Solution – Serie 2

Equilibrium of a nucleus

1. Write the equilibrium condition for a spherical nucleus of radius R, containing gas and vapor, within a liquid at rest at the pressure p_∞.

Static equilibrium of a spherical nucleus:

$$p_\infty = p_v + p_g - \frac{2S}{R} \qquad (1)$$

2. Assuming an isothermal transformation and a constant mass of gas, find the equation of the curve $p_\infty(R)$ as a function of the parameters p_{g0} and R_0.

Isothermal process:

$$p_{g0}V_0 = p_g V$$

$$p_{g0} * \frac{4}{3}\pi R_0^3 = p_g * \frac{4}{3}\pi R^3$$

$$p_g = p_{g0}\left(\frac{R_0}{R}\right)^3$$

$$\xrightarrow{\text{substituting in 1}} \qquad p_\infty = p_{g0}\frac{R_0^3}{R^3} - \frac{2S}{R} + p_v$$

3. Find the radius R_c at which the curve $p_\infty(R)$ is minimum. Compute the corresponding pressure at this minimum. This pressure is the critical pressure p_c (or Blake threshold pressure).

Critical Radius:

$$p_\infty = p_{g0}\frac{R_0^3}{R^3} - \frac{2S}{R} + p_v \qquad \xrightarrow{\text{to find the critical value of } R} \qquad \frac{dp_\infty}{dR} = 0$$

$$-3p_{g0}R_0^3R^{-4} + 2SR^{-2} = 0$$

$$R^2 = \frac{3p_{g0}R_0^3}{2S} \quad \Longrightarrow \quad R_c = \sqrt{\frac{3p_{g0}R_0^3}{2S}}$$

Critical pressure:

$$p_c = p_\infty(R = R_c) = p_{g0}R_0^3\frac{1}{R_c^3} - \frac{2S}{R_c} + p_v$$

And we have:

$$R_c = \sqrt{\frac{3p_{g0}R_0^3}{2S}} \Longrightarrow p_{g0}R_0^3 = \frac{2S}{3}R_c^2$$

Therefore:

$$p_c = \frac{2S}{3} * \frac{1}{R_c} - \frac{2S}{R_c} + p_v = p_v - \frac{4S}{3R_c}$$

4. Let us consider 3 bubbles of radius R_0 =1mm, 1µm and 1nm, in equilibrium at $p_{\infty 0} =$ 1 bar. Compute the corresponding pressure of the gas p_{g0} inside each bubble (we assume a temperature of 20°C → $p_v = 2300\ Pa$, $S = 0.072\ \ N/m$).

Static equilibrium:

$$p_\infty = p_v + p_g - \frac{2S}{R} \Rightarrow p_{g0} = p_{\infty 0} - p_v + \frac{2S}{R_0}$$

$R_0\ [m]$	10^{-3}	10^{-6}	10^{-9}
$p_{g0}\ [kPa]$	97.844	241.7	144097.7

Notice how the non-condensable gas pressure increases significantly as the bubble radius decreases towards nanoscale. This is due to the surface tension term, which also increases significantly.

5. Find the critical radius and the critical pressure corresponding to each bubble. Are the bubbles in a stable equilibrium?

Critical radius and critical pressure for 3 bubbles:

$$R_c = \sqrt{\frac{3p_{g0}R_0^3}{2S}} \quad , \quad p_c = p_v - \frac{4S}{3R_c}$$

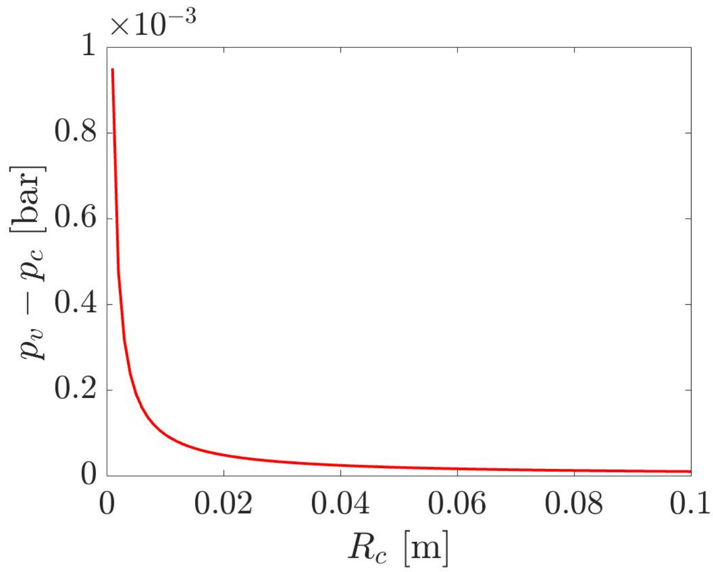
$R_0\ [m]$	$R_c\ [m]$	$p_c\ [kPa]$
10^{-3}	0.045	2.298
10^{-6}	$2.24 * 10^{-6}$	-40.48
10^{-9}	$1.73 * 10^{-9}$	-55405

All these bubbles are in stable equilibrium because they all satisfy the condition $R_0 < R_c$.

6. The difference $p_v - p_c$ represents the static delay to cavitation of a nucleus. Compute this quantity for each bubble. Sketch the general form of the curve representing the static delay to cavitation as a function of the critical radius.

Static delay to cavitation: $(p_v - p_c) = \frac{4S}{3R_c}$

$R_0\ [m]$	10^{-3}	10^{-6}	10^{-9}
$(p_v - p_c)\ [kPa]$	0.0021	42.78	55406.8



7. Using *Matlab* or *Excel*, plot the curve $p_\infty(R)$ corresponding to each bubble.

The $p_\infty(R)$ curves: