Cavitation & Interface Phenomena

Autumn semester 2022

Exercises – Series 3

Dynamics of a spherical bubble

3.1. Establish the Rayleigh-Plesset equation for a spherical bubble in infinite volume of liquid.

3.2. By neglecting the effects of viscosity and surface tension and assuming that the bubble is filled only with vapor (Rayleigh equation), derive the expression for the interface velocity and compute the collapse time T_i (Rayleigh time).

3.3. By taking the initial radius R_0 and the Rayleigh time T_i as the length and time scales ($r = r/R_0$, $t^* = t/T_i$, write the Rayleigh equation in non-dimensional form.

3.4. Demonstrate that this non-dimensional equation can be written as:

$$\ddot{r} + \frac{K}{r^4} = 0$$

With a constant K (to be determined) and the dots denote derivatives with respect to the non-dimensional time t^*.

3.5. Consider the collapse of a cavitation bubble with an initial radium R_0=1 cm, in an infinite volume of liquid at pressure $p_{ref} = 1\ bar$. The bubble contains non-condensable gas at the pressure $p_{g0} = 100\ Pa$. Using *Matlab,* solve numerically the evolution of the radius with the time according to:

a. The Rayleigh equation (compare the collapse time with the analytical solution)
b. The Rayleigh-Plesset equation

hint: use the Ordinary Differential Equation (ODE) solvers in Matlab to solve numerically the differential equation.

3.6. A more realistic model, which takes into account liquid compressibility, is proposed by Keller and Miksis. The equation of motion reads:

$$\ddot{R} = \frac{(p_g + p_v - p_\infty)(1+\tilde{v}) + R\dot{p}_g / c - (3-\tilde{v})\dot{R}^2 \rho / 2}{(1-\tilde{v})\rho R}$$

where $\tilde{v}(t) \equiv \dot{R}(t)/c$ and *c* is the speed of sound in the liquid.

Assuming an adiabatic evolution of the pressure of non-condensable gas in the bubble:

$$p_g = p_{g0}\left(\frac{R_{max}}{R}\right)^{3\gamma}$$

Implement this set of equations in your program and compare with previous results.

3.7. Compute the pressure p_g, the temperature T and the interface velocity $\dot{R}$ during the collapse using the different models (Rayleigh, Rayleigh-Plesset, Keller-Miksis).

3.8. Compare the values previously obtained if the pressure of non-condensable gas is changed to $p_{g0} = 1000\ Pa$. Repeat the operation with $p_{g0} = 10'000\ Pa$.

3.9. By neglecting the effects of viscosity & surface tension and assuming that the bubble is filled only with vapor, plot the pressure distribution in the liquid when *R/R₀=0.2*. Repeat the operation when *R/R₀=0.1* and *R/R₀=0.05.*

3.10. Demonstrate that at any location in the surrounding liquid, the local velocity exhibits a maximum when:

$$\frac{R}{R_0} = 0.25^{\frac{1}{3}} \approx 63\%$$

Give an expression of this maximum velocity at any location *r*.

3.11. Check the validity of the analytical solution for the collapse of a bubble proposed by *Nikolay A Kudryashovand Dmitry I Sinelshchikov:*

$$u\ddot{u} + \frac{3}{2}\dot{u}^2 + \xi^2 = 0$$

$$\pm t = \sqrt{\frac{2}{3}}\frac{1}{\xi}\left[\frac{1}{u^3} - 1\right]^{1/2} F\left\{\frac{1}{2}, \frac{4}{3}; \frac{3}{2}; 1 - \frac{1}{u^3}\right\}$$

t and *u* stand for non-dimensional time and radius.
How does the computation time required to evaluate the analytical solution compare with the one for numerical solution?

References

- **Cavitation lecture notes**
- Obreschkow, D., Bruderer, M. & Farhat, M. **Analytical approximations for the collapse of an empty spherical bubble**. Phys. Rev. E 85, 66303 (2012).
- Kudryashov, N. A. & Sinelshchikov, D. I. **Analytical solutions of the Rayleigh equation for empty and gas-filled bubble**. J. Phys. A: Math. Theory 47, 405202 (2014).