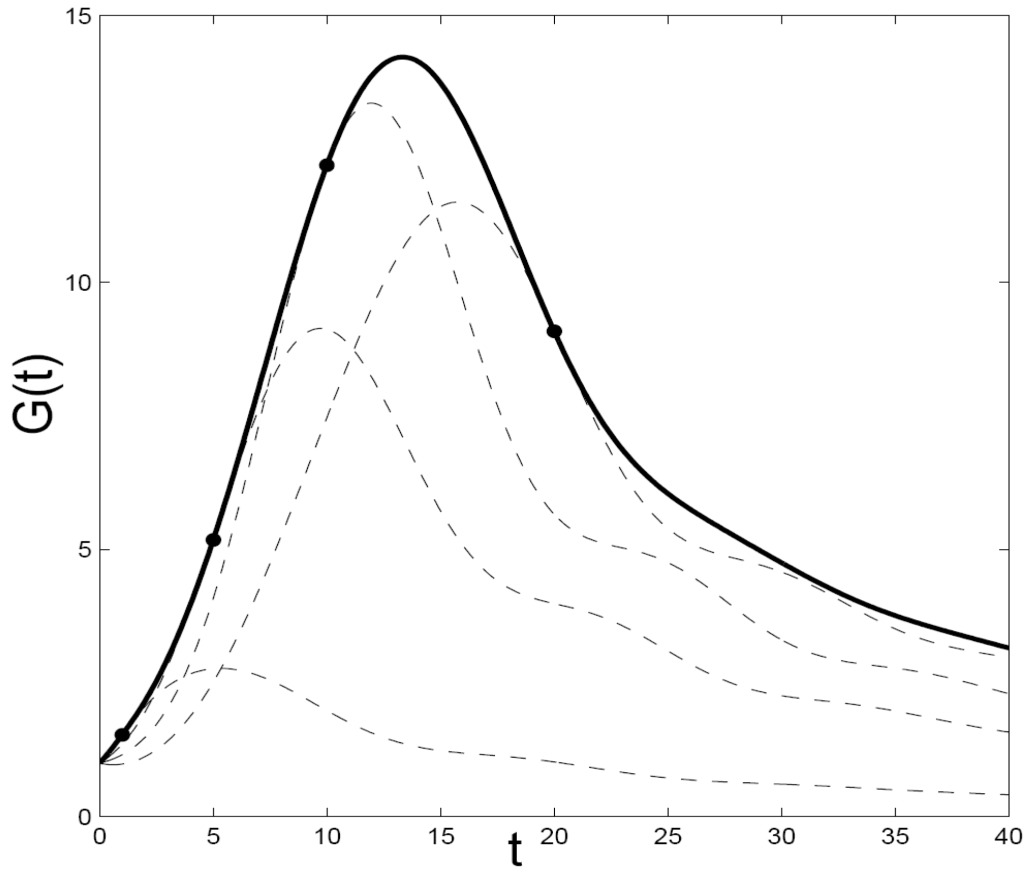
Linearly stable until Re=5772, but the transition is observed experimentally close to Re=1000-2000!

Plane Poiseuille flow - stability



Spectrum for plane Poiseuille flow for $\alpha = 1, \beta = 0, Re = 10000$.

Orr mechanism (2D)

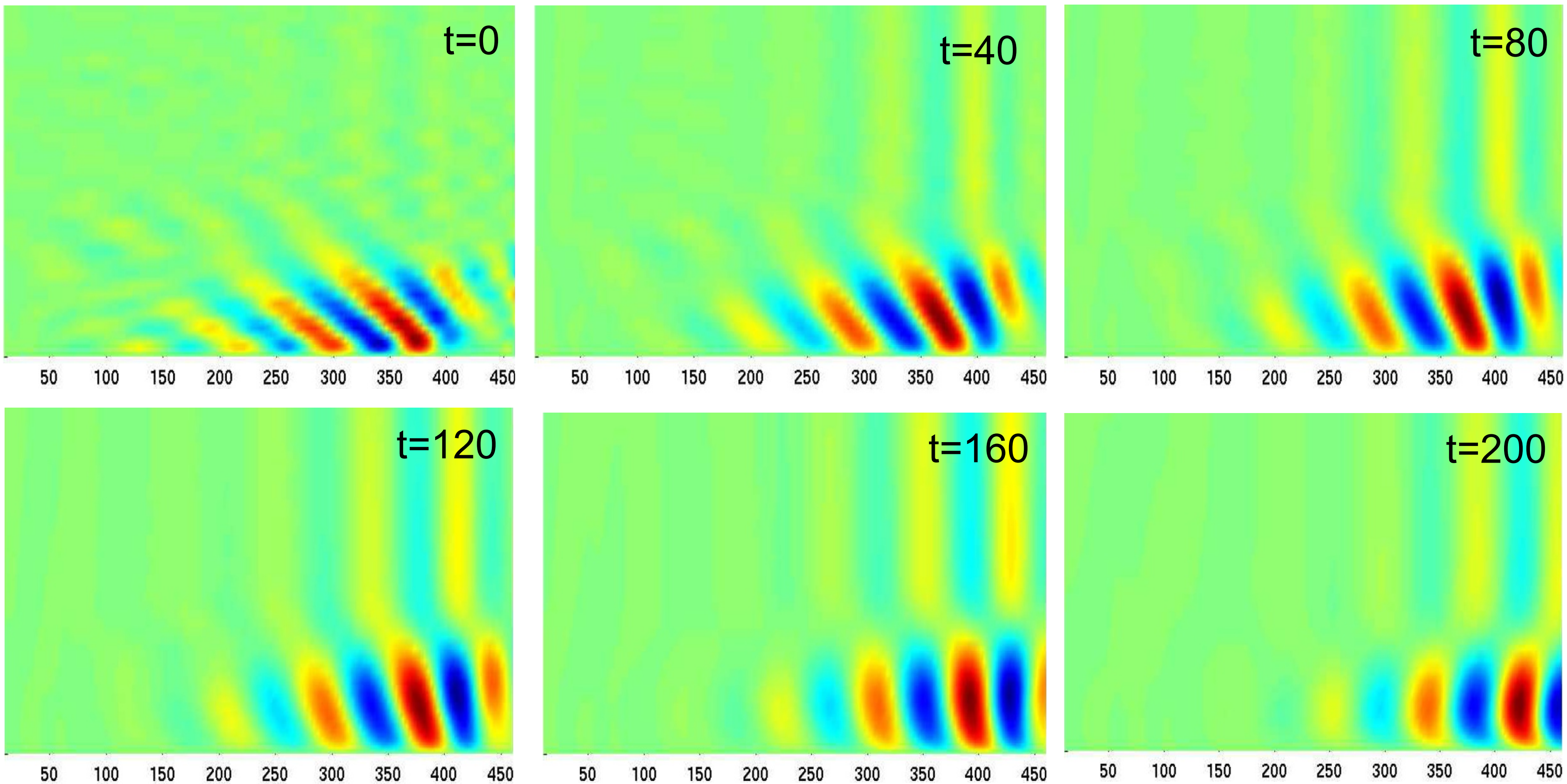


Amplification $G(t)$ for Poiseuille flow with $\mathrm{Re} = 1000, \alpha = 1$ (solid line)

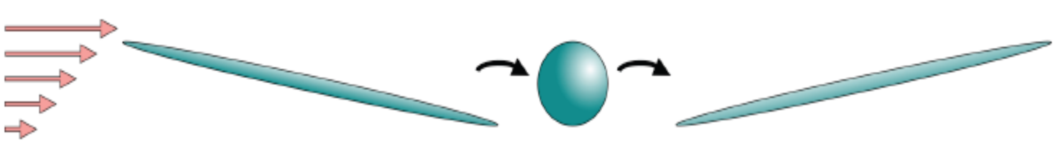
« By-pass transition »

Trefethen et al (1993), Buttler & Farrell (1993) , Schmid & Henningson (2001)

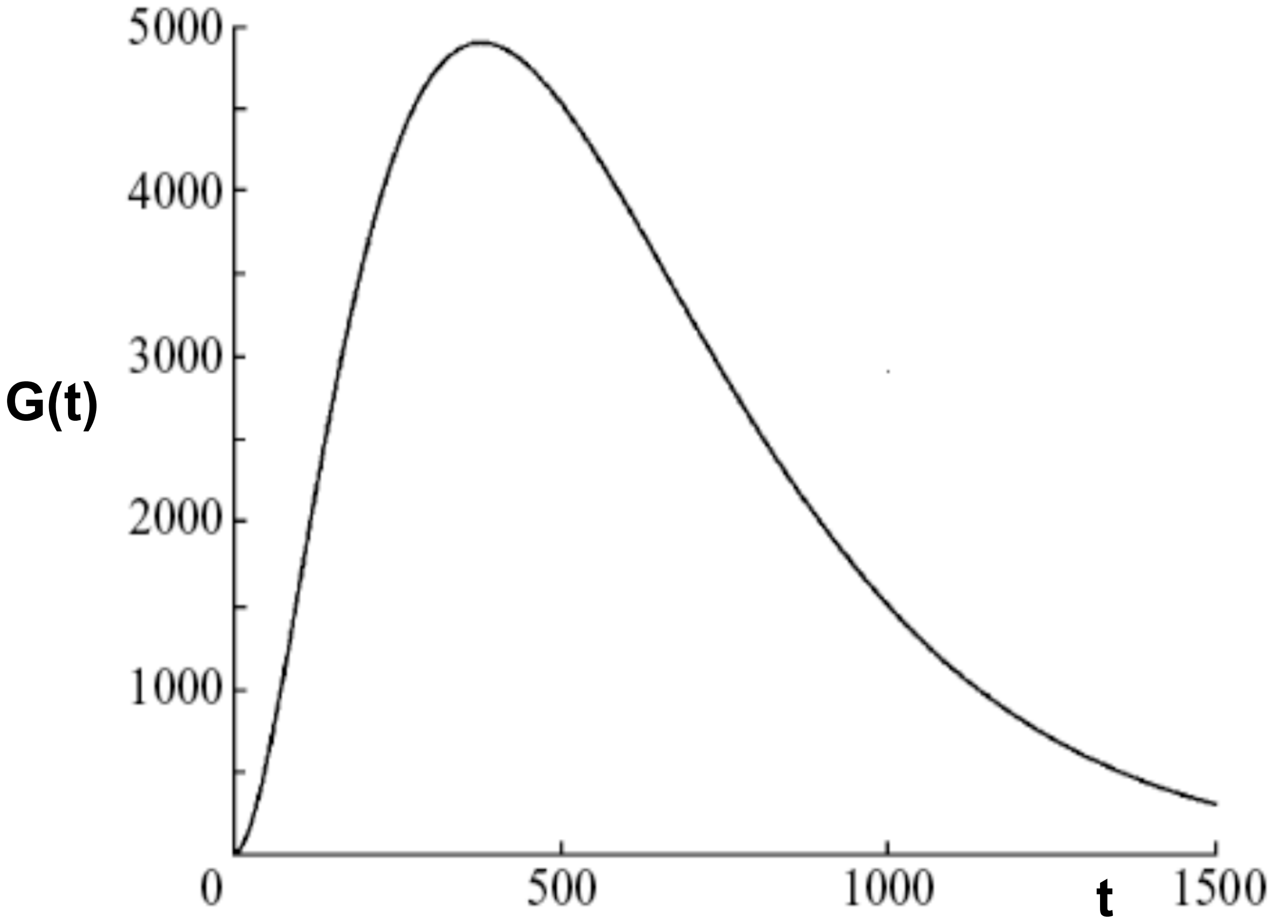
Optimal perturbation (here in boundary layer)



Orr mechanism



Plane Poiseuille flow
Lift-up mechanism (3D)



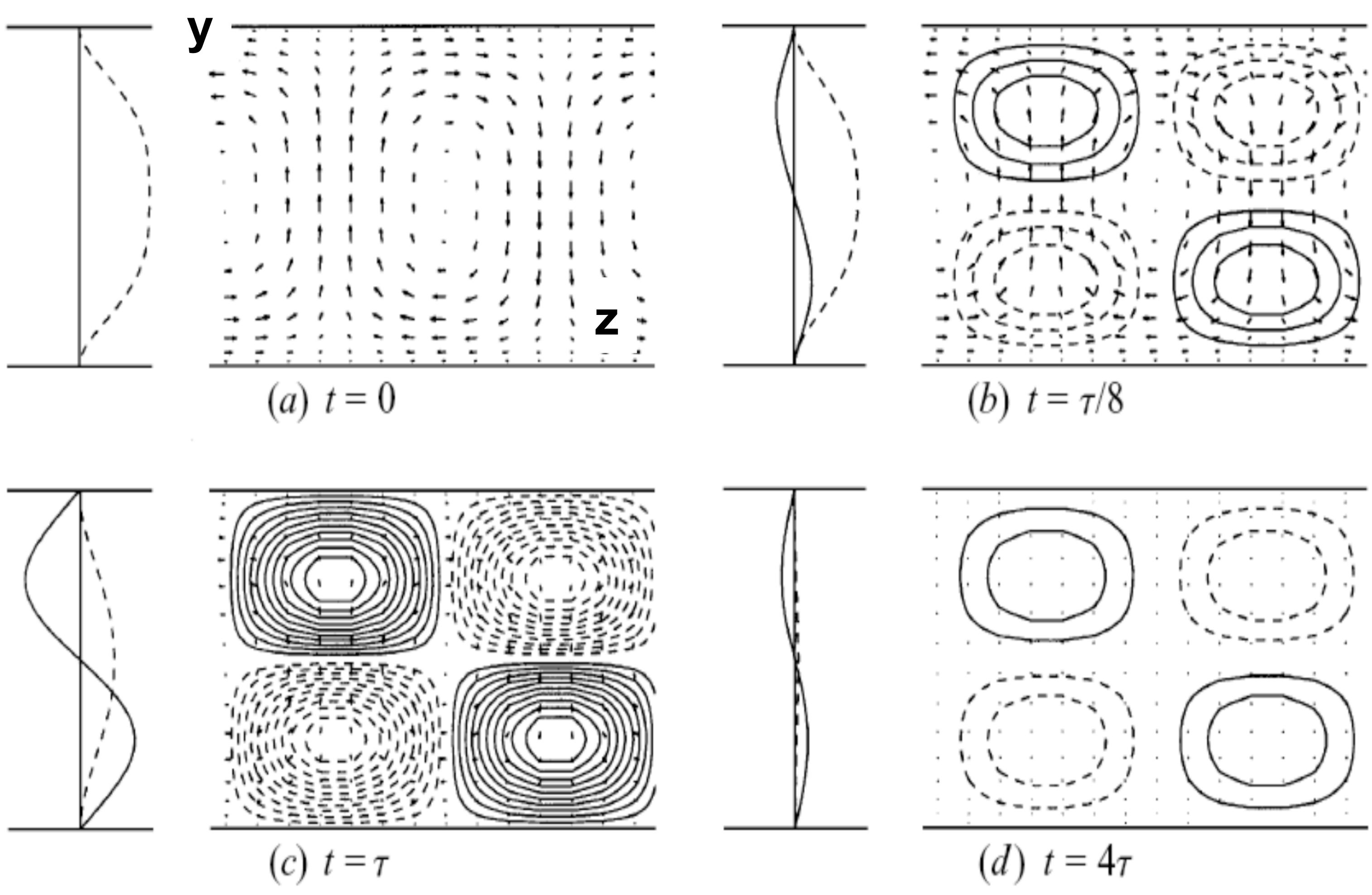
Re=5000, α=0, β=1

« By-pass transition »

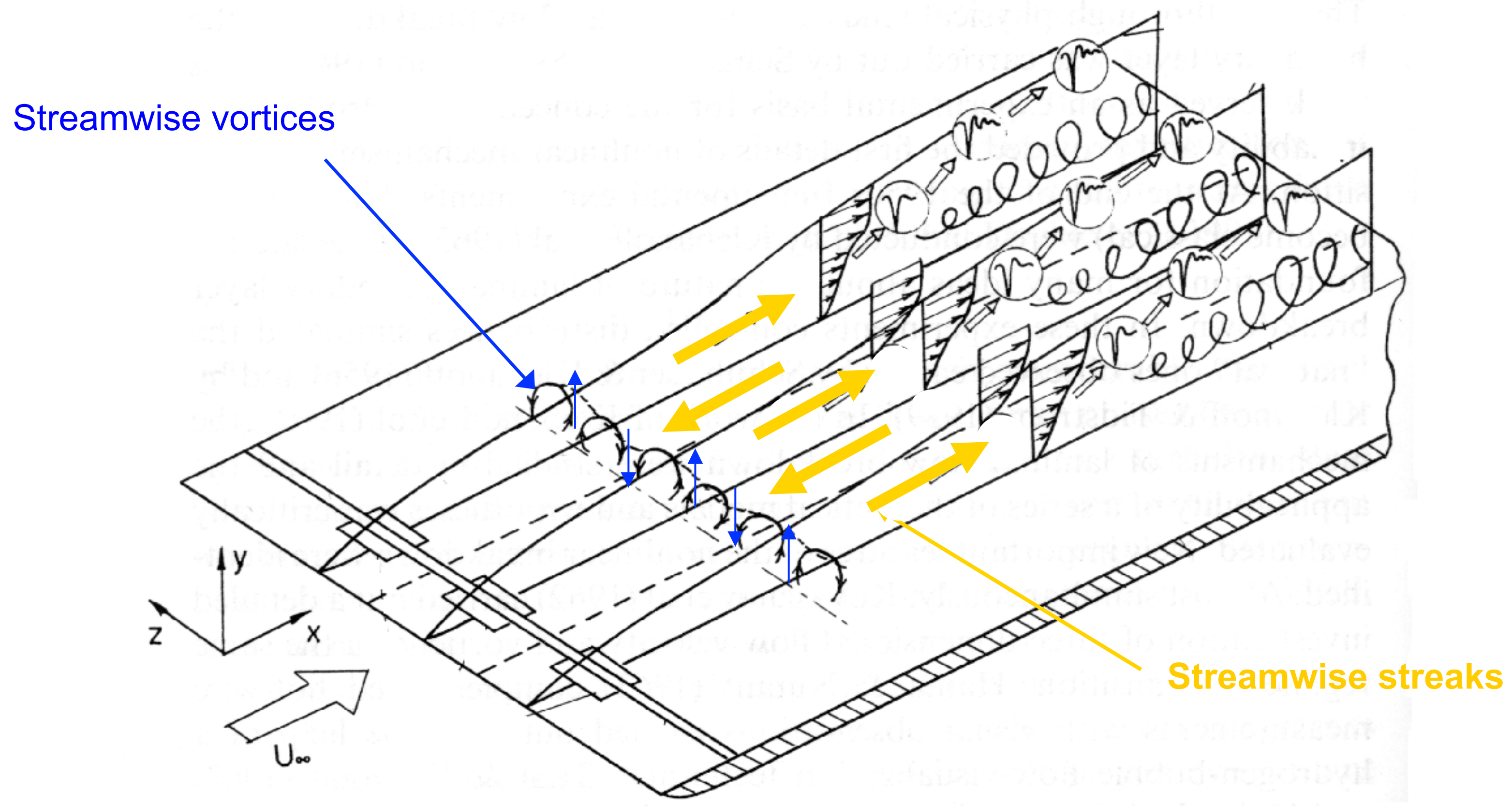
Trefethen et al (1993), Buttler & Farrell (1993) , Schmid & Henningson (2001)

Lift-up mechanisms

Optimal transformation of vortices into streaks



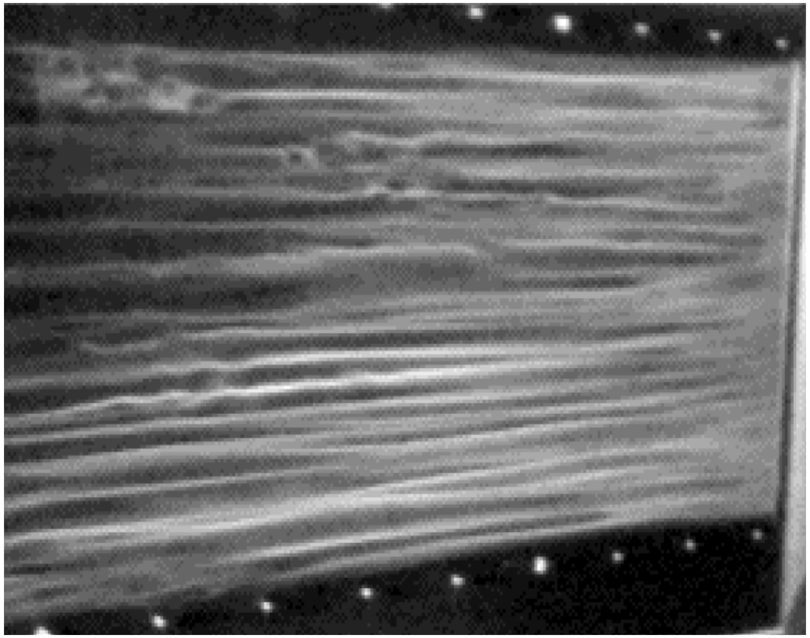
Lift-up mechanism



Schmid & Henningson.(2001)

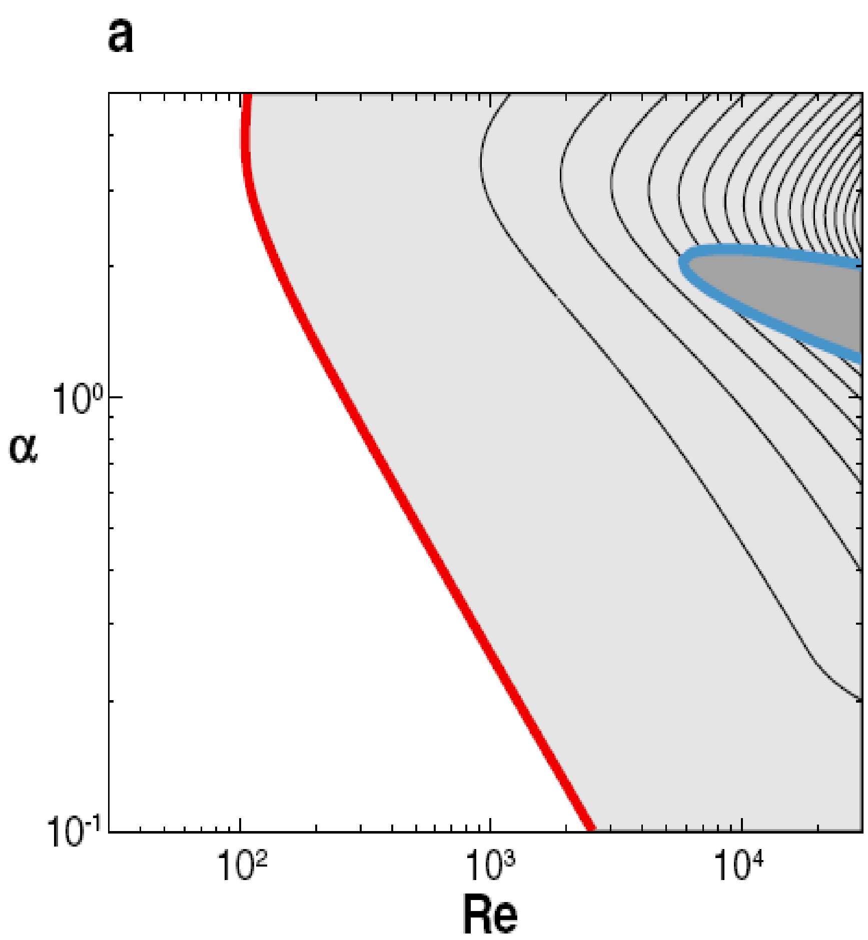
Lift-up mechanism

Optimal transformation of vortices into streaks



Alfredson & Matsubara (1996), streaky structures in the boudary layer

Isolines of maximum transient amplification
β=0



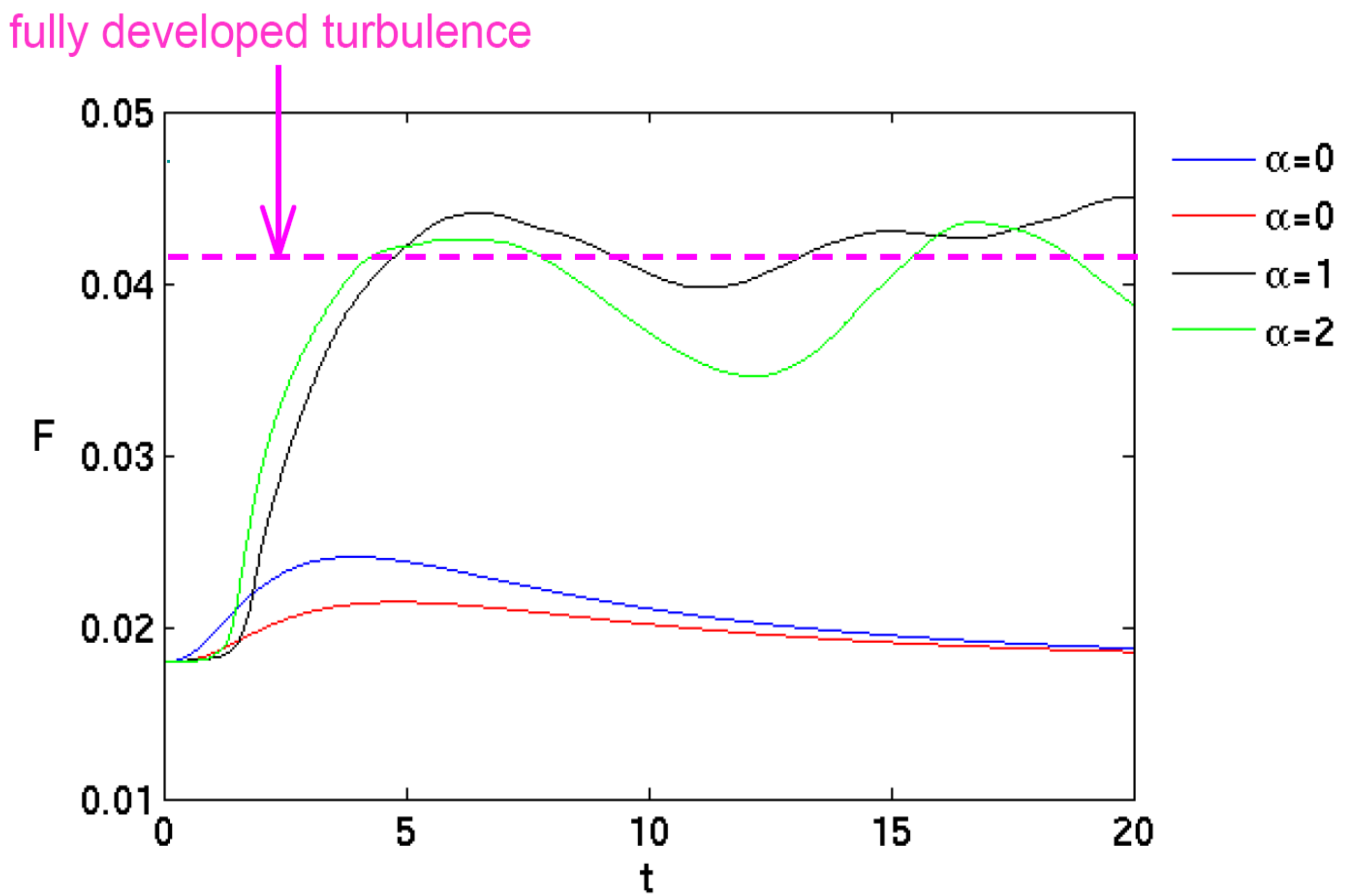
« By-pass transition »

Trefethen et al (1993), Buttler & Farrell (1993) , Schmid & Henningson (2001)

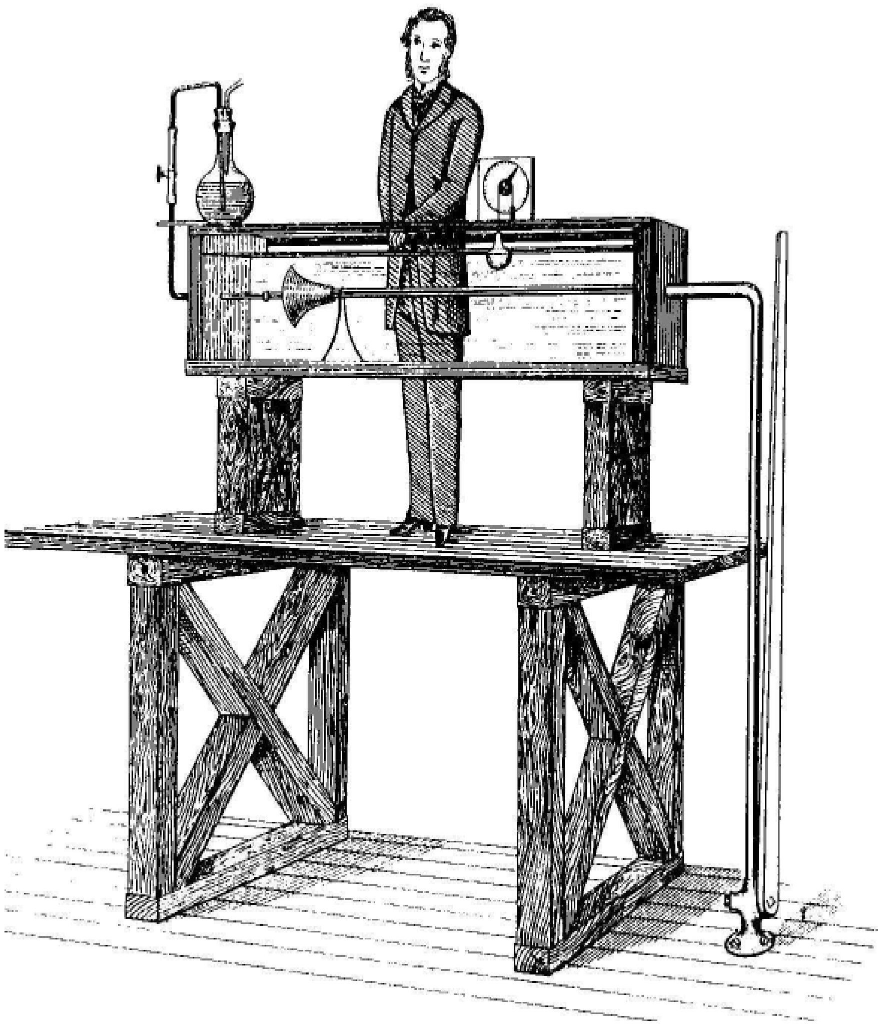
	$G_{\max}$ (10^{-3})	t_{max}	α	β
Plane Poiseuille	0.20 Re^2	0.076 Re	0	2.04
Couette	1.18 Re^2	0.117 Re	35/Re	1.6
Pipe	0.07 Re^2	0.048 Re	0	1
Boundary layer	1.50 Re^2	0.778 Re	0	0.65

Lift up mechanism in a square duct in the stable regime

Bye-pass transition to turbulence

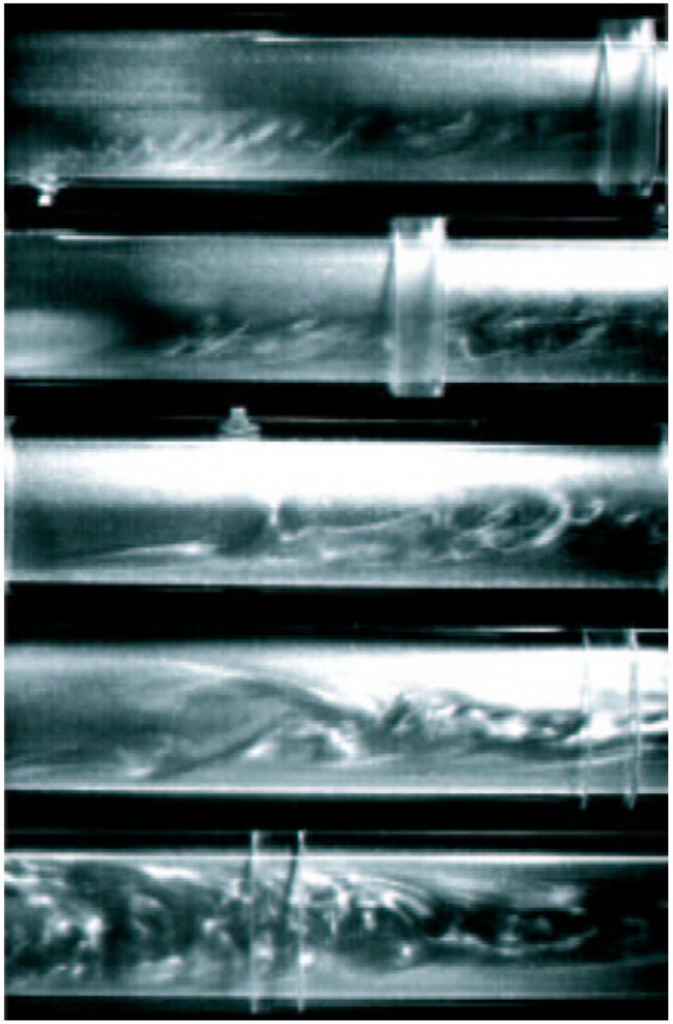


Bottaro et al. (2008)



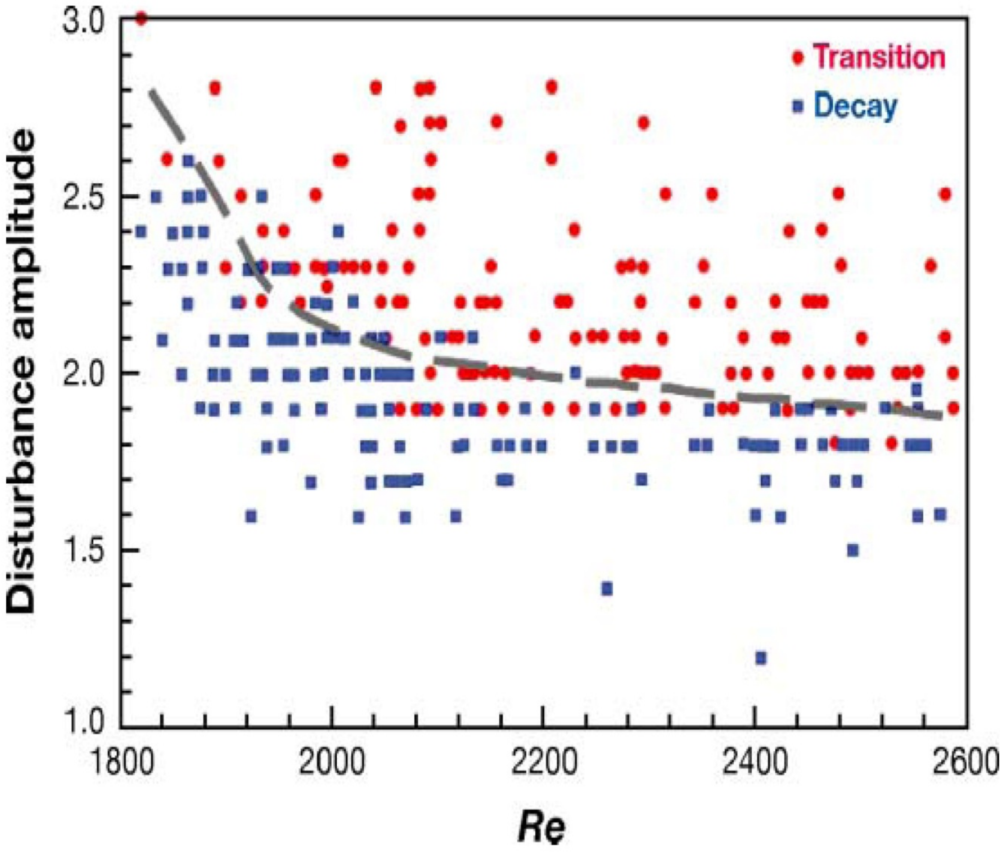
5.1 – *Osborne Reynolds en 1883 derrière son expérience à Manchester.*

Transition in Cylindrical Pipe Poiseuille flow



Mullin (2008)

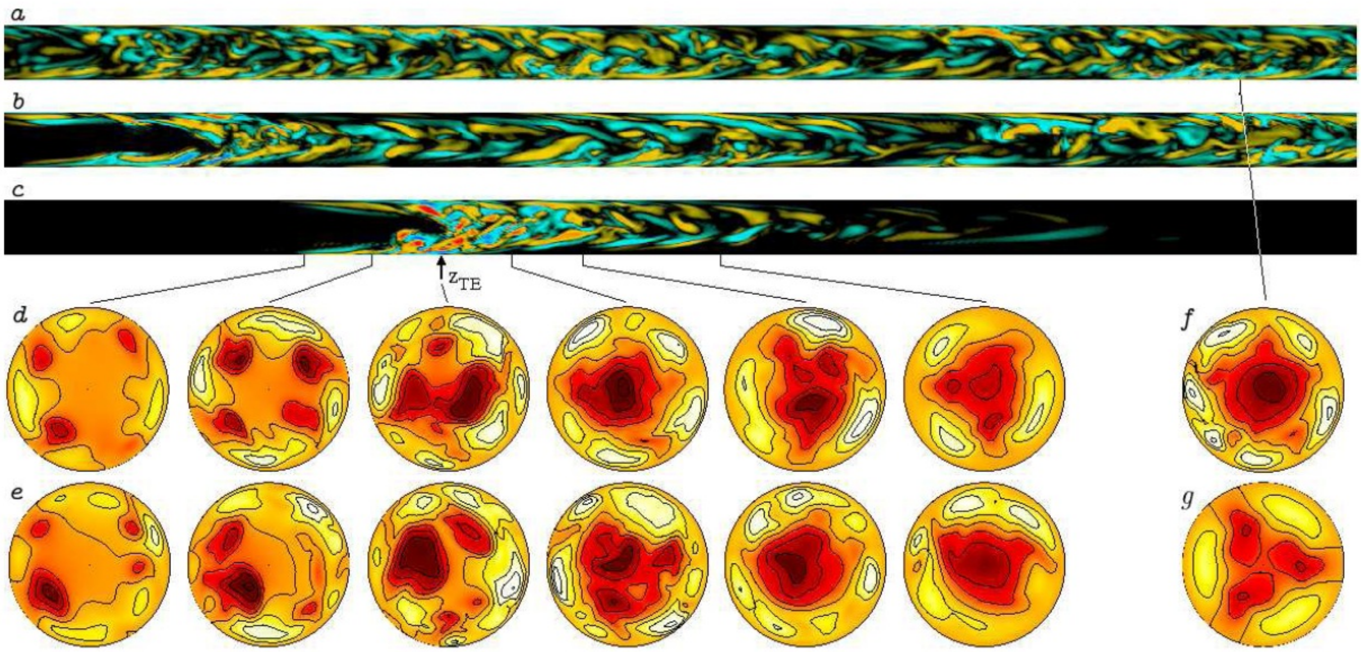
Transition in Cylindrical Pipe Poiseuille flow



Mullin (2008)

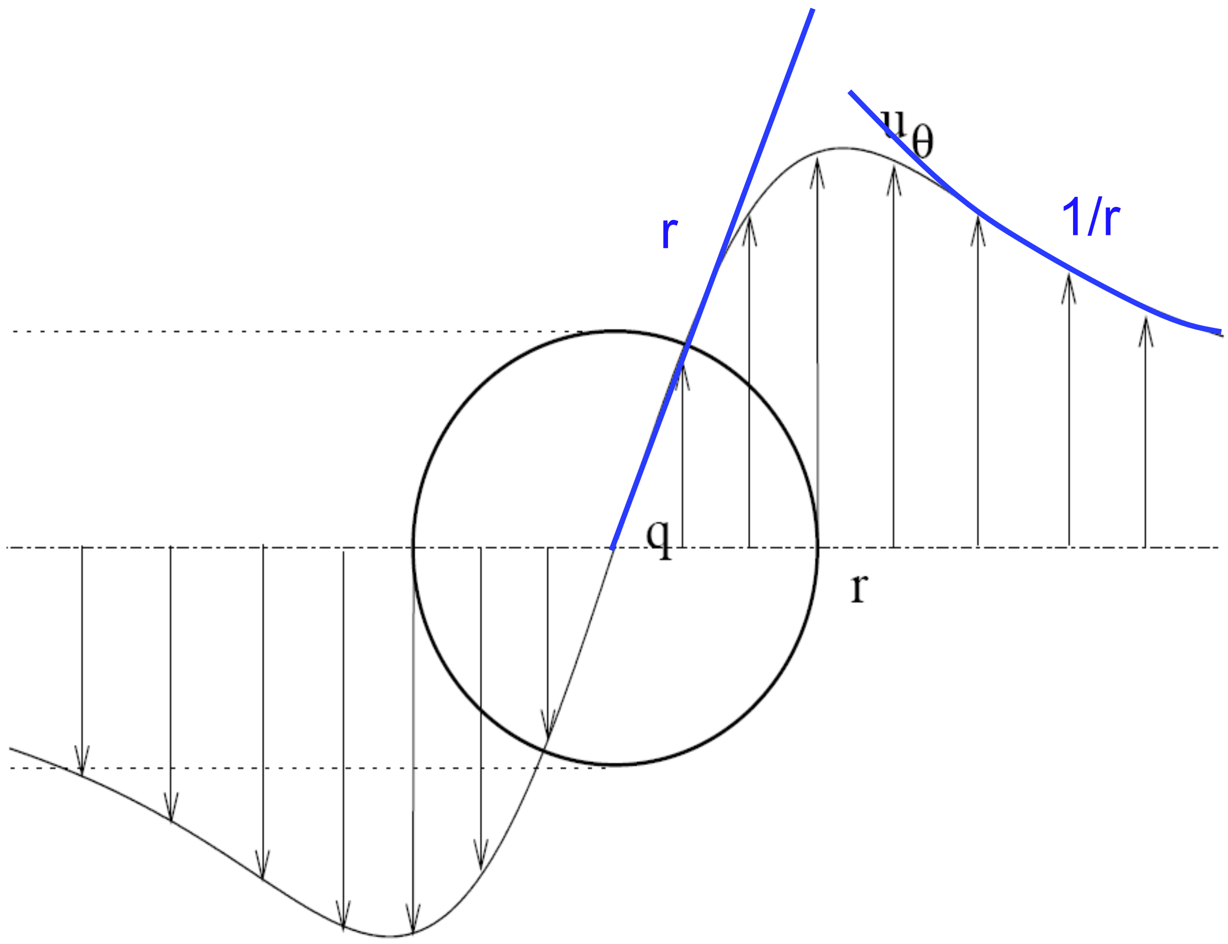
Transition in Cylindrical Pipe Poiseuille flow

Numerical study



Kerswell (2008)

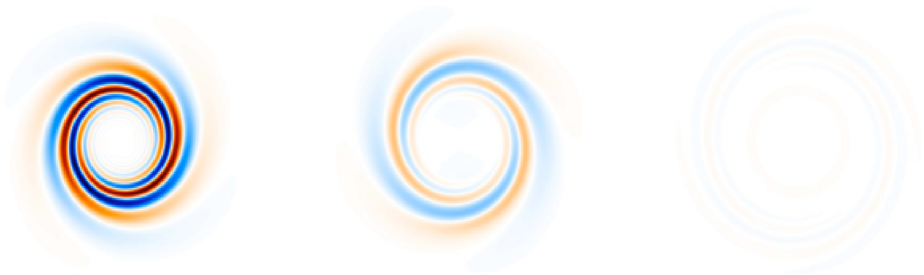
Tourbillon de Lamb-Oseen



Stable lineairement!

TOWARDS AN ELLIPTICAL STATE

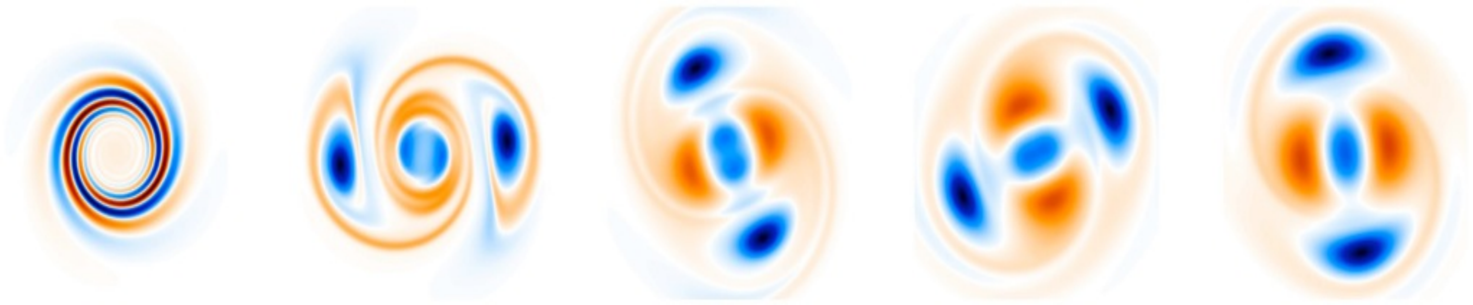
Nonlinear evolution of the optimal perturbation with initial amplitude below a given threshold...



Re = 1000

TOWARDS AN ELLIPTICAL STATE

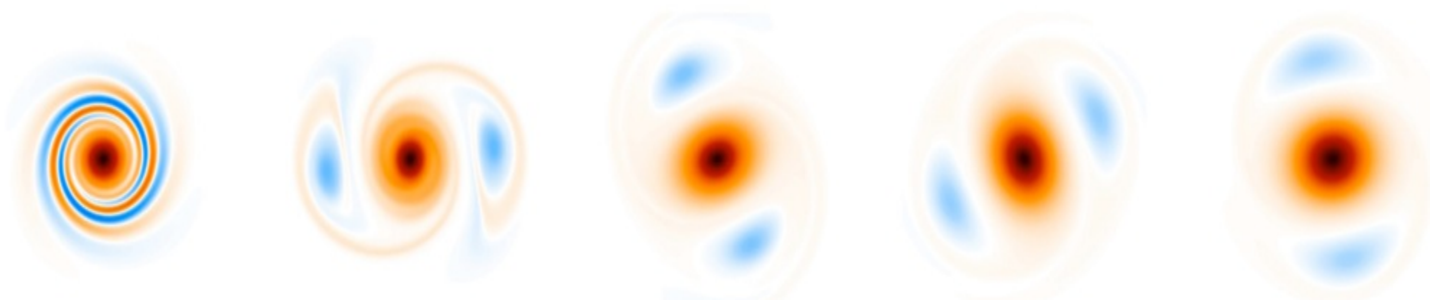
… and now ***ABOVE*** *the threshold*



Re = 1000

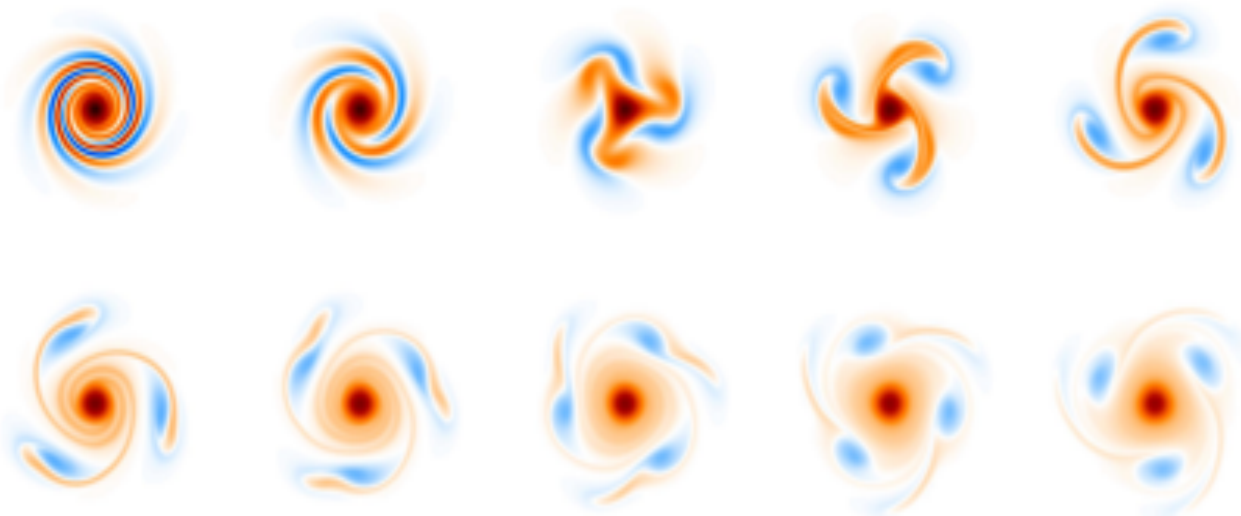
TOWARDS AN ELLIPTICAL STATE

Reconstructed flow

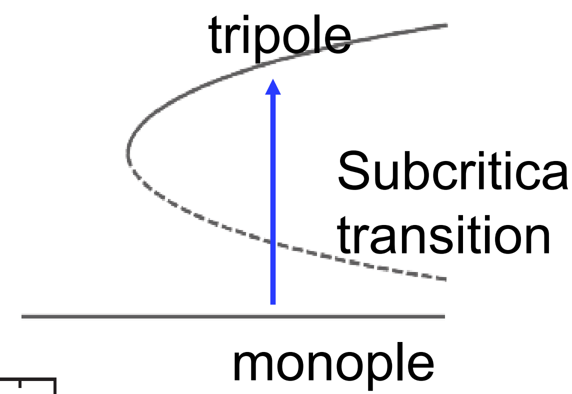


Re = 1000

Tripolar vortices?



BASIN OF ATTRACTION'S SHRINKAGE



Lamb-Oseen vortex' bifurcation diagram