Solution – Serie 4– Cavitation bubble dynamics: Image Processing

1. Write a computer program to extract the frames from the AVI movie and analyze them to retrieve the evolution of the non-dimensional radius as a function of non-dimensional time.

The non-dimensional radius and the non-dimensional time can be obtained by different image processing tools. Here is a simple procedure:

a. Binarize the raw images to obtain white pixels inside the bubble (non-condensable gas and vapor) and black pixels outside the bubble (liquid domain).

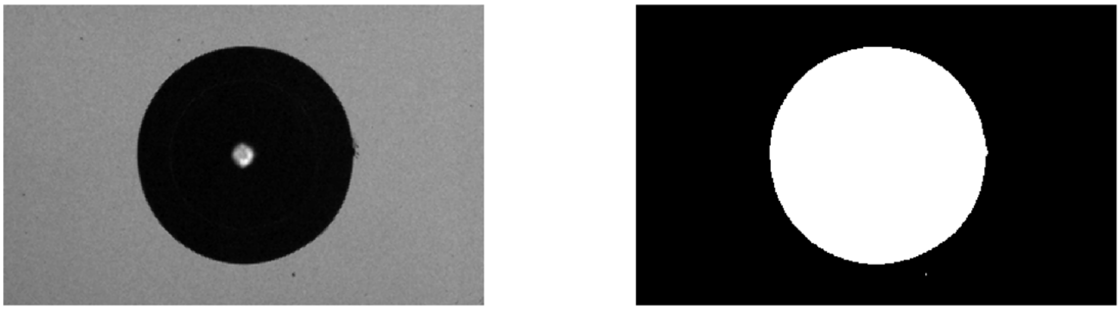

Figure 1: Left: Raw image, Right: Binarized image.

b. Then, for each frame n, sum up the white pixels inside the bubble to obtain the projected area A_n (i.e. a disk) of the spherical bubble.

Then, the radius at each frame n can be retrieved with the formula $R_n = \sqrt{A_n/\pi}$.

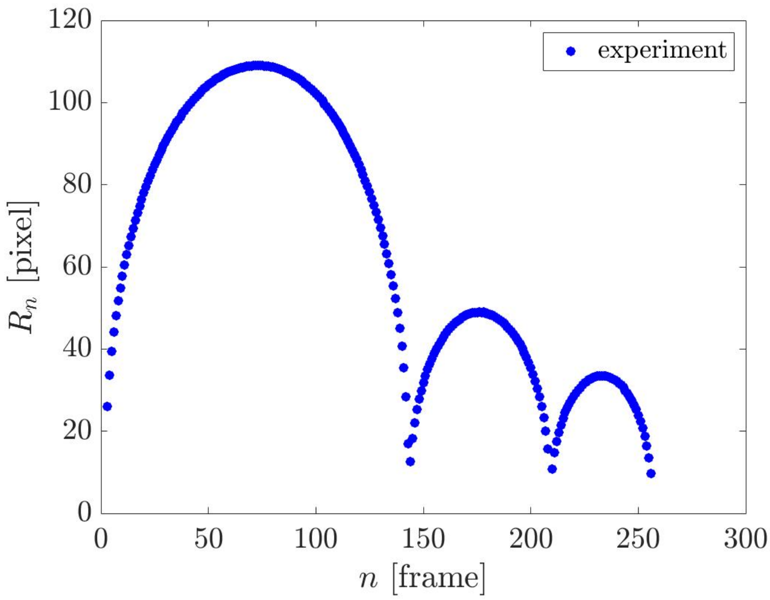

Figure 2: Experimental data.

We can retrieve the frame number n_{max} corresponding to the maximum radius R_{max} as well as the frame number $n_{Collapse}$ corresponding to the occurrence of the first collapse. The non-dimensional radius R_n^* and time t_n^* read:

$$R_n^* = \frac{R_n}{R_{max}} \quad and \quad t_n^* = \frac{t_n}{T_R} = \frac{n - n_{max}}{n_{Collapse} - n_{max}}$$

Where T_R is the collapse time, counted from the time corresponding to the maximum radius.

2. Compare the experimental data related to the collapse phase with Rayleigh model.

Since we don't know yet the length and time scales, we can still compare the data with Rayleigh model in non-dimensional way, from the maximum radius to the first collapse. To this end, we use the following non-dimensional Rayleigh equation:

$$\ddot{R}^* + \frac{\xi^2}{{R^*}^4} = 0 \quad \text{with initial conditions} \quad R^*(0) = 1, \dot{R}^*(0) = 0$$
$$\text{and } \xi \approx 0.915$$

We use the 'ode45' function in *Matlab* to compute the solution of this nonlinear ODE. The result is plotted in red in the following figure:

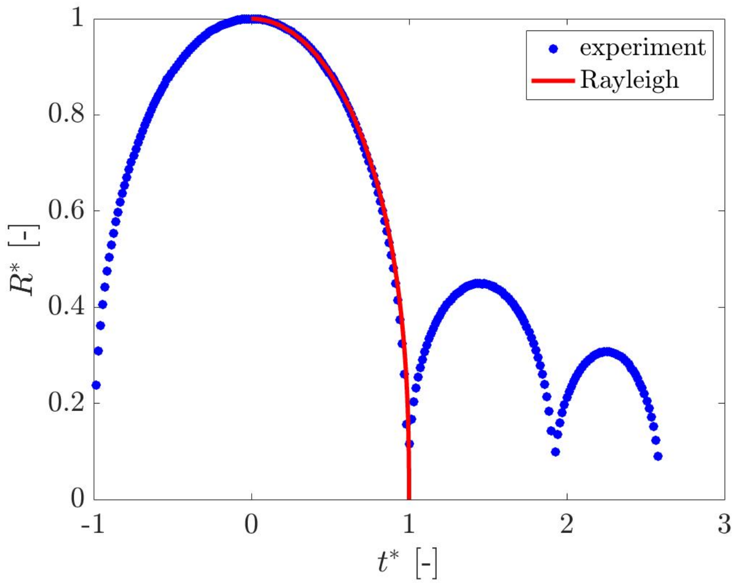

Figure 3: Comparison of the experimental data with the non-dimensional Rayleigh model.

The Rayleigh model fits well with the experimental data during the collapse phase. This simple analytical model cannot predict the rebound because it assumes the bubble filled with vapor only.

3. By tracking the faint shockwaves in the movie, determine the frame rate of the camera. Evaluate the dimensional values of the collapse time and maximum radius of the bubble. (Hypothesis: The speed of sound in water is 1500 m/s).

A shock wave is emitted when the bubble is generated by the laser pulse (c.f. course). After its emission, the shock wave travels through the water and is first reflected by the mirror, which is the closest obstacle from the center of the bubble.
Whether visually or by applying a simple processing on the frames, it is easy to track the passage of the first shockwave on Frame N° 35, as illustrated below.

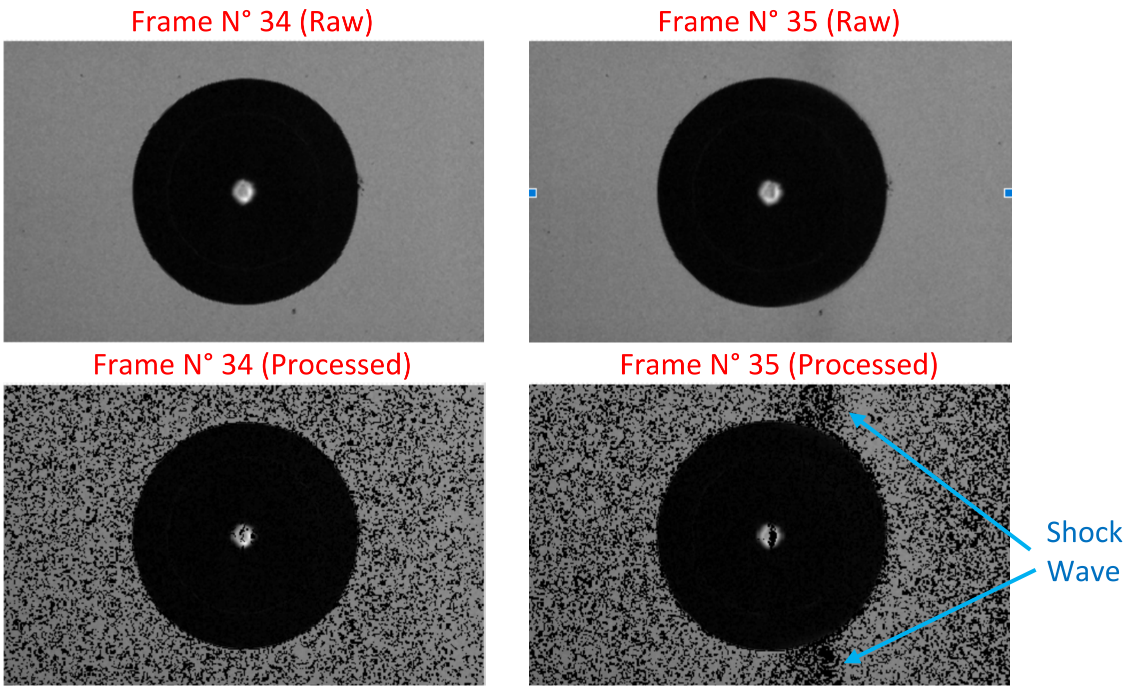

Figure 4: Shock wave detection

This information can also be retrieved by monitoring the sum of pixels intensity along a specific line and plotting it as a function of the frame number. The passage of the shockwave is then easy to identify as it corresponds to a drop in pixel intensity (see figure below).

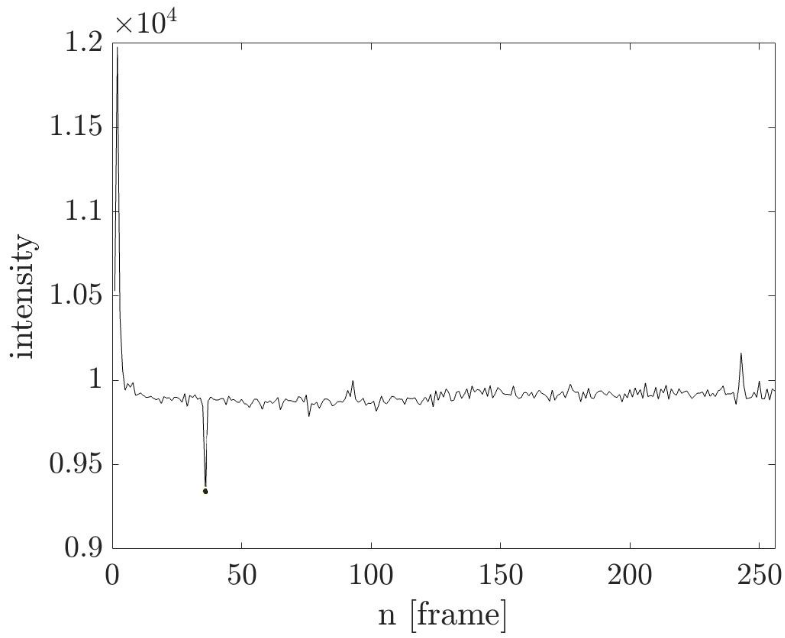

Figure 5: Summed intensity of all pixels of an arbitrary line as a function of frame number.

We assume that the shock waves travel at the speed of sound in water ($c = 1500\ m/s$). The distance traveled by the shock wave back and forth from the bubble to the mirror is $L = 2 \times 51 = 102\ mm$. Therefore, the shockwave must reach the center of the container at time $T = L/c = 68\ \mu s$.

During this time, 34 frames have elapsed (In fact, <u>we should eliminate the first frame since the bubble appears only on the second frame)</u>. This gives a frame rate of $FR = \frac{34}{68} = 500'000$ fps. This was exactly the actual frame rate of the camera! This method appears to be very accurate.

We can now compute the collapse time, between the maximum and the first minimum of the bubble radius:

$$T_R = \frac{n_{Collapse} - n_{max}}{FR} = 142\ \mu s$$

The maximum bubble radius can be obtained using the theoretical collapse time (Rayleigh time).

$$R_{max} = \frac{T_R}{\xi}\sqrt{\frac{p_\infty - p_v}{\rho}} \approx 1.5\ mm$$

With $p_\infty = 10^5\ Pa$, $p_v = 2300\ Pa$ and $\rho = 1000\ kg/m^3$.

4. Compare the experimental data related to the collapse phase with Rayleigh-Plesset model. The partial pressure of the non-condensable gas is 100 Pa.

We use the Rayleigh-Plesset model (see course chap 2.2) by setting the partial pressure of the non-condensable gas to $p_{g0} = 100\ Pa$. The water dynamic viscosity is $\mu = 1 \times 10^{-3} Pa.s$, the surface tension is $S = 72\ mN/m$ and the adiabatic coefficient of air is $\gamma = 1.4$. Again, the function 'ode45' in *Matlab* enables to compute the solution of this nonlinear ODE. The result is plotted in green in the following figure:

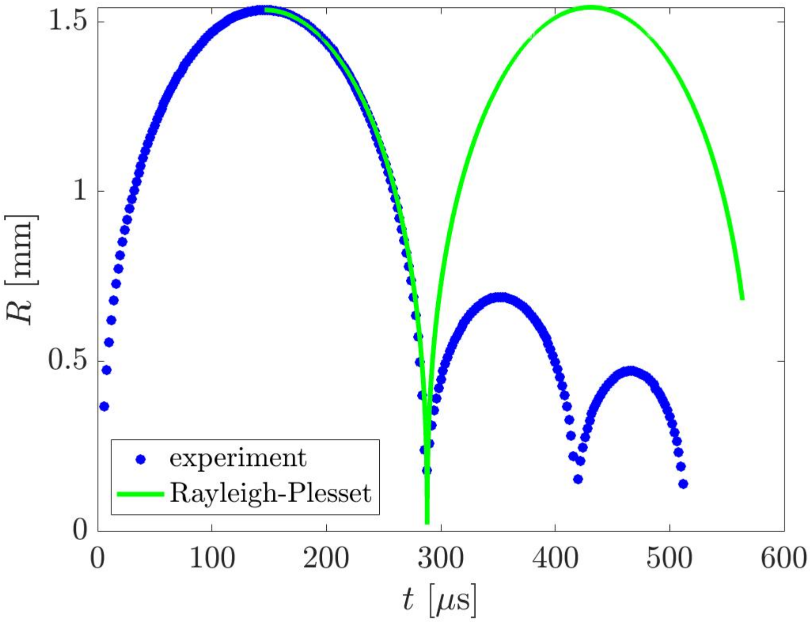

Figure 6: Comparison of the experimental data with the Rayleigh-Plesset model.

The Rayleigh-Plesset model fits the experimental data well during the collapse phase, but is unable to represent the amplitude and duration of the rebounds.

5. Compare the experimental data related to the collapse phase with Keller-Miksis model. Use the rebound bubble to determine the partial pressure of the non-condensable gas, which leads to the best fit of the data.

We use the Keller-Miksis (K-M) model (see course chap 2.2) by setting the partial pressure of the non-condensable gas from $p_{g0} = 50$ to $90\ Pa$, the adiabatic coefficient of air is $\gamma = 1.4$ and the speed of sound is $c = 1500\ m/s$. Again, the function 'ode45' in *Matlab* enables to compute the solution of this nonlinear ODE. The result is presented in the figure below.

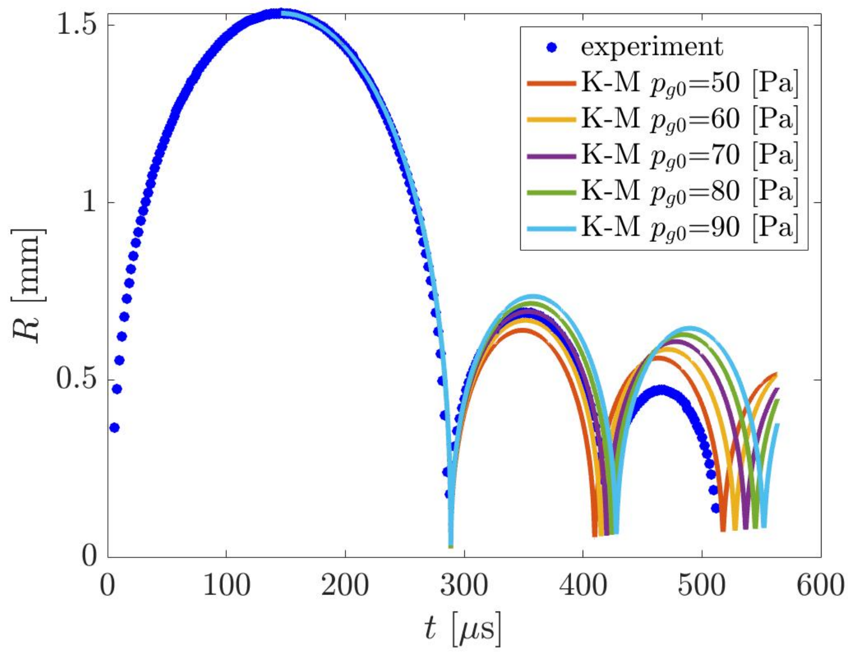

Figure 7: Comparison of the experimental data with the Keller-Miksis (K-M) model for different p_{g0}.

The experimental data are best fitted with the Keller-Miksis model taking $p_{g0} \approx 70\ Pa$.
The inclusion of this partial pressure value leads to a good approximation of the collapse and first rebound of the bubble. It's also important to note that the value of p_{g0} has almost no effect on the collapse phase.

6. Produce a video to illustrate the evolution of the pressure field around the bubble during the collapse phase.

According to Rayleigh model, the non-dimensional pressure in the liquid phase during the collapse phase is expressed as:

$$\frac{p(r,t) - p_\infty}{p_\infty - p_v} = \frac{R}{3r}\left(\frac{R_{max}^3}{R^3} - 4\right) - \frac{R^4}{3r^4}\left(\frac{R_{max}^3}{R^3} - 1\right)$$

The plot of this non-dimensional pressure is presented in the figure below at chosen times.
One may easily produce a movie to present the evolution of the pressure in time. It is also possible to superpose this computed pressure field on the bubble movie directly.

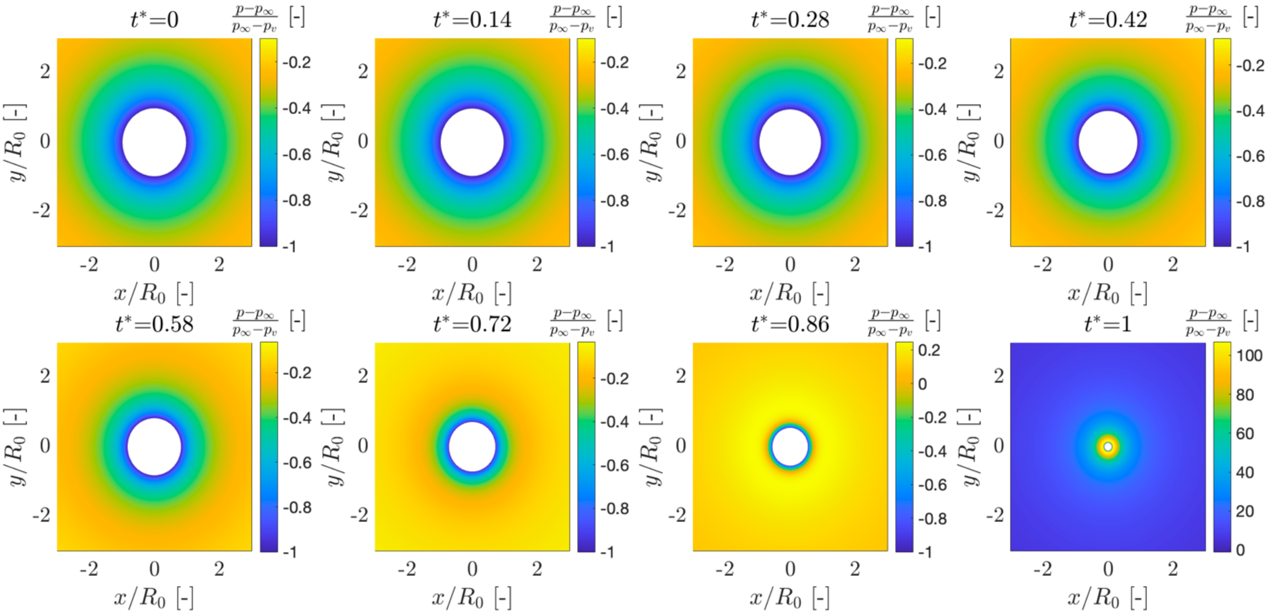

Figure 8: Evolution of the non-dimensional pressure field around the bubble as function of the non-dimensional time during the collapse phase of the bubble.