Solution- Serie 5. Estimating the length of a leading edge cavity.

1. Using the applet Xfoil or JavaFoil (open software), compute the pressure distribution on the hydrofoil for an upstream velocity of 20 m/s and incidence angles of 0°, 2° and 4°.
2. Compute the pressure distributions for an upstream velocity of 40 m/s (same incidence angles). The cavitation number is σ = 0.8 for all these cases.

The pressure coefficient C_p of a foil depends on its geometry and on the incidence angle α. However, for sufficiently large Reynolds numbers (here $Re = \frac{\rho C_{ref} c_0}{\mu} \geq 2.2 \times 10^6$), and for low incidence angles, it does not depend on the Reynolds number. Since JavaFoil is a potential flow solver (limit $Re \to \infty$), the computation of C_p does not require any Reynolds number input. For the foil geometry NACA0009 and incidence angles $\alpha = 0°, 2°, 4°$ the pressure coefficients are plotted in Figure 1 below:

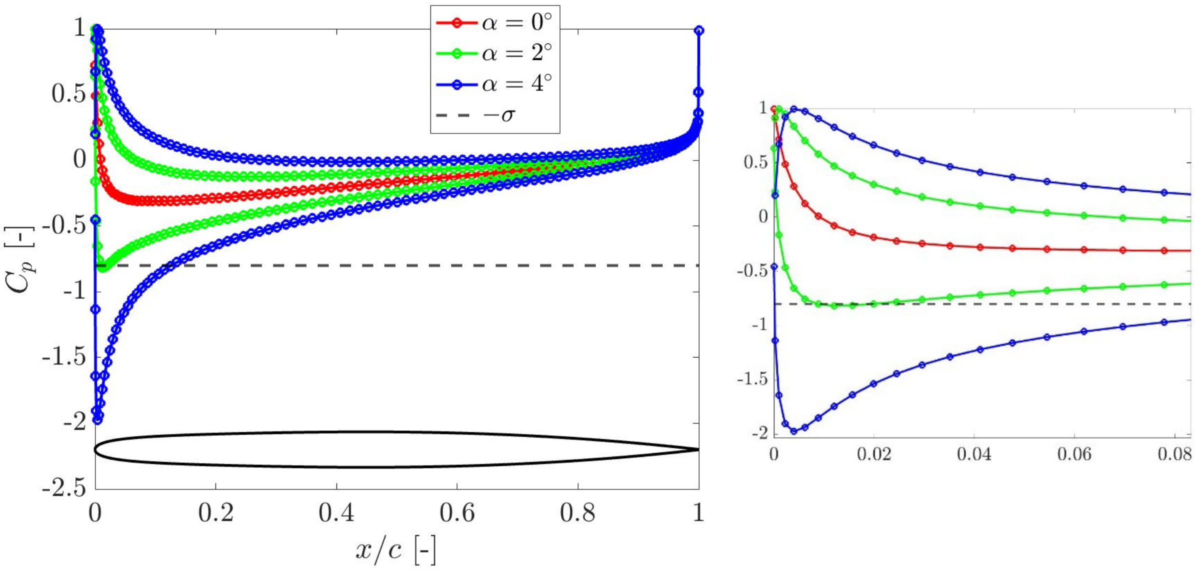

Figure 1. Left: NACA0009 pressure coefficients for various incidence angles. Right: Zoom around the leading edge of the foil.

For a given abscissa, there are two ordinates corresponding to the pressure coefficient at the pressure side (higher C_p) and suction side (lower C_p). Note that for the $\alpha = 0°$ case, the pressure side and suction side curves are superimposed (symmetric foil geometry). From the zoomed image on the right, we can observe that for $\alpha > 0°$ the stagnation point ($C_p = 1$) is not located at $x = 0$ but slightly shifted downstream on the pressure side of the foil. Cavitation occurs under the condition $p < p_v \Leftrightarrow C_p < -\sigma$ corresponding to the region located below the horizontal black dashed line in Figure 1.

The pressure $p(\boldsymbol{x})$ at $\boldsymbol{x} = (x, y)$ on the hydrofoil pressure/suction side can be derived from the pressure coefficient $C_p(\boldsymbol{x})$ as follows:

$$C_p(\boldsymbol{x}) = \frac{p(\boldsymbol{x}) - p_{ref}}{\frac{1}{2}\rho C_{ref}^2} \Longrightarrow p(\boldsymbol{x}) = \frac{1}{2}\rho C_{ref}^2 C_p(\boldsymbol{x}) + p_{ref}$$

Knowing C_{ref} and given the cavitation number $\sigma = 0.8$, we can retrieve the upstream pressure p_{ref}:

$$\sigma = \frac{p_{ref} - p_v}{\frac{1}{2}\rho C_{ref}^2} \Longrightarrow p_{ref} = \frac{1}{2}\rho C_{ref}^2 \sigma + p_v$$

This leads to $p_{ref} = 162'300\ Pa$ for $C_{ref} = 20 m/s$ and $p_{ref} = 642'300\ Pa$ for $C_{ref} = 40 m/s$.
The vapor pressure is taken as $p_v = 2300\ Pa$ at ambient temperature.
The absolute pressure distribution on the foil suction side is plotted for all incidence angles and upstream velocities in Figures 2, 3, 4 below.

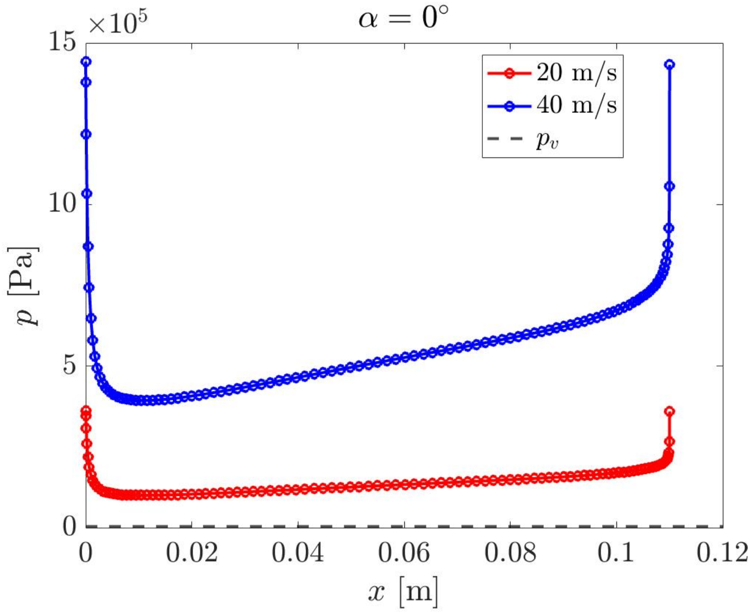

Figure 2: Pressure distribution along the foil suction side for $\alpha = 0°$.

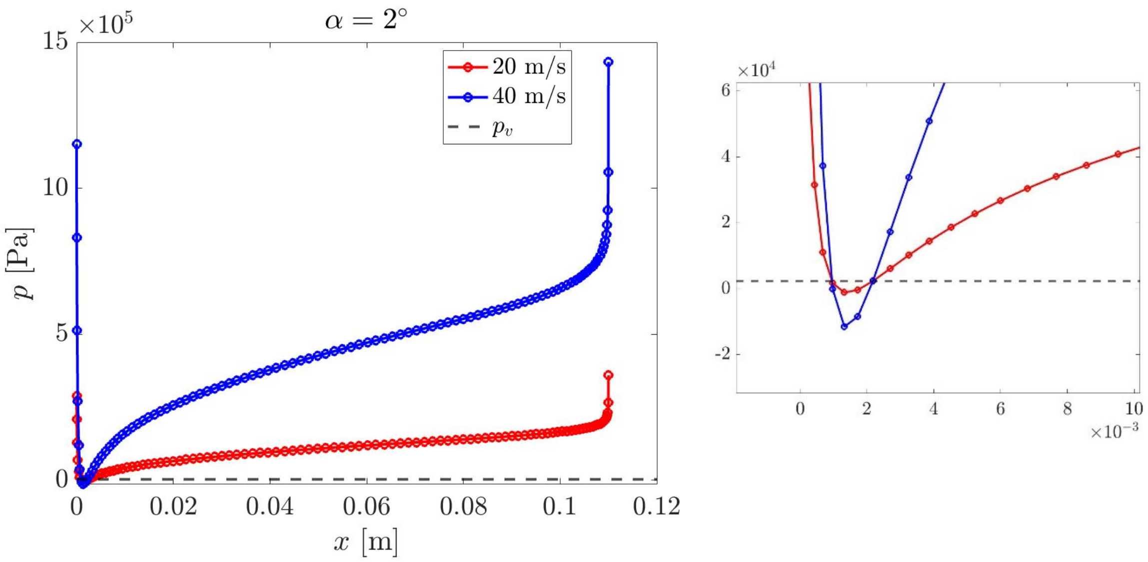

Figure 3. Left: Pressure distribution along the foil suction side for $\alpha = 2°$. Right: Zoom around cavitating region.

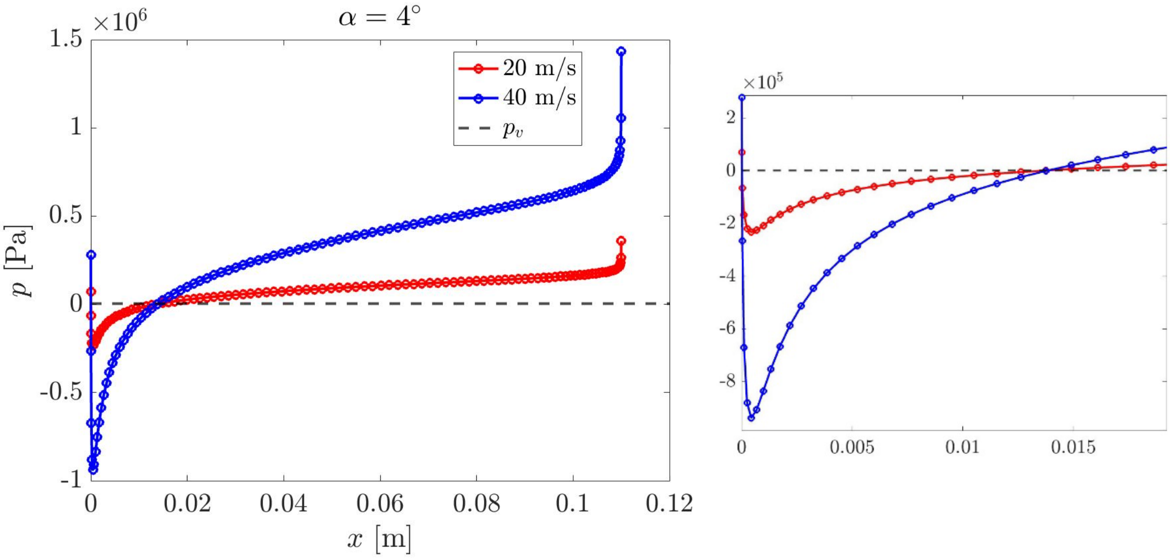

Figure 4. Left: Left: Pressure distribution along the foil suction side for $\alpha = 4°$. Right: Zoom around cavitating region.

From these figures, we can see that for a given incidence angle, the locations where $p(\boldsymbol{x}) < p_v$ are the same for both upstream velocities. This result is the consequence of an identical pressure coefficient distribution and cavitation number in both cases.

3. With the computed C_p distribution, find the pressure $p(t)$ experienced by a nucleus travelling on the foil suction side for 20 m/s and 40 m/s upstream velocity and incidence angles of 0°, 2° and 4°.

Considering the flow as incompressible and neglecting the viscous losses, the Bernoulli's equation along a streamline reads:

$$p_{ref} + \frac{1}{2}\rho C_{ref}^2 = p(\boldsymbol{x}) + \frac{1}{2}\rho \boldsymbol{C}^2(\boldsymbol{x}) \Longrightarrow C_p(\boldsymbol{x}) = 1 - \frac{\boldsymbol{C}^2(\boldsymbol{x})}{C_{ref}^2}$$

And then, the tangential velocity component $C(\boldsymbol{x}) = |\boldsymbol{C}(\boldsymbol{x})|$ at each point can be obtained through:

$$C(\boldsymbol{x}) = C_{ref}\sqrt{1 - C_p(\boldsymbol{x})}$$

We now want to track the motion of a particle traveling with the flow along the suction side of the foil.
Knowing the velocity $\boldsymbol{C}(\boldsymbol{x}_i)$ at each point $\boldsymbol{x}_i$ on the suction side of the foil, we are able to deduce the times t_i at which the particle passed through these successive points.
This can be made by using the following differential form:

$$|d\boldsymbol{x}| = |\boldsymbol{C}(\boldsymbol{x})|dt$$

And after temporal discretization we get:

$$|\Delta \boldsymbol{x}_i| \approx |\boldsymbol{C}(\boldsymbol{x}_i)|\Delta t_i \qquad i = 0,1, \ldots, N_p - 1$$

With $\boldsymbol{x}_i = \boldsymbol{x}(t_i)$ and N_p is the number of points along the foil suction side. This is equivalent to:

$$|\boldsymbol{x}_{i+1} - \boldsymbol{x}_i| = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \approx |\boldsymbol{C}(\boldsymbol{x}_i)|(t_{i+1} - t_i)$$

Which results in:

$$t_{i+1} = \frac{\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}}{|\boldsymbol{C}(\boldsymbol{x}_i)|} + t_i \quad \text{with} \quad t_0 = 0s$$

After performing the computation, we get a time variable discretized with a non-constant time step and, consequently, the path line of the fluid particles along the foil suction side.

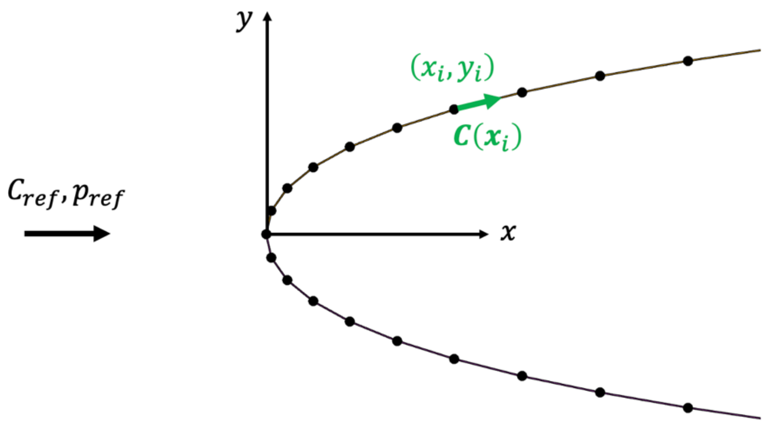

Figure 5: Spatial discretization along the hydrofoil pressure and suction sides.

Knowing the pressure at each time step, we recover the transient pressure signal $p(t)$ seen by a nucleus travelling on the foil suction side.
This transient pressure is shown below for all incidence angles and upstream velocities.

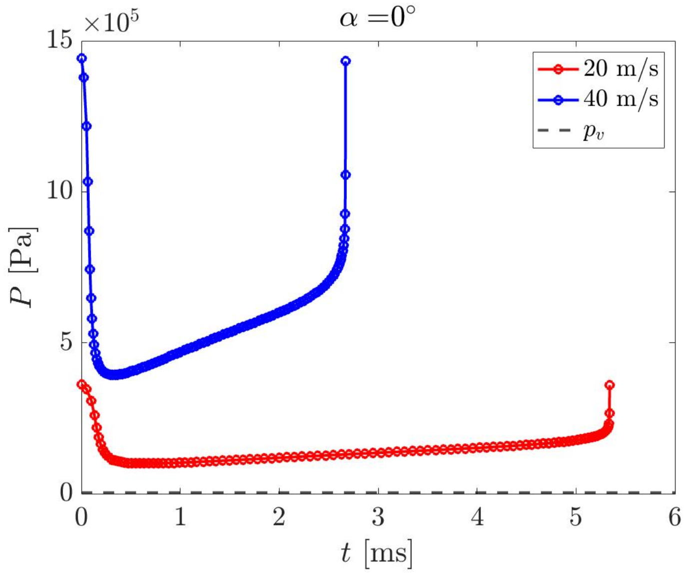

Figure 6 : The pressure felt by a nucleus travelling from leading to trailing edge of the hydrofoil ($\alpha = 0°$).

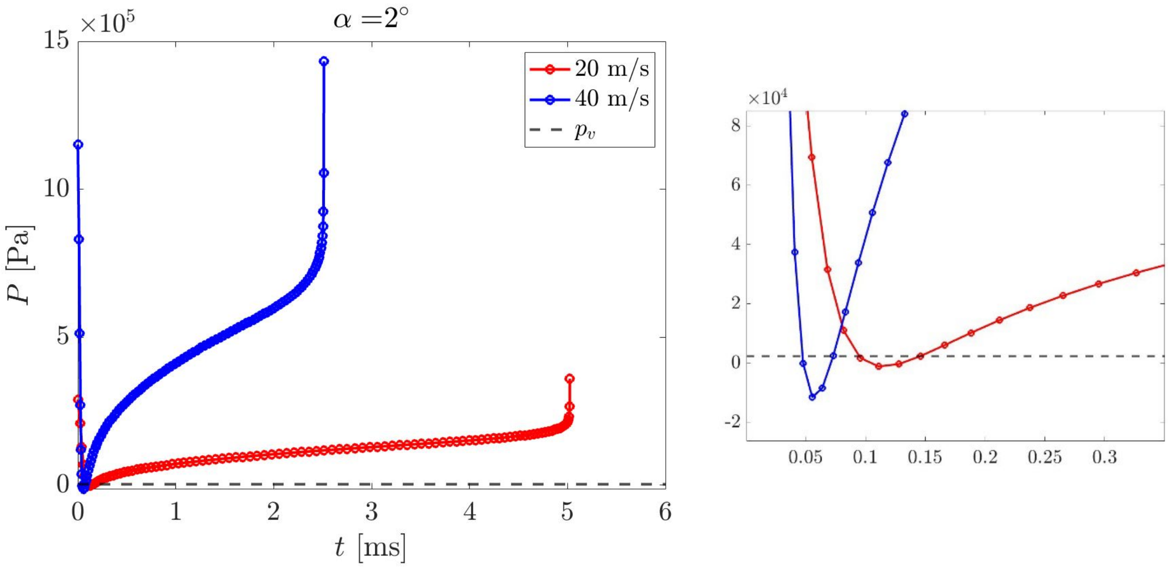

Figure 7. Left: The pressure felt by a nucleus travelling from leading to trailing edge of the hydrofoil ($\alpha = 2°$).
Right: Zoom around minimum pressure.

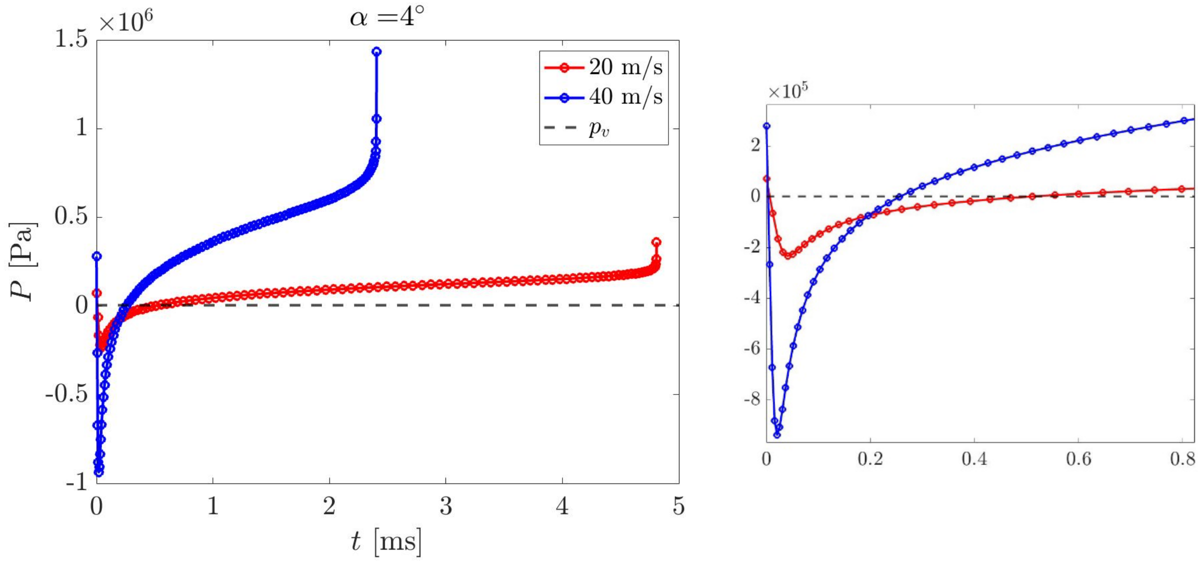

Figure 8. Left: The pressure felt by a nucleus travelling from leading to trailing edge of the hydrofoil ($\alpha = 4°$). Right: Zoom around minimum pressure.

From these figures, we can see that for a given incidence angle, the temporal start and duration at which $p(t) < p_v$ are different for the two upstream velocities.

4. Using the code that you have developed in the part 3 on bubble dynamics, compute the radius evolution of a nucleus evolving under the transient pressure computed above.

The relevant cases for cavitation occurrence are $\alpha = 2°$ and $\alpha = 4°$. Indeed, in those cases, there exist a region on the foil suction side where $p(\boldsymbol{x}) < p_v$. Therefore, we only focus on those cases to compute the bubble's radius evolution with time $R(t)$.
To do so, we imposed the transient pressure function as forcing pressure at infinity $p_\infty(t) = p(t)$ in the Rayleigh model (which considers an unbounded fluid domain).
Since the cavity does not necessarily begin to grow at the foil leading edge, a fluid particle will travel during a certain time on the foil suction side before cavitation starts. Therefore, the time integration should start at the first time when $p(t) < p_v$ on the particle trajectory, which we define as t_{cav}.
Moreover, the initial radius of the nucleus should be small, i.e $R(t_{cav}) = \varepsilon$ with $\varepsilon \approx 10^{-7}\, m$ for the numerical simulation to be successful. The problem to solve takes the form:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p_v - p_\infty(t)}{\rho} \quad \text{with initial conditions } R(t_{cav}) = \varepsilon \text{ and } \dot{R}(t_{cav}) = 0$$

This can be solved by using the function 'ode45' in *Matlab*. For this, it is required to transform this second order nonlinear ODE into a first order differential system of the form:

$$\dot{\boldsymbol{y}}(t) = \boldsymbol{f}(\boldsymbol{y}, t)$$

With $\boldsymbol{y}(t) = (y_1(t), y_2(t)) = (R(t), \dot{R}(t))$.
The ODE can be now rewritten as:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = \dfrac{1}{y_1}\left(\dfrac{p_v - p_\infty(t)}{\rho} - \dfrac{3}{2} y_2^2\right) \end{cases}$$

In the function handle that evaluates the right-hand side of the ODE, the transient pressure obtained for successive time steps (non-constant time steps) should be interpolated on the internal time steps that the solver uses. In *Matlab*, this function can be written as:

```
function dydt = Rayleigh(t,y,t_calc,p_inf,pv,rho)
dydt=zeros(2,1);
p_inf_int=interp1(t_calc,p_inf,t,'spline');  % Interpolation of pressure
onto internal time of the solver
dydt(1)=y(2);
dydt(2)=1/y(1)*((pv-p_inf_int)/rho-3/2*y(2)^2); % Rayleigh ODE
end

% t_calc is the calculated time (non-constant time steps)in question 3
% p_inf is the forcing pressure at infinity defined at each time
% t is the internal time of the solver
```

The temporal evolution of the radius $R(t)$ as well as its corresponding spatial distribution $r(\boldsymbol{x})$ on the foil suction side are given hereafter for the two incidence angles and the two upstream velocities considered.

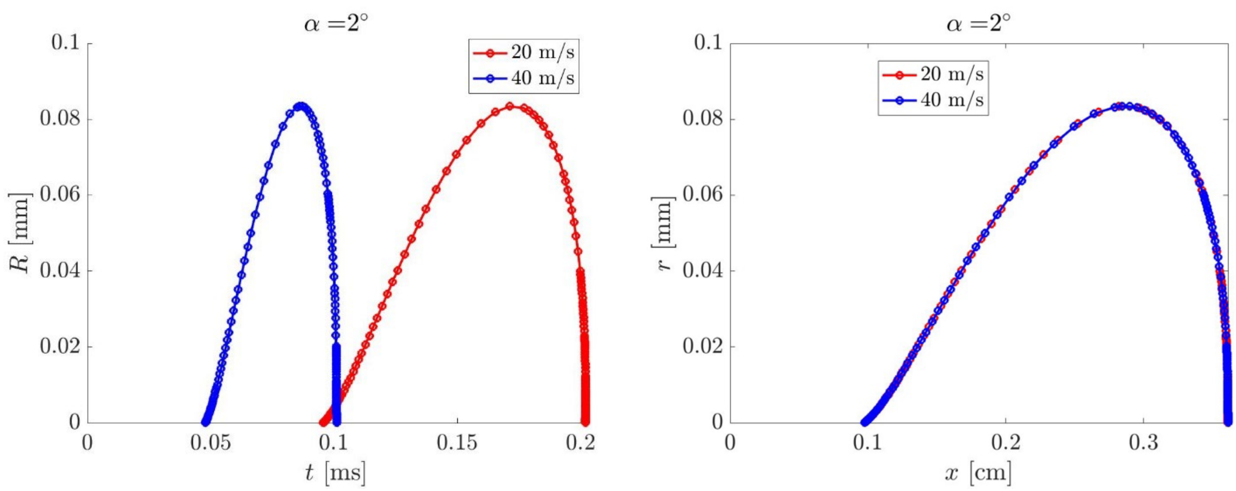

Figure 9. Case $\alpha = 2°$. Left: Radius evolution with time. Right: Radius spatial distribution over the foil suction side.

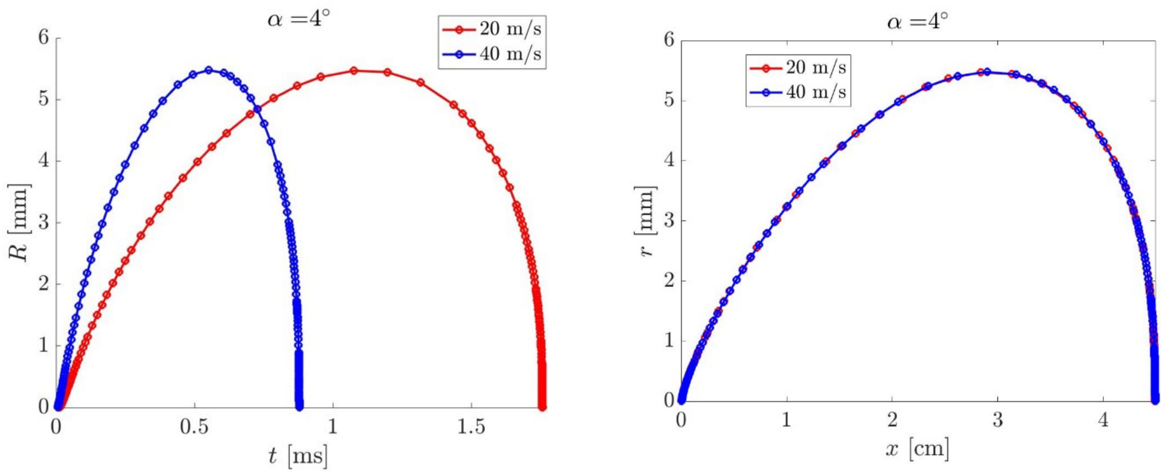

Figure 10. Case $\alpha = 4°$. Left: Radius evolution with time. Right: Radius spatial distribution over the foil suction side.

In the temporal domain, the results show that for a given incidence angle, even if the maximum radius is the same, the radius evolution depends on the upstream velocity. In the spatial domain, for a given incidence angle, the radius spatial distribution over the foil suction side is independent of the upstream velocities. The latter result is the consequence of an identical pressure coefficient distribution and cavitation number in both cases.

5. Finally, estimate the shape of the cavity by assuming that its thickness corresponds to the radius of the nucleus at a particular time.

The cavity thickness on the foil suction side is given hereafter for the two incidence angles (it is similar for the two upstream velocities).

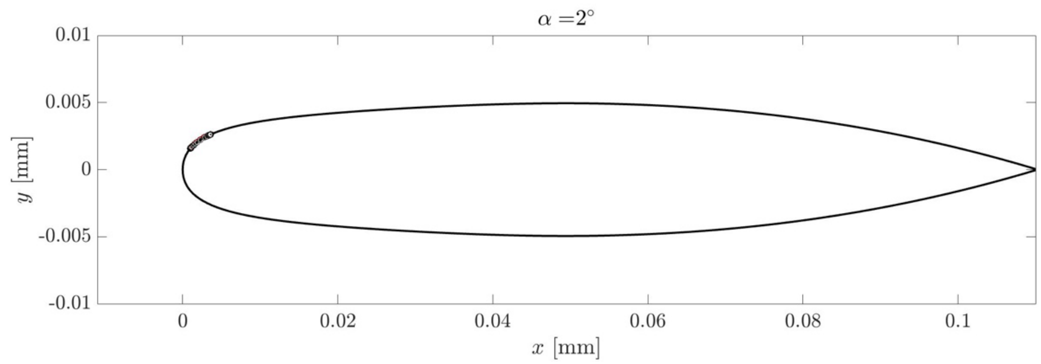

Figure 11: Cavity shape over the hydrofoil suction side for $\alpha = 2°$.

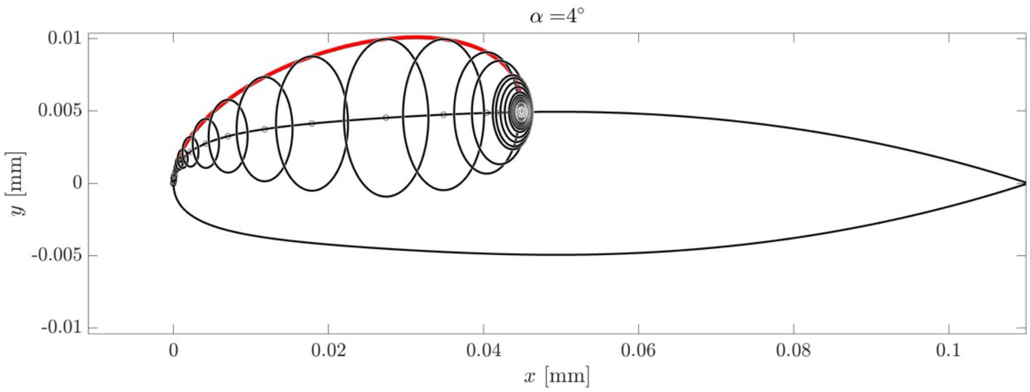

Figure 12: Cavity shape over the hydrofoil suction side for $\alpha = 4°$.

For the $\alpha = 2°$ case, the cavitation starts at an abscissa of $974.6\ \mu m$, the maximum cavity thickness is $83.36\ \mu m$, and the cavitation pocket extends over a length of $2.638\ mm$ in the x direction.
For the $\alpha = 4°$ case, the cavitation starts an abscissa of $27.5\ \mu m$, the maximum cavity thickness is 5.47 mm, and the cavitation pocket extends over a length of $4.49\ cm$ in the x direction.