

Exercise on the stability of dynamical systems

- Exercise 1: stability of the bifurcated solutions

Above the critical value of the rotation rate $\omega = \omega_0$ (where ω_0 is the natural frequency of the pendulum $\omega_0 = \sqrt{g/R}$, the bead on the hoop system discussed in class reaches one of the two symmetric equilibrium states $\pm\theta_s$. We remind that the fundamental governing equation writes

$$\ddot{\theta} = -\omega_0^2 \sin(\theta) + \omega^2 \sin(\theta) \cos(\theta) \quad (1)$$

Determine and solve the equation governing θ_s , as outlined in class. Demonstrate then that the two symmetric equilibrium states existing above criticality ($\omega > \omega_0$) are linearly stable.

- Exercise 2: equilibria of a pendulum held by a torsional spring

We consider a pendulum departing from the upward vertical by an angle θ held at its basis by a torsional spring of stiffness C the rest state of which is attained when the pendulum makes a right angle with the plane making an angle α with the horizontal. We choose m , l and $\sqrt{l/g}$ as characteristic mass, length and time scales. In addition a viscous torque is opposing the motion with a non dimensional relaxation rate τ (corresponding to a dimensional viscous damping torque of $-mgl/\tau \dot{\theta}$).

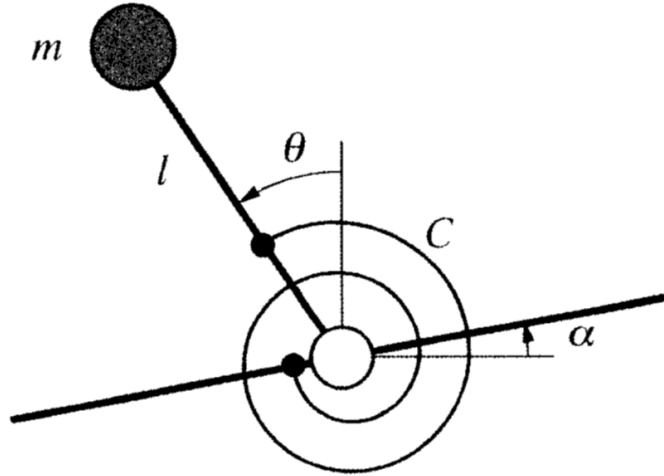


Figure 1: Sketch of the pendulum held by a torsional spring

- Show first that the dimensionless dynamical equation governing the position of the pendulum θ writes

$$\ddot{\theta} + \frac{1}{\tau} \dot{\theta} = \sin(\theta) - \omega^2(\theta - \alpha), \quad (2)$$

where you will also show that $\omega^2 = \frac{C}{mgl}$.

- By fixing $\omega^2 = 0.9$ in the rest of the exercise, determine numerically the equilibrium of solution(s) as a function of α as well as their stability.

- Draw a bifurcation diagram as a function of α , using the traditional convention of full (resp. dashed) lines representing stable (resp. unstable) states.