Stating an optimization problem

Different types of optimisation problems

Optimisation problem

$$\min_{X_{variable}} f(X_{variable}, \pi_{parameters})$$ Objective function

$$h(X_{variable}, \pi_{parameters}) = 0$$

$$X_{variable} - \pi_{specification} = 0$$

m : Model
+
Specifications

n : Decision Variables

$$g(X_{variable}) \leq 0$$

$$X_{variable,min} \leq X_{variable} \leq X_{variable,max}$$

Constraints

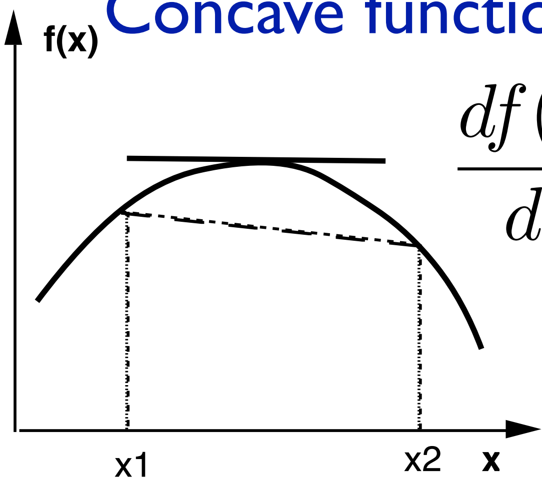
Degrees of freedom : n-m

Solving method

The goal of the optimisation is to fix the value of the n-m degrees of freedom

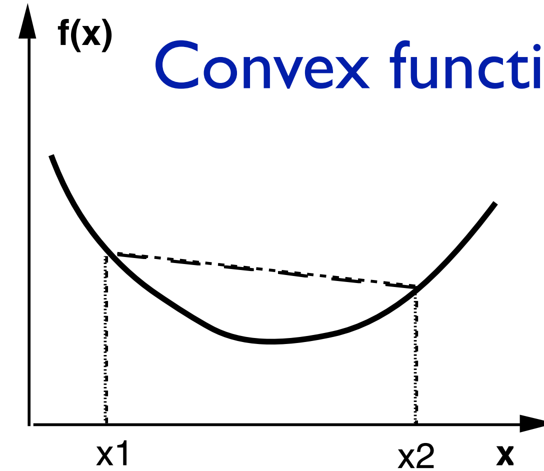
Optimum definition : min or max f(x)

Concave function



$$\frac{df(x)}{dx} = 0$$

Convex function



Concavity condition

$$\forall\ x_1, x_2 \in R,\ \forall\ \theta \in [0,1]:\ f[\theta\ x_1 + (1-\theta)\ x_2] \geq \theta\ f(x_1) + (1-\theta)\ f(x_2)$$

Hessian matrix $\quad : \underline{\underline{H}}(\underline{x}) = \nabla^2 f(\underline{x}) = \left[\dfrac{\partial^2 f(\underline{x})}{\partial x_i \partial x_j}\right]$ positive definite matrix

i.e. $\forall x \neq 0 : \underline{x}^T \underline{\underline{H}}\, \underline{x} > 0 \Rightarrow$ eigen values > 0

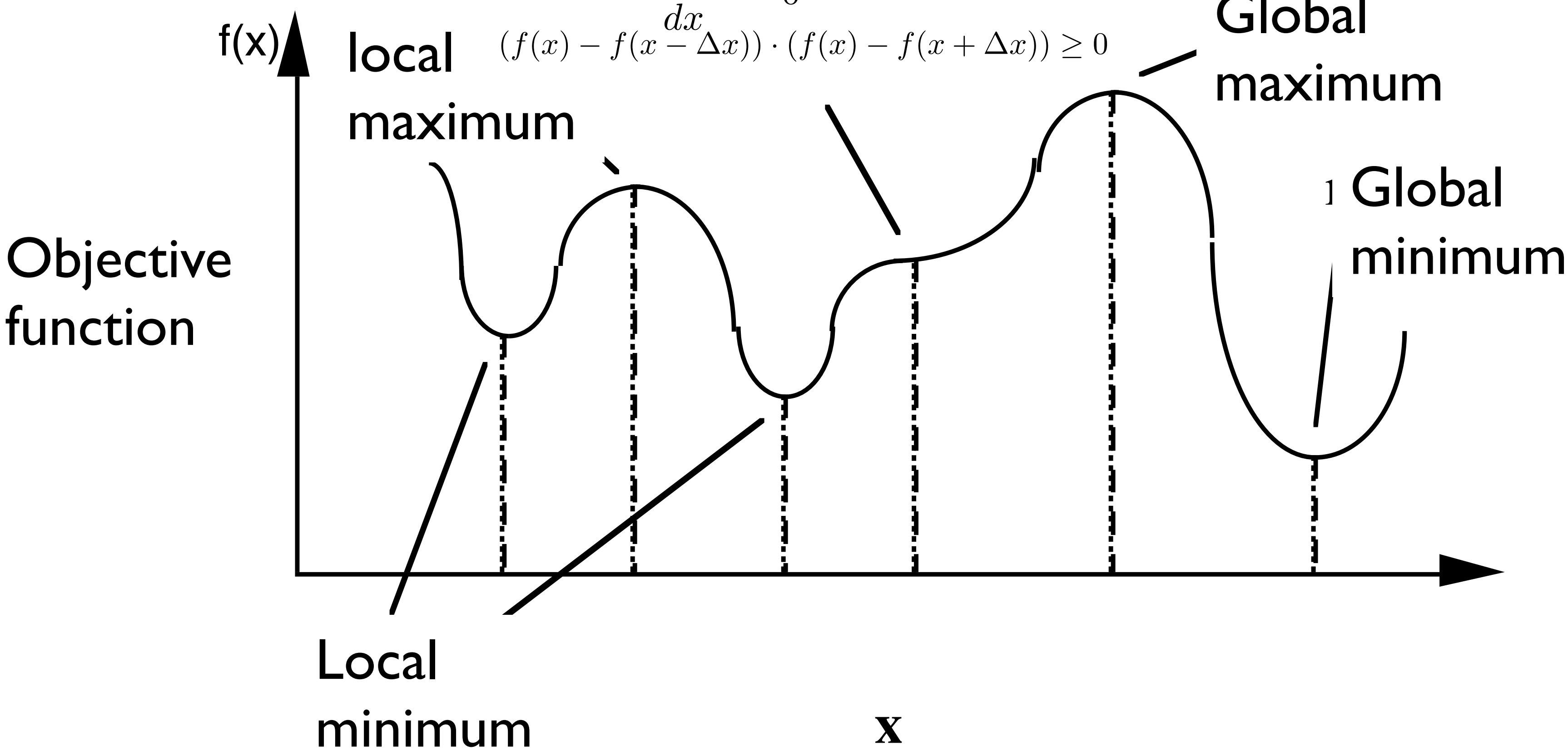
Definition of the optimum : min or max f(x)

$$max(f(x) : x|\frac{df(x)}{dx} = 0, x_{min} \leq x \leq x_{max})$$

Inflection point

$$\frac{df(x)}{dx} = 0$$

$$(f(x) - f(x - \Delta x)) \cdot (f(x) - f(x + \Delta x)) \geq 0$$



$$\frac{df(x)}{dx} = 0$$

$$(f(x) - f(x - \Delta x)) \cdot (f(x) - f(x + \Delta x)) < 0$$

Constraints

- Constrained optimisation problems
  - Inequality on the decision variables

    $$X_{min} \leq X \leq X_{max}$$

  - Inequality constraints
    - Linear $A \cdot X + B \leq 0$
    - Non linear $G(X) \leq 0$
  - Equality contraints
    - Linear $A \cdot X + B = 0$
    - Non linear $H(X) = 0$

Different optimisation problems

Continuous Variables

$$x_i^{\min} \leq x_i \leq x_i^{\max} \qquad \forall i = 1,\ldots,n_x$$

Linear
Objective and equations

$$\min C = \sum_{i=1}^{n_x} a_i * x_i + \sum_{j=1}^{n_y} b_j * y_j$$

$$soumis\ à \qquad AX + BY \begin{smallmatrix} = \\ \leq \\ \geq \end{smallmatrix} RHS$$

$$(m \times n_x) \times n_x + (m \times n_y) \times n_y = m$$

LP

MILP

ILP

NLP

MINLP

INLP

Non linear
Objective and equations

$$\min C = C(X,Y)$$

$$soumis\grave{a} \qquad H(X,Y) = 0$$

$$G(X,Y) \geq 0$$

Discrete (integer) variables

$$y_j\, \varepsilon \left\{0,1,2,3,\ldots,n_j\right\} \qquad \forall j = 1,\ldots,n_y$$

Definitions

- LP : Linear Programming
- NLP : Non Linear Programming
- MILP : Mixed Integer Linear Programming
- MINLP : Mixed Integer Non Linear Programming
- ILP : Integer Linear Programming
- INLP : Integer Non Linear Programming

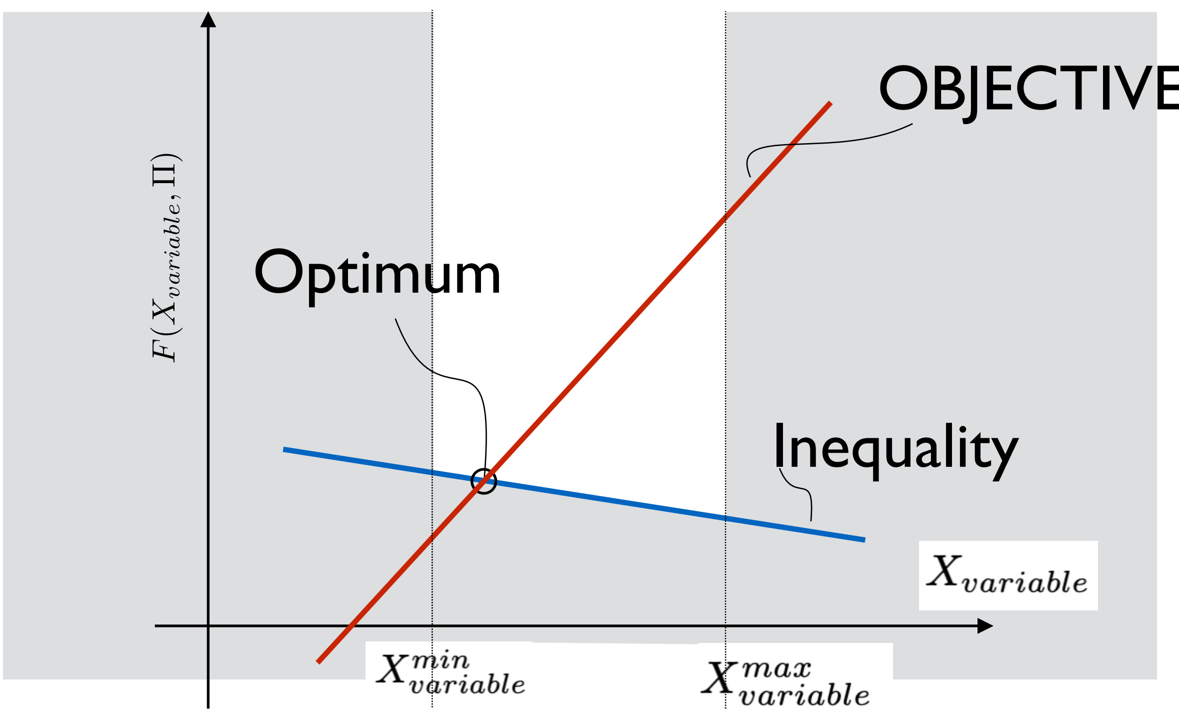
Linear programming problems

$$\min_{X_{variable}} OBJECTIVE = X_{variable} \cdot \Pi_{objective}$$

$$F(X_{variable}, \Pi) \equiv X_{variable} \cdot \Pi_{equality} = \Pi_{RHS_{equality}}$$

$$G(X_{variable}, \Pi) \equiv X_{variable} \cdot \Pi_{inequality} \geq \Pi_{RHS_{inequality}}$$

$$X_{variable}^{min} \leq X_{variable} \leq X_{variable}^{max}$$



In linear programming, the minimum is always defined by the activation of a set of inequality constraints that become equality constraints