

Annualising the investment

Prof François Marechal

$$TotalCost[\$/year] = OPEX + CAPEX + Tax$$

$$OPEX = \sum_{p=1}^{n_p} \left(\sum_{r=1}^{n_r} \dot{m}_{r,p}^+ c_{r,p}^+ + \dot{E}_p^+ c_{e,p}^+ - \dot{E}_p^- c_{e,p}^- + \sum_{u=1}^{n_u} f_{u,p} c m_u \right) d_p$$

$$CAPEX = \sum_{u=1}^{n_u} \frac{1}{\tau(n_{y,u}, i)} (I(S_u))$$

$$Tax = \gamma^{CO_2^+} \cdot \left(\sum_{p=1}^{n_p} \left(\sum_{r=1}^{n_r} \dot{m}_{r,p}^+ \epsilon_r^{CO_2} + \dot{E}_p^+ \epsilon_{e,p}^{CO_2^+} - \dot{E}_p^- \epsilon_{e,p}^{CO_2^-} \right) d_p \right)$$

- Why ?
 - The Investment value (I [\$]) to be compared with annual income and expenses (OPEX [\$ /year])
 - Money value of today (CAPEX) to be compared with future income or expenses (OPEX)

Expected Cost in [CHF] of an investment (I) after n years of operation under an interest rate of i

$$I^*(i, n_y) = I(1 + i)^{n_y} \quad [\$]$$

Where :

I : Investment in [\\$]

i : interest rate

n_y : expected lifetime [year]

$I^*(i, n_y)$: Value of I after n_y years with the interest rate of i

The interest rate is the one that is expected by the stakeholders of the company
Typically higher than the one proposed by a bank and higher than inflation rate

Value in [CHF] of “n” annual paiements B [CHF/year] after n_y years with an interest rate of i

$$B^*(i, n_y) = \sum_{r=1}^{n_y} B(1+i)^{r-1} = B \frac{(1+i)^{n_y} - 1}{i}$$

Each year : I'm investing my paiement at the interest rate i up to the end of the lifetime of the equipment

B [CHF/year] : is the annual profit = Sales - Expenses

B is made each year when the process is operational

We will assume that n is the same lifetime of the equipment

- If the paiement B is the annual income of the project

$$B = \sum_o c_o \cdot \dot{m}_o^- - \sum_i c_i \cdot \dot{m}_i^+$$

- Present value V [in CHF today] of an annual income B after n years under an interest rate of i

$$V^*(i, n_y) = B^*(i, n_y) \rightarrow V[CHF] = B \frac{(1 + i)^{n_y} - 1}{i(1 + i)^{n_y}}$$

This is the equivalent investment in today's CHF of the expected annual income during n years with an interest rate of i

- $B[CHF_{year_{project}}/year]$ can be compared with the equivalent investment I $[CHF_{year_{project}}]$ made today :

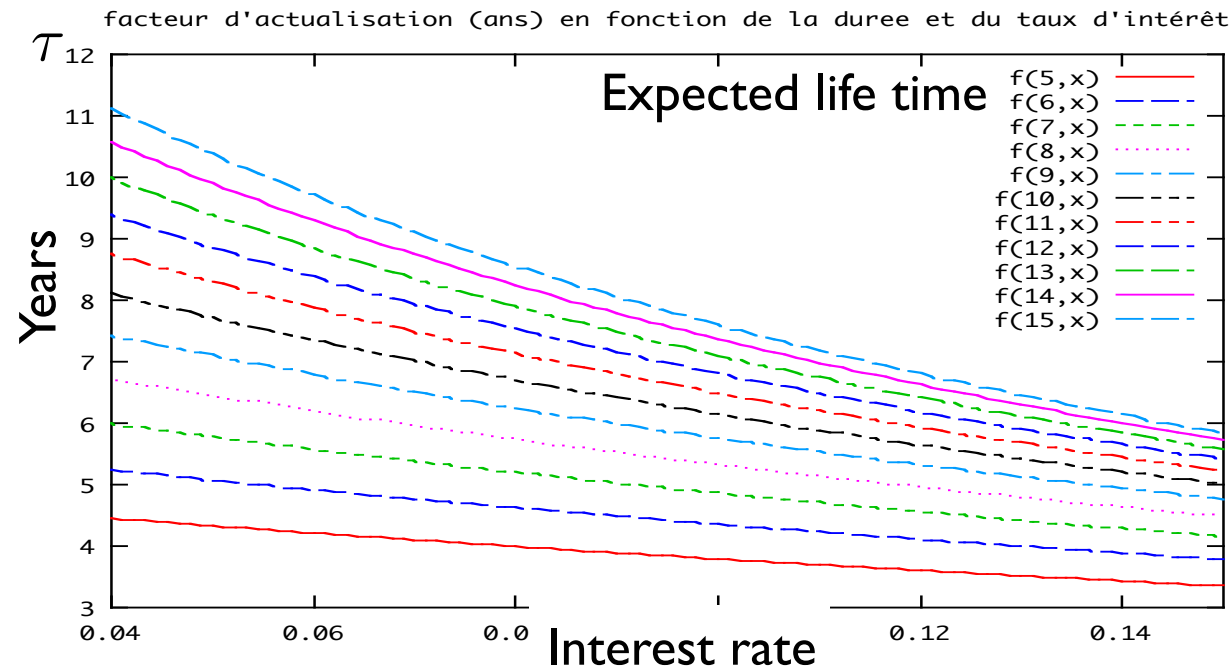
$$I^*(i, n_y) = IC^*(i, n_y) \rightarrow IC[CHF/year] = I \frac{i(1+i)^{n_y}}{(1+i)^{n_y} - 1}$$

This is the equivalent income in today's CHF/year of the investment I during n years with an interest rate of i

- Net present value : $NPV[CHF_{year_{project}}] = B \frac{(1+i)^n - 1}{i(1+i)^n} - I \geq 0$

- Annualised Cost :

$$Profit[CHF_{year_{project}}/year] = B - \frac{i(1+i)^n}{(1+i)^n - 1} \cdot I = B - \frac{1}{\tau} \cdot I \geq 0$$



Over how many years the investment has to be “amortised”

Sales	$\Delta Sales[CHF/year]$
$\int_{t_0}^{t_f} \left(\sum_{P=1}^{n_P} \Delta \dot{M}_P^-(t) c_P^-(t) + \Delta E^-(t) c_e^- \right) dt$	
Operating costs	$\Delta Cost[CHF/year]$
$\int_{t_0}^{t_f} \left(\sum_{R=1}^{n_R} \Delta \dot{M}_R^+(t) c_R^+(t) + \sum_{F=1}^{n_F} \Delta \dot{M}_F^+(t) c_F^+(t) + \Delta E^+(t) c_e^+ + \sum_{W=1}^{n_W} \Delta \dot{M}_W^-(t) c_W^-(t) \right) dt$	
Profit [CHF/year]	
$\Delta B[CHF/year] = \Delta Sales[CHF/year] - \Delta Cost[CHF/year]$	
Investments : CAPEX [CHF/year]	
$\Delta IC[CHF/year] = \Delta I \frac{i(1+i)^n}{(1+i)^n - 1}$	

$$Profit[\$/year] = \Delta OPEX + \Delta Tax - \Delta CAPEX$$

$$\Delta OPEX + \Delta Tax \quad [\$ / year] = (OPEX_{ref} - OPEX_{new}) + (Tax_{ref} - Tax_{new}) \geq 0$$

$$\Delta CAPEX \quad [\$ / year] = CAPEX_{ref} - CAPEX_{new} \leq 0$$

$$\text{Rate of Return [years]} = \frac{\Delta I}{\Delta B} \quad \text{How many years are needed to recover the investment Risk !}$$

$$\text{Net Present Value [\$]} = \Delta B \frac{(1+i)^{n_y} - 1}{i(1+i)^{n_y}} - \Delta I \geq 0 \quad \text{After } n_y \text{ years}$$

$$\text{Annualised Profit } \left[\frac{\$}{\text{year}}\right] = \Delta B - \Delta I \frac{i(1+i)^{n_y}}{(1+i)^{n_y} - 1} = \Delta B - \frac{I}{\tau} \geq 0$$

$$\text{Internal interest rate } [\%] \quad i^* | \Delta B \frac{(1+i^*)^{n_y} - 1}{i^*(1+i^*)^{n_y}} - \Delta I = 0$$

$$\Delta I \quad [\$] \quad \text{Additional investment}$$

$$\Delta B \quad \left[\frac{\$}{\text{year}}\right] \quad \text{Annual expected profit from the investment}$$

$$n_y \quad [\text{year}] \quad \text{Expected life time of the equipment}$$

$$\Delta I \quad [\$] = - \Delta CAPEX \quad [\$/\text{year}] \cdot \tau(i, n_y) \quad [\text{year}]$$

!!! Annual expected benefit is assumed with the energy price of today !!!