The Energy Supply System

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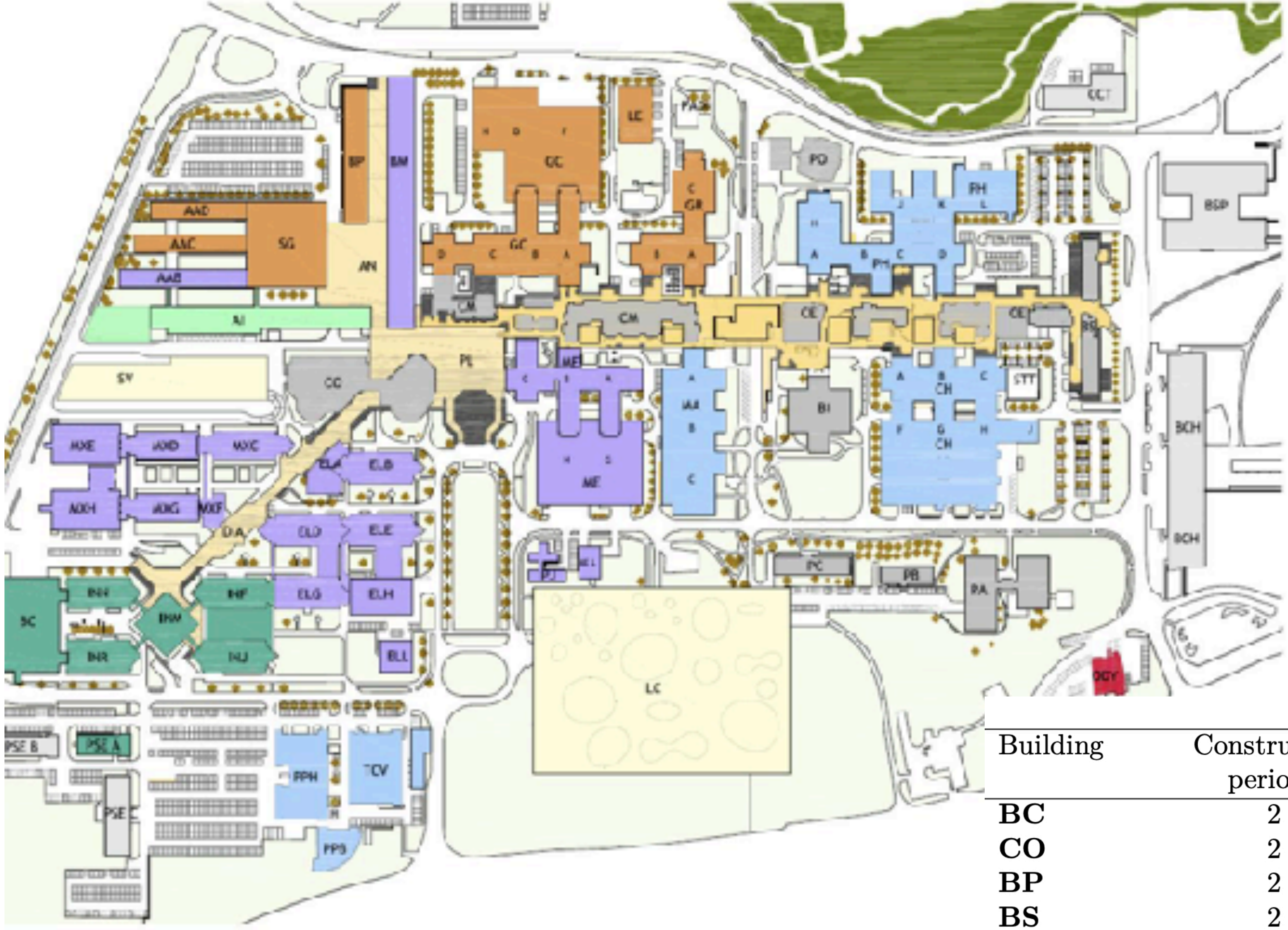
IPESE
Industrial Process
and Energy Systems
Engineering

Part1 : defining the building needs

Buildings stock



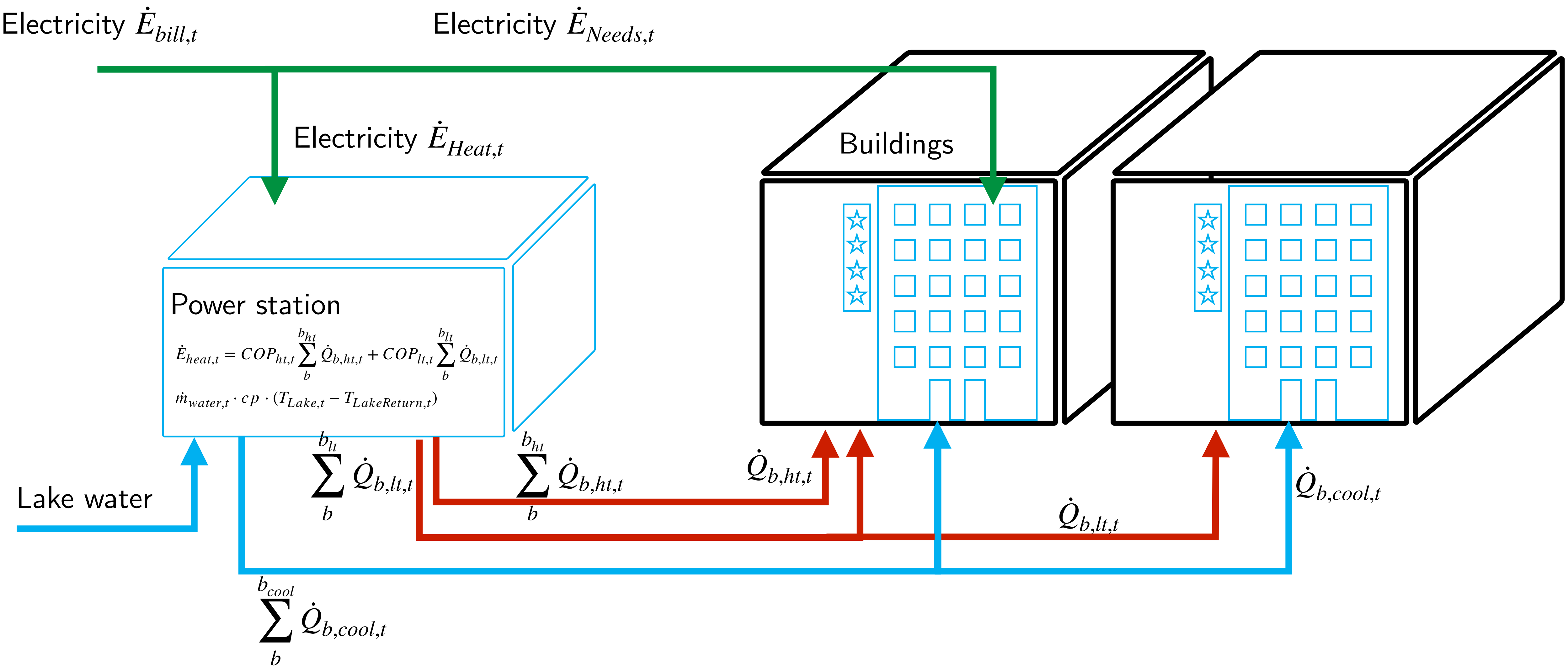
Measures

Table 1.1: EPFL Buildings

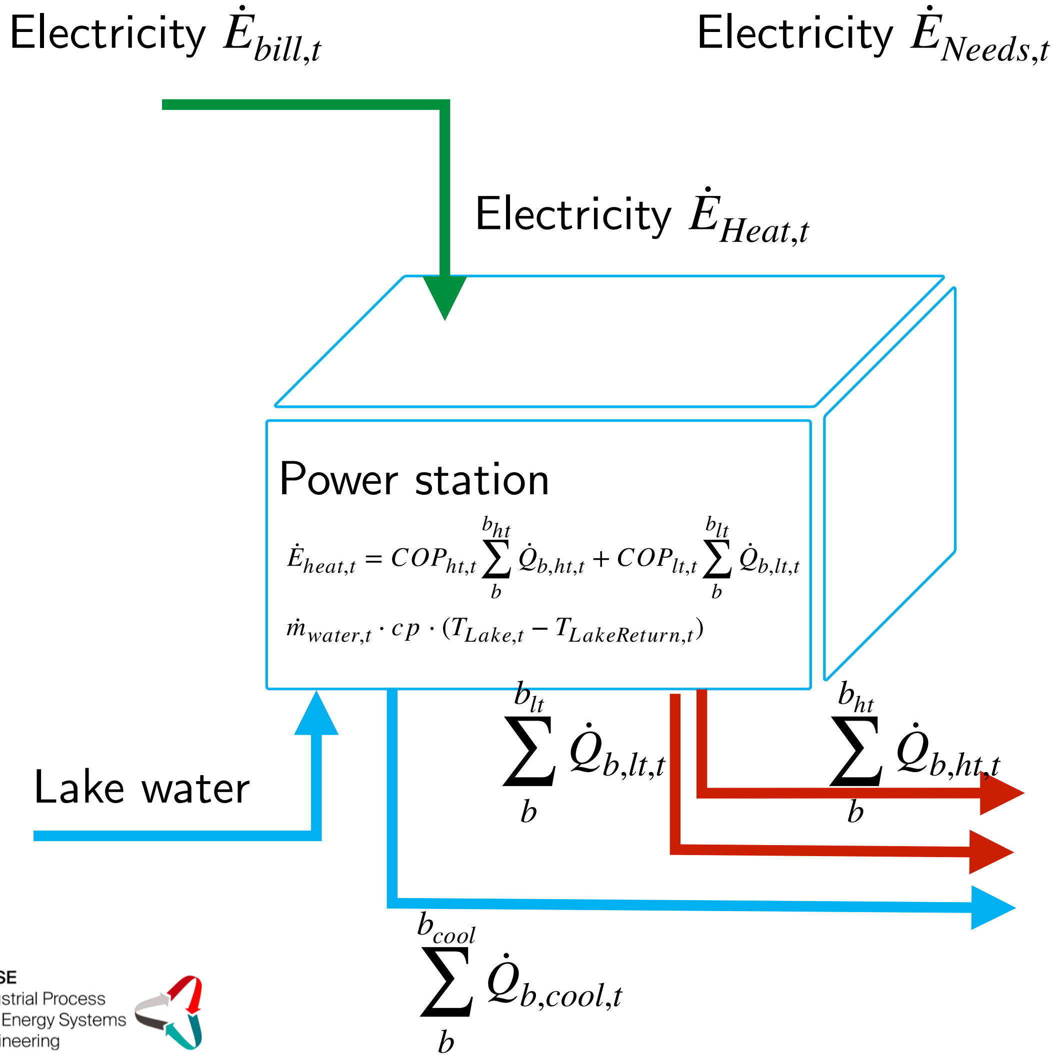
Building	Construction period[a]	Heated surface A_{th} [m^2]	Annual heat demand Q_{th} [kWh]	Annual electricity demand Q_{el} [kWh]
BC	2	17480	418,491	1,603,596
CO	2	11901	477,008	943,653
BP	2	10442	457,861	691,031
BS	2	10267	509,183	350,860
TCV	2	6095	318,209	2,067,675

- The president has first asked his colleagues to make some suggestions of the options :
  - Prof of Architecture : you have to refurbish the buidlings
  - Prof of Heat exchangers : recover the heat of the data centers and the hot air in buildings
  - Prof of Compressor : use multi-level heat pumps, choose the right fluid
  - Prof of Photovoltaic : use PV panels
  - Prof of Fuel cells : use a solid oxide fuel cell
  - Prof of Bio-engineering : use biomethane
  - Prof of Geology : use deep geothermal sources
  - Prof of Water Treatment : use of the heat of the waste water
  - Prof of Energy system : look at the system integration
  - Prof of power systems : use my battery
  - Prof of Climate and economics : take into account the global warming
  - Vice-president finance : I do not have money for that
  - ETH board : demonstrate the sustainability

The energy system of EPFL



The energy conversion system



- For each distribution system d (e.g. ht)
  - Demand : $\dot{Q}_{d,t} = \sum_{b}^{n_b} \dot{Q}_{b,d,t} \qquad \forall t \in lifetime$
- For each unit u (e.g. heat pump}
  - Supply by unit u : $\sum_{u=1}^{n_u} \dot{Q}_{u,d,t} = \dot{Q}_{d,t} \quad \forall t \in lifetime$
  - and $\dot{Q}_{u,t} = \sum_{d}^{n_d} \dot{Q}_{u,d,t}$
- Cost of the supply
  - Size of the conversion equipment :
    $$\sum_{u=1}^{n_u} \max_{t \in lifetime} (\dot{Q}_{u,d,t}) = \max_{t \in lifetime} (\dot{Q}_{d,t})$$
  - $\dot{Q}_{u,max} = \max_{t \in lifetime} (\dot{Q}_{u,t})$
  - Buy the resources : $\sum_{r=1}^{n_{res}} \sum_{u}^{n_u} \left(\int_{t_0}^{lifetime} c_{r,t} \cdot m_{r,u,t} \cdot \dot{Q}_{u,t} \cdot dt \right)$

Energy conversion OPEX

$$\sum_{r=1}^{n_{res}} \sum_{u}^{n_u} \left(\int_{t_0}^{lifetime} c_{r,t} \cdot m_{r,u,t} \cdot \dot{Q}_{u,t} \cdot dt \right) \quad [CHF/lifetime]$$

- $c_{r,t}$ $[CHF/unit_r]$ cost of one unit of resource r at time t
  - e.g. kg of water, kg of fuel, kJ of fuel or kWh of electricity
  - $c_{r,t} \leq 0$ for products (e.g. electricity production)
- $m_{r,u,t}$ $[unit_r/kJ_{th}]$ unit of resource r used to deliver one $[kJ_{th}]$ of heat by unit u at time t
- $\dot{Q}_{u,t}$ $[kW/unit_r]$ heat delivered by unit u at time t
- $lifetime$ $[s]$ expected lifetime of the project

OPEX in [CHF/year]

- OPerating EXpenditure (we assume a typical year of operation) :

  Cost of resources $\sum_{r=1}^{n_{res}} \sum_{u}^{n_u} \left(\int_{t_0}^{year} c_{r,t} \cdot m_{r,u,t} \cdot \dot{Q}_{u,t} \cdot dt \right) \quad [CHF/year]$

  with $m_{r,u,t}$ [kg/MJ] is the resource consumption per unit of heat $Q_{u,t}$

  \+ Maintenance [CHF/year]

  \+ Men Power [CHF/year]

  \+ Taxes [CHF/year] :

  fixed : e.g. based on installed power

  proportional : $\sum_{r=1}^{n_{res}} \sum_{u}^{n_u} \left(\int_{t_0}^{year} t_{r,t} \cdot m_{r,u,t} \cdot \dot{Q}_{u,t} \cdot dt \right) \quad [CHF/year]$

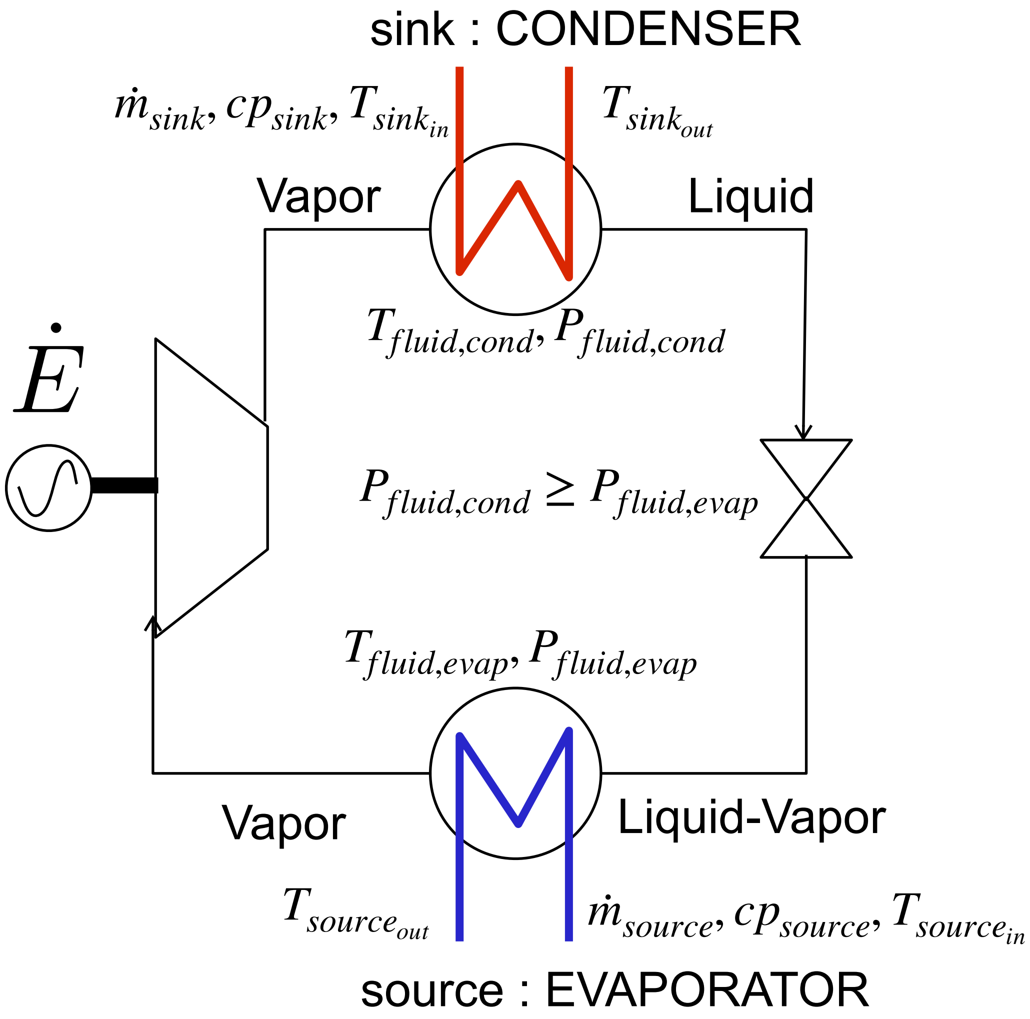
  with $t_{r,t} \quad [CHF/unit_r]$ tax per unit of r

  e.g. $t_{r,t} = \tau_{CO_2}[CHF/kg_{CO_2}] \cdot m_{CO_2,r}[kg_{CO_2}/unit_r]$

Calculating $m_{r,u,t} \quad [unit_r/kJ_{th}]$ for a Cogeneration unit

- resource use is calculated with a multiplication factor of a reference production $m_{r,u,t} \quad [unit_r/kJ_{th}]$
  - Thermal efficiency : $\eta_{th,u} = \dfrac{\dot{Q}_{th,u}}{\dot{m}_{r,u}} \quad [kJ_{th}/kJ_r]$
  - Electrical efficiency : $\eta_{e,u} = \dfrac{\dot{E}_u}{\dot{m}_{r,u}} \quad [kJ_e/kJ_r]$
- $\dot{E}_{u,t} = \dfrac{\eta_{e,u}}{\eta_{th,u}} \cdot \dot{Q}_{u,t}$

Calculating $m_{r,u,t}$ $[kJ_e/kJ_{th}]$ for a heat pump : approximation



$$\dot{Q}_{sink} = \dot{Q}_{u,t} = \dot{m}_{sink,t} \cdot cp_{sink} \cdot (T_{sink_{out,t}} - T_{sink_{in,t}})$$

$$\dot{E}_{u,t} = \frac{\dot{Q}_{u,t}}{COP_{u,t}}$$

$$COP_{u,t} = \eta_{COP} \cdot COP_{th,u,t}$$

$\eta_{COP} = 50\,\%$ or is a fitted curve

$$COP_{th,u,t} = \frac{\tilde{T}_{sink,t}}{\tilde{T}_{sink,t} - \tilde{T}_{source,t}}$$

$$\tilde{T}_{sink,t} = \frac{T_{sink_{out,t}} - T_{sink_{in,t}}}{ln(T_{sink_{out,t}}) - ln(T_{sink_{in,t}})}$$

$$\tilde{T}_{source,t} = \frac{T_{source_{out,t}} - T_{source_{in,t}}}{ln(T_{source_{out,t}}) - ln(T_{source_{in,t}})}$$

$$\dot{Q}_{source,t} = \dot{m}_{source,t} \cdot cp_{source} \cdot (T_{source_{in,t}} - T_{source_{out,t}})$$

$$\dot{Q}_{source,t} = \dot{Q}_{u,t} - \dot{E}_{u,t}$$

Total Cost (TOTEX in [CHF/year])

- TOTal EXpenditure :

$$TOTEX \quad [CHF/year] = OPEX \quad [CHF/year] + CAPEX \quad [CHF/year]$$

it assumes that the unit will be operated with the same power profile over the lifetime of the equipment

We assume a mean year and the associated costs as being representative of the lifetime of the project

Mixed Integer Linear Porgamming problem

Objective function

The objective is to minimize the **total annualized cost** function:

$$TOTEX = OPEX + CAPEX + ENVEX \qquad [CHF/year] \tag{1}$$

where:

- **Annual operating cost**:

Define the flows =>

$$OPEX = \int_0^{\mathrm{t^{op}}} \sum_r^{\mathbf{R}} \mathrm{c}_r(t)\dot{m}_r(t)dt + \sum_u^{\mathbf{U}} \mathrm{c}_u S_u + \int_0^{\mathrm{t^{op}}} \mathrm{c}_e^+(t)\dot{E}^+(t)dt - \int_0^{\mathrm{t^{op}}} \mathrm{c}_e^-(t)\dot{E}^-(t)dt \tag{2}$$

$$\approx \sum_t^{\mathbf{T}} \sum_r^{\mathbf{R}} \mathrm{c}_{r,t}\dot{m}_{r,t}\Delta\mathrm{t}_t + \sum_u^{\mathbf{U}} \mathrm{c}_u^{\mathrm{mt}} S_u + \sum_t^{\mathbf{T}} \mathrm{c}_{e,t}^+ \dot{E}_t^+ \Delta\mathrm{t}_t - \sum_t^{\mathbf{T}} \mathrm{c}_{e,t}^- \dot{E}_t^- \Delta\mathrm{t}_t \qquad [CHF/year] \tag{3}$$

- **Annualized investment cost**:

Define the sizes =>

$$CAPEX = \frac{1}{\tau}\sum_u^{\mathbf{U}} I(s_u) \approx \frac{1}{\tau}\sum_u^{\mathbf{U}} \left(\mathrm{c}_u^{\mathrm{inv1}} y_u + \mathrm{c}_u^{\mathrm{inv2}} s_u\right) \qquad [CHF/year] \tag{4}$$

- **Annual cost related to emissions**:

$$ENVEX = \mathrm{c}_{co_2} \sum_t^{\mathbf{T}} \left[\dot{m}_{co_2,t}\Delta\mathrm{t}_t + \mathrm{k}_{co_2,t}\left(\dot{E}_t^+ - \dot{E}_t^-\right)\Delta\mathrm{t}_t\right] \qquad [CHF/year] \tag{5}$$

Energy and mass balances

- **Heat distribution** $\forall t \in \mathbf{T}\,,\;\; \forall d \in \mathbf{D}$ with $T_{d+1,t} \geq T_{d,t}$

$$\sum_{u}^{\mathbf{U}} \dot{\mathrm{q}}_{u,d} f_{u,t} - \dot{Q}_{d,t} = \varnothing \tag{13}$$

Balance demand and supply =>

$$\dot{Q}^{\mathrm{res}}_{d+1,t} + \dot{Q}_{d,t} - \sum_{b}^{\mathbf{B}} \dot{\mathrm{Q}}_{b,d,t} = \dot{Q}^{\mathrm{res}}_{d-1,t} \tag{14}$$

$$\dot{Q}_{d,t} = \dot{\mathrm{m}}_{d,t}\mathrm{cp}_{d}(\mathrm{T}^{\mathrm{s}}_{d,t} - \mathrm{T}^{\mathrm{r}}_{d,t}) \tag{15}$$

$$\dot{Q}^{\mathrm{res}}_{0,t} = \varnothing, \quad \dot{Q}^{\mathrm{res}}_{n_{d+1},t} = \varnothing \tag{16}$$

$$\dot{Q}^{\mathrm{res}}_{d,t} \geq \varnothing \tag{17}$$

Technology constraints and modeling equations

- **Inequality constraints** $\forall t \in \mathbf{T}\,,\ \ \forall u \in \mathbf{U}$

Choosing the option =>

$$\mathrm{f}_u^{\min} y_u \leq f_u \leq \mathrm{f}_u^{\max} y_u \tag{6}$$

$$\mathrm{f}_u^{\min} y_{u,t} \leq f_{u,t} \leq \mathrm{f}_u^{\max} y_{u,t} \tag{7}$$

$$f_{u,t} \leq f_u \tag{8}$$

- **Modeling equations**

  example: heat pump

  u = heat pump

$$\dot{\mathrm{e}}_u^{+} = \frac{\dot{\mathrm{q}}_u^{-}}{\mathrm{COP}} \tag{9}$$

$$\mathrm{COP} = \left(\frac{\tilde{\mathrm{T}}^{\mathrm{sink}}}{\tilde{\mathrm{T}}^{\mathrm{sink}} - \tilde{\mathrm{T}}^{\mathrm{source}}} \right) \eta_{\mathrm{carnot}} \tag{10}$$

η_{Carnot} : fitted curve $\eta_{Carnot}(T_{sink}, T_{source}, \dot{Q})$

The heat pump COP can be calculated as a function of the temperature at each time step t and distribution system d considered:

u = heat pump, $\forall t \in \mathbf{T}\,,\ \ \forall d \in \mathbf{D}$

$$\dot{\mathrm{e}}_{u,d,t}^{+} = \frac{\dot{\mathrm{q}}_{u,d,t}^{-}}{\mathrm{COP}} \tag{11}$$

Constant for the optimisation =>

$$\mathrm{COP}_{d,t} = \left(\frac{\tilde{\mathrm{T}}_{d,t}^{\mathrm{sink}}}{\tilde{\mathrm{T}}_{d,t}^{\mathrm{sink}} - \tilde{\mathrm{T}}_{d,t}^{\mathrm{source}}} \right) \eta_{\mathrm{carnot}} \tag{12}$$

- **Electricity balance** $\forall t \in \mathbf{T}$

Flows of resources =>

$$\dot{E}_t^{+} - \dot{E}_t^{-} + \sum_{u}^{\mathbf{U}} f_{u,t}\dot{\mathrm{e}}_u^{-} - \sum_{u}^{\mathbf{U}} f_{u,t}\dot{\mathrm{e}}_u^{+} - \sum_{b}^{\mathbf{B}} \dot{\mathrm{E}}_{b,t} = \varnothing \tag{18}$$

- **Resource and material balance** $\forall t \in \mathbf{T}\,,\ \forall r \in \mathbf{R}$

Flows of resources =>

$$\dot{m}_{r,t} = \sum_{u}^{\mathbf{U}} f_{u,t}\dot{\mathrm{m}}_{r,u} \tag{19}$$

example: emissions of CO_2 $\forall t \in \mathbf{T}\,,\ r = co_2$

Flows of emissions =>

$$\dot{m}^{-}_{\mathrm{co}_2,t} = \sum_{u}^{\mathbf{U}} f_{u,t}\dot{\mathrm{m}}_{\mathrm{co}_2,u} \tag{20}$$

The goal is to calculate the set of variables $f_{u,t},\ y_{u,t},\ f_u,\ y_u,\ \dot{E}_t^{-},\ \dot{E}_t^{+},\ \dot{m}_{r,t},\ \dot{Q}_{d,t},\ \dot{Q}_{d,t}^{\mathrm{res}}$ that minimizes the objective TOTEX.

Variables (to be calculated) and parameters (fixed)

Index and set	Description
$t \in \mathbf{T}$	Time $\mathbf{T} = \{t_1 \dots t_{n_t}\}$
$u \in \mathbf{U}$	Utility $\mathbf{U} = \{\text{boiler, heat pump, refrigeration, ...}\}$
$r \in \mathbf{R}$	Resource $\mathbf{R} = \{\text{hydrogen, natural gas, chocolate, ...}\}$
$d \in \mathbf{D}$	Distribution system $\mathbf{D} = \{d_1 \dots d_{n_d}\}$
$b \in \mathbf{B}$	Building $\mathbf{B} = \{b_1 \dots b_{n_b}\}$

<= SETS : are groups of the same type and scopes in the problem

Parameter	Description
t^{op}	Total operating time per year [h/year]
$\Delta \mathrm{t}_t$	Duration of time interval t [h]
$\mathrm{c}_{r,t}$	Specific cost of resource r at time t [CHF/kg]
$\mathrm{c}^{+}_{e,t}$	Price for purchased electricity at time t [CHF/kWh]
$\mathrm{c}^{-}_{e,t}$	Price for sold electricity at time t [CHF/kWh]
$\mathrm{c}^{\mathrm{inv1}}_{u}$	Fixed investment cost of utility u [CHF]
$\mathrm{c}^{\mathrm{inv2}}_{u}$	Variable investment cost of utility u [CHF/size]
$\mathrm{c}^{\mathrm{mt}}_{u}$	Maintenance specific cost of utility per yea u [CHF/size/year]
$\frac{1}{\tau}$	Annualization factor of investment [1/year]
$\mathrm{f}^{\mathrm{min}}_{u}$	Minimum sizing factor of utility u [-]
$\mathrm{f}^{\mathrm{max}}_{u}$	Maximum sizing factor of utility u [-]
$\mathrm{k}_{co_2,t}$	CO_2 equivalent emissions of the grid at time t [ton_{co_2}/kWh]
$\dot{\mathrm{m}}_{r,u}$	Reference mass flow of resource r in utility u [kg/s]
$\dot{\mathrm{q}}^{-}_{u}$	Reference heat load produced by utility u [kW]
$\dot{\mathrm{q}}^{+}_{u}$	Reference heat load consumed by utility u [kW]
$\dot{\mathrm{e}}^{-}_{u}$	Reference electrical power produced by utility u [kW]
$\dot{\mathrm{e}}^{+}_{u}$	Reference electrical power consumed by utility u [kW]
$\mathrm{T}_{d,t}$	Logarithmic mean temperature of the distribution system d at time t [K]
$\mathrm{T}^{s}_{d,t}$	Supply temperature of the distribution system d at time t [K]
$\mathrm{T}^{r}_{d,t}$	Return temperature of the distribution system d at time t [K]
$\dot{\mathrm{m}}_{d,t}$	Mass flow in the distribution system d [kg/s]
cp_{d}	Heat capacity of the fluid in the distribution system d [kJ/K/kg]
$\dot{\mathrm{q}}_{u,d}$	Reference heat load of utility u fed in the distribution system d [kW]
$\dot{\mathrm{Q}}_{b,d,t}$	Heat load demand of building b at time t in the distribution system d [kW]
$\dot{\mathrm{E}}_{b,t}$	Electricity consumption of building b at time t [kW]
COP	Coefficient Of Performance of the heat pump [-]
$\tilde{\mathrm{T}}_{sink}$	Logarithmic mean temperature of the heat sink of the heat pump [K]
$\tilde{\mathrm{T}}_{source}$	Logarithmic mean temperature of the heat source of the heat pump [K]
η_{carnot}	Carnot efficiency [-]

<= Parameters calculated before solving the MILP problem

Variables => min and max bounds

Variable	Description
f_u	Sizing factor of utility u [-]
$f_{u,t}$	Sizing factor of utility u at time t [-]
y_u	Binary variable to use or not utility u [-]
$y_{u,t}$	Binary variable to use or not utility u at time t [-]
$\dot{E}^{+}_{t}$	Purchased electrical power at time t [kW]
$\dot{E}^{-}_{t}$	Sold electrical power at time t [kW]
$\dot{m}_{r,t}$	Mass flow of resource r at time t [kg/s]
$\dot{Q}_{d,t}$	Total utility heat load at time t in the distribution system d [kW]
$\dot{Q}^{\mathrm{res}}_{d,t}$	Heat load cascaded from distribution system d to $d+1$ at time t [kW]

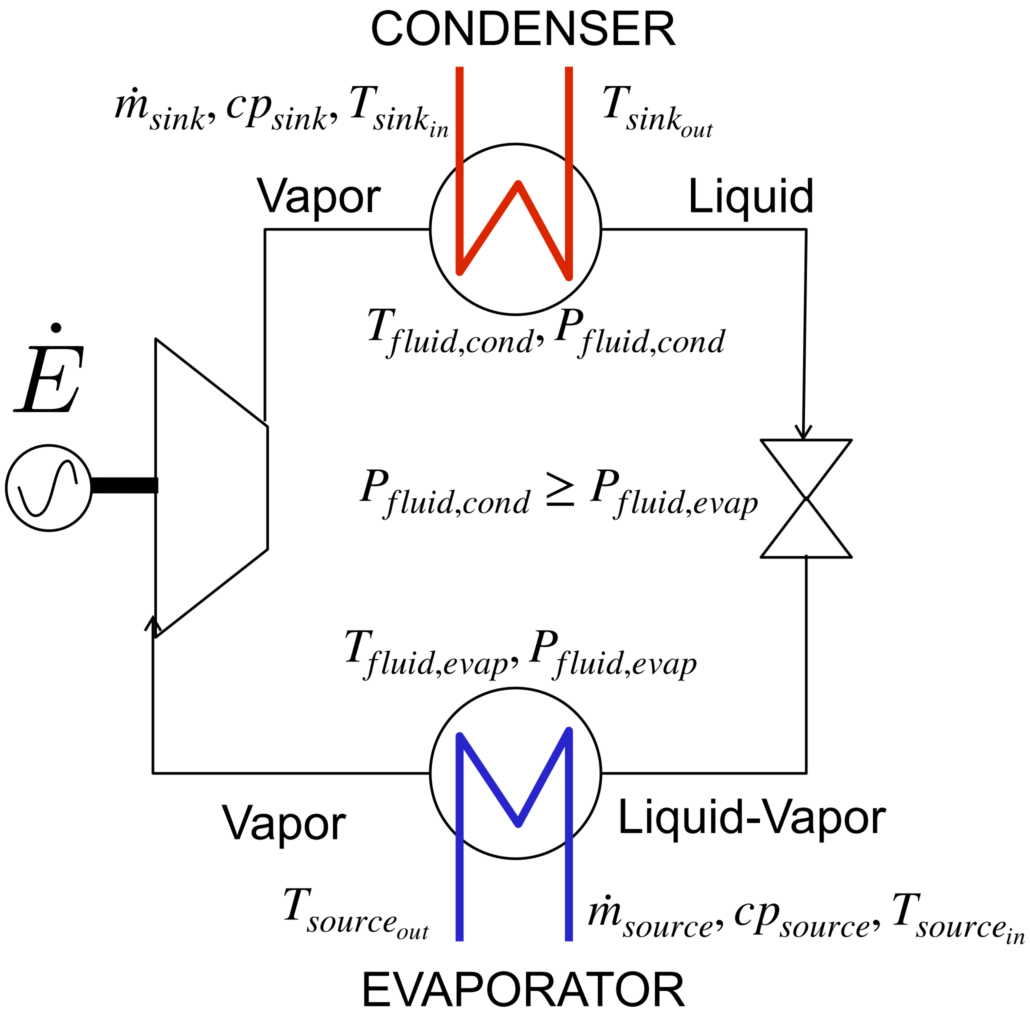
CAPEX energy conversion system [CHF/year]

- CAPital EXpenditure

$$CAPEX \quad [CHF/year] = \sum_{u=1}^{n_u} \frac{1}{\tau_u} \cdot i_{u_{\dot{Q}_{u,max}}} \cdot \dot{Q}_{u,max}$$

  - $i_{u_{\dot{Q}_{u,max}}} \quad [CHF/kW]$ specific investment of unit u for size $\dot{Q}_{u,max}$
    - $\dot{Q}_{u,max} = \max_{t \in lifetime} (\dot{Q}_{u,t})$ [kW] size of unit u
    - MILP problem representation : $\dot{Q}_{u,t} \cdot f_{u,t} \leq \dot{Q}_{u,max} \qquad \forall t$
  - $\frac{1}{\tau_u} = \frac{1}{\tau(i, lifetime_u)} \quad [year^{-1}]$: annualisation factor of unit u

Calculating $i_{u_{\dot{Q}_{u,max}}}$ $[CHF/kW]$ for a heat pump



Sizing : condenser, evaporator & compressor

$$\dot{Q}_{u,t} = \dot{Q}_{sink,t} = \dot{m}_{sink,t} \cdot cp_{sink} \cdot (T_{sink_{out},t} - T_{sink_{in},t})$$

$$\dot{Q}_{sink,t} = \dot{m}_{fluid,t} \cdot (h(T_{fluid,cond_{in},t}, P_{fluid,cond_{in},t} - h(T_{fluid,cond_{out},t}, P_{fluid,cond_{out},t})$$

$$A_{condenser} = \max_{t \in lifetime}\left(\left(\frac{1}{u_{sink}} + \frac{1}{u_{fluid,cond}}\right)\frac{\dot{Q}_{sink,t}}{\frac{(T_{fluid,cond,t} - T_{sink_{in},t}) - (T_{fluid,cond,t} - T_{sink_{out},t})}{ln(T_{fluid,cond,t} - T_{sink_{in},t}) - ln(T_{fluid,cond,t} - T_{sink_{out},t})}}\right)$$

$$A_{evaporator} = \max_{t \in lifetime}\left(\left(\frac{1}{u_{source}} + \frac{1}{u_{fluid,evap}}\right)\frac{\dot{Q}_{source,t}}{\frac{(T_{source_{out},t} - T_{fluid,evap,t} - (T_{source_{in},t} - (T_{fluid,evap,t})}{ln(T_{source_{out},t} - T_{fluid,evap,t}) - ln(T_{source_{in},t} - T_{fluid,evap,t})}}\right)$$

$$\dot{E}_{max} = \max_{t \in lifetime}\left(\frac{\dot{Q}_{u,t}}{COP_{u,t}}\right)$$

$$\dot{Q}_{source,t} = \dot{m}_{source,t} \cdot cp_{source} \cdot (T_{source_{in},t} - T_{source_{out},t})$$

$$\dot{Q}_{source,t} = \dot{m}_{fluid,t} \cdot (h(T_{fluid,evap_{out},t}, P_{fluid,evap_{out},t} - h(T_{fluid,evap_{in},t}, P_{fluid,evap_{in},t})$$

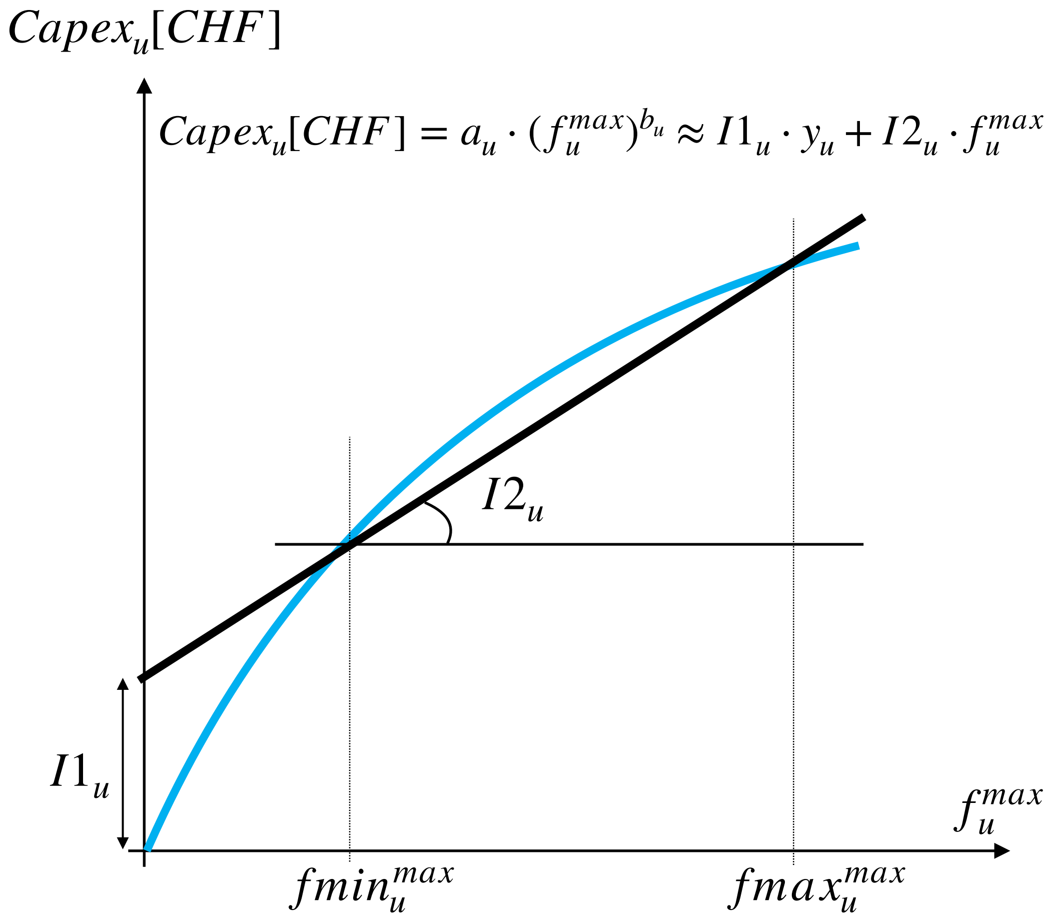
$$\dot{Q}_{source,t} = \dot{Q}_{sink,t} - \dot{E}_{u,t}$$

$$CAPEX_u = \frac{1}{\tau}(I_{HTX}(A_{condenser}) + I_{HTX}(A_{Evaporator}) + I_{compressor}(\dot{E}_{max}))$$

Investment linearised

$$CAPEX[CHF/year] = \sum_{u=1}^{n_u} \frac{1}{\tau(n_{y,u}, i)} (I1_u y_u + I2_u f_u^{max})$$

$\frac{1}{\tau(n_{y,u}, i)}$	$[\frac{1}{year}]$	annualisation factor of unit u
$n_{y,u}$	$[year]$	expected life time of unit u
$I1_u$	$[CHF]$	fixed investment of unit u
y_u	$[-]$	existence unit u
$I2_u$	$[CHF]$	proportional investment cost of unit u
f_u^{max}	$[-]$	size of unit u



$$fmin_u^{max} \cdot y_u \leq f_u^{max} \leq fmax_u^{max} \cdot y_u$$

$$f_{u,t} \leq f_u^{max} \qquad \forall t \in periods$$

Options for the building b

- The building b has n_{o_b} options to suppl the heat (e.g. air heat recovery)
  - Heat load of the building on distribution system d

$$\dot{Q}_{b,d,t} = \sum_{o_b}^{n_{o,b}} y_{o_b} \cdot \dot{Q}_{o_b,d,t}$$

  - Choosing the option

$$\sum_{o_b}^{n_{o_b}} y_{o_b} = 1$$

  - Electricity consumption

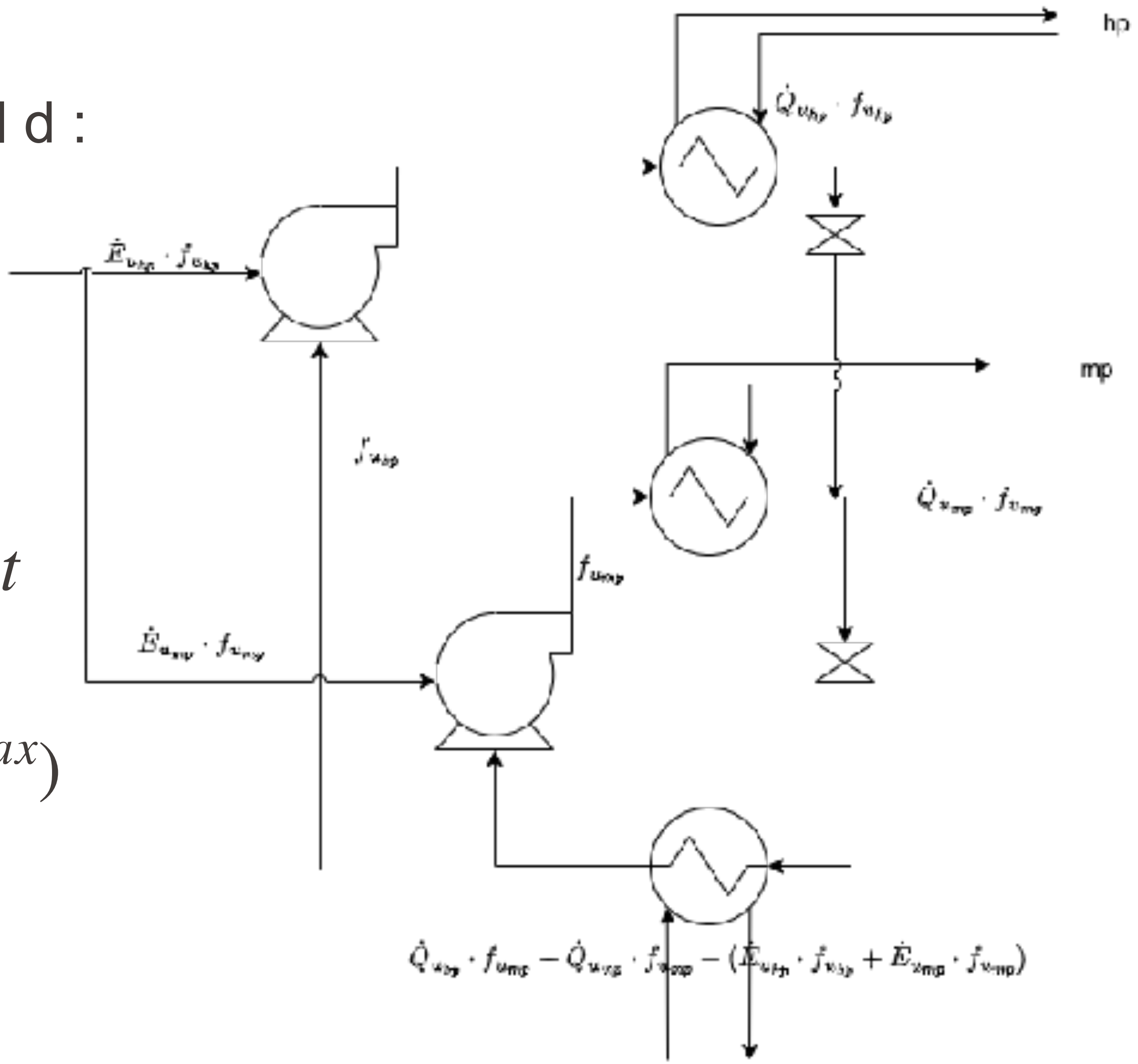
$$\dot{E}_b = \sum_{o_b}^{n_{o,b}} y_{o_b} \cdot \dot{E}_{o_b,t}$$

  - Capital cost

$$CAPEX_b = \sum_{o_b}^{n_{o_b}} CAPEX_{o_b} \cdot y_{o_b}$$

Option for multi-stage heat pump

- Heat supplied by u on distribution level d :
  $$\dot{Q}_{u,d,t} = f_{u_d,t} \cdot \dot{Q}_{u_d,hp,t}$$
- Electricity in the compressor of u :
  $$\dot{E}_{u,t} = \sum_{d}^{n_{d,u}} f_{u_d,t} \cdot \dot{Q}_{u_d,d,t} \cdot COP_{u_d,d,t}$$
- Size of the unit u : $\dot{E}_u^{max} \geq \dot{E}_{u,t} \qquad \forall t$
- CAPEX of unit u :
  $$CAPEX_u = \frac{1}{\tau_u} \cdot (I1_u \cdot y_u + I2_u \cdot \dot{E}_u^{max})$$



Multi objective optimisation with MILP problem

- As the cost of energy is really uncertain, it is difficult to claim that there is one cost optimal solution
- What if government is imposing a CO2 tax ?

Generating ordered list of system configurations

- Integer cut constraint on the equipment set $\{y_u\}$
  - assuming that we know already the solution k
  - The problem k + 1 is defined by adding to the previous MILP problem the integer cut constraint

$$Problem^{k+1}:$$

$$Problem^{k}$$

$$\sum_{i=1}^{n_y}(2y_i^k - 1) * y_i \leq \sum_{i=1}^{n_y} y_i^k$$

where y_i^k value of y_i in solution of problem k

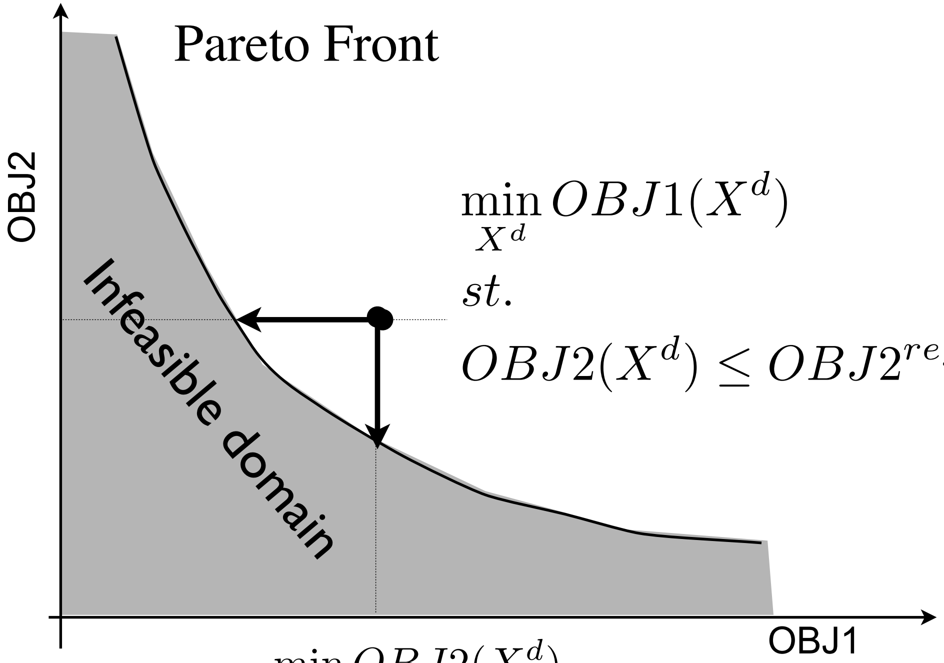
Example of multi-objective optimisation

- Cost of reducing the emissions
  - OBJ1 = total cost
  - OBJ2 = emissions
- Investment to increase the renewable energy usage
  - OBJ1 = investment
  - OBJ2 = RES
- Thermo-economic trade-off
  - OBJ1 = investment
  - OBJ2 = operating cost

Multi objective optimization

Trade-off between one objective and another one

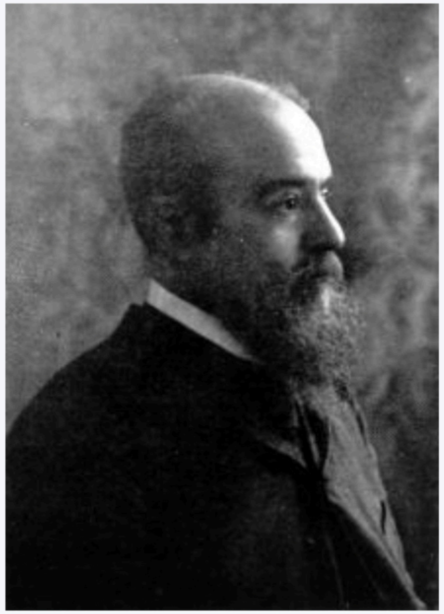
Generation of the Pareto Front



$$\min_{X^d} OBJ1(X^d)$$
$$st.$$
$$OBJ2(X^d) \leq OBJ2^{ref}$$

$$\min_{X^d} OBJ2(X^d)$$
$$st.$$
$$OBJ1(X^d) \leq OBJ1^{ref}$$

Vilfredo Pareto



Born	15 July 1848 Paris, France
Died	19 August 1923 (aged 75) Céligny, Switzerland
Nationality	Italian
Institutions	University of Lausanne
Field	Microeconomics Socioeconomics

source : wikipedia

Muti-objective optimisation

- Single objective parametric
  - Weighting factor

$$X_d(w) : min_{X_d}(1-w) \cdot OBJ_1(X_d) + w \cdot OBJ_2(X_d)$$
$$\forall w \in [0,1]$$

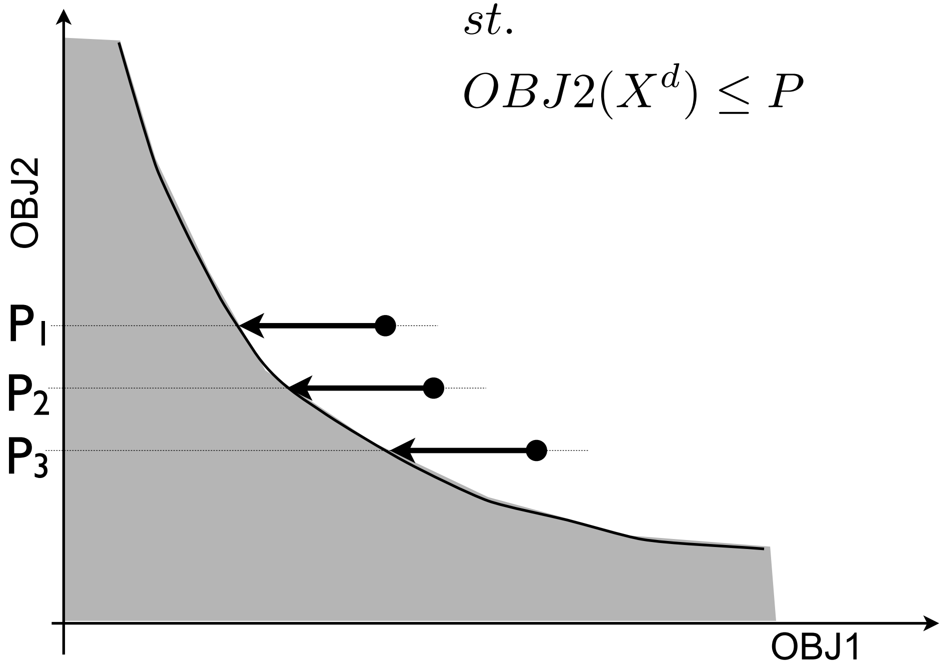
  - Note : if OBJ1 is a cost function

$$\frac{w}{1-w} \text{ is a tax on } OBJ_2(X_d)$$

e.g. CO_2 tax if OBJ_2 is the CO_2 emissions and OBJ_1 the total cost

Parametric programming

$$
\begin{aligned}
for \quad P \quad &\in \quad \{OBJ2(X_d|\min_{X^d} OBJ1(X^d))..OBJ2(X_d|\min_{X^d} OBJ2(X^d))\} \\
X^d(P) \quad &: \quad \min_{X^d} OBJ1(X^d) \\
& \quad\; st. \\
& \quad\; OBJ2(X^d) \leq P
\end{aligned}
$$



Note : the Lagrange multiplier of the inequality constraints gives the slope of the Pareto curve