Cost Annualization

Advanced energetics course

Complementory document

Nasibeh Pouransari

Francois Marechal

Laboratory for Industrial Energy Systems, LENI, STI, EPFL

Contents

1 What is cost annualization?

The basic concepts and equations of cost annualization concept will be explained through three examples.

1.1 Example 1

Let's imagine that you have 10,000 CHF and you want to deposit it at the bank for a period of 4 years. The interest rate of the bank is 10% per year. How much money do you have in the bank at the end of this 4 years ?

Solution:

Time [year]	0	1	2	3	4
Money [CHF]	10000	10000 + 10000 (0.1)	11000 + 11000 (0.1)	12100 + 12100 (0.1)	13310 + 13310 (0.1)
SUM [CHF]	10000	11000	12100	13310	14461

Then using the known polynomial of $(1+a)^n$, the result can be written as $F = 10000(1+0.1)^4 = 14641$ $_{[CHF]}$

Depositing I [CHF] with the interest rate of i, the available amount of money after n years (F) is: $\rightarrow$ $\quad F = I(1+i)^n$ $_{[CHF]}$

1.2 Example 2

Now, imagine that you receive a salary of 10,000 CHF each year and you want to deposit it at the bank during four years with the interest rate of 10%. How much money do you have in the bank at the end of this four years ?

Solution:

To solve this problem, we can use the result of example 1. each year the bank receive an amount of 10,000 CHF, and its value till the fourth year can be calculated. for example after the first year, there is 10,000 CHF and at the end of 4 years it would be $10000(1+0.1)^4$. In the first year another 10,000 CHF will be injected to the bank that its value at the end of 4 years would be $10,000(1+0.1)^2$. There would be the same procedure for the rest as well, as you can see below:

Time [year]	0	1	2	3	4
Money [CHF]		10000	10000	10000	10000 (1+0.1)^3 10000 (1+0.1)^2 10000 (1+0.1)^1 10000
SUM [CHF]		10000			46410

Then using the known series of $1 + a + a^2 + a^3 + ... + a^n = \frac{a^{n+1}-1}{a-1}$, we are able to sum all of this part and rewrite them as $10000(1+0.1)^3 + 10000(1+0.1)^2 + 10000(1+0.1) + 10000 = 10000\left(\frac{(1+0.1)^4-1}{1+0.1-1}\right)$

Depositing B [CHF] at the end of each year with the interest rate of i, the available amount of money after n years (B) is: $\rightarrow$ $\quad P = B\left(\frac{(1+i)^n-1}{i}\right)$ $_{[CHF]}$

1.3 Example 3

Finally, imagine the same situation as example 1, but by considering constant annual reimbursement. How much money do you receive per year if you want to take out all the money during a period of 4 years?

Solution:

Notice: The sum of the annual reimbursement would be equivalent to the value of 10,000 CHF investment at the end of 4 years.

Firstly, we calculate the value of the money that you have invested at the bank after 4 years. using the result of first example: $F = 10000 \times (1 + 0.1)^4 = 14641$ $_{[CHF]}$.

Secondly, we imagine that you receive B CHF per year, then with the help of result of example 2, after 4 years the value of the money that you have received from the bank is: $P = B\left(\frac{(1+0.1)^4-1}{0.1}\right)$ $_{[CHF]}$.

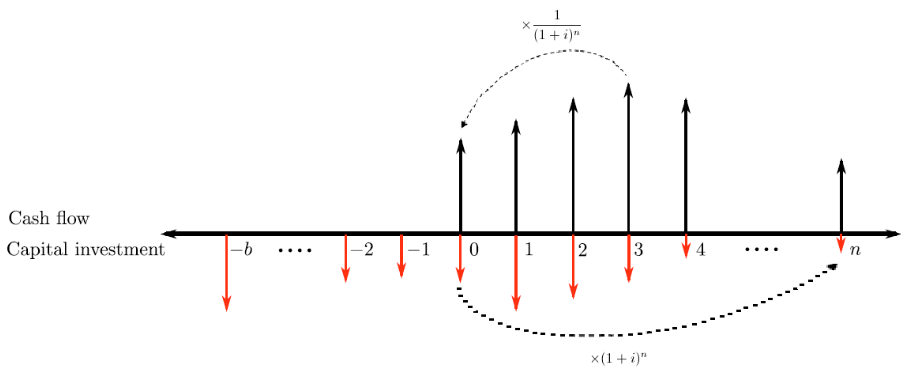
In order to solve the problem, the value you received from the bank should be equal to the value you have invested in the bank, therefore: $F = P \rightarrow \quad B = 10000\left(\frac{(1+0.1)^4 \times 0.1}{(1+0.1)^4-1}\right) = 3154.7$ $_{[CHF/year]}$.

Depositing A [CHF] with the interest rate of i results into the receipt of B [CHF] per year for a period of n years $\rightarrow \quad B = A\left(\frac{(1+i)^n \times i}{(1+i)^n-1}\right)$ $_{[CHF/year]}$

2 What is the Net Present Value (NPV)?

Present Value: The present value refers to the value of money at the time that is defined as a reference or starting time. For example, a company that is started at the year 2000, will calculate all its benefits at the present value of the year 2000, If it gains B CHF at 2011, then the present value of that would be (regarding the example 1) as $\frac{B}{(1+i)^{(2011-2000)}}$.

Net Present Value (NPV): The net present value is the total of the present value of all cash flows minus the present value of all capital investments.



In this figure, the black arrows show the amount of benefit we obtain each year, and the red arrows show the capital investment of each year. If we want to calculate the value of the investment or benefit of each year at the startup time (year zero), we have to multiply it by the factor of $\frac{1}{(1+i)^n}$ and if we want to calculate the amount of each of them at n years after we have to multiply it by the factor of $(1+i)^n$. Therefore, when the values are evaluated based on the startup year :

$$NPV = \sum_{j=1}^{n} A_j \left(\frac{1}{(1+i)^j}\right) - \sum_{j=-b}^{n} TCI_j \left(\frac{1}{(1+i)^j}\right)$$

Where:

B_j	Cash flow in year j $_{[CHF/year]}$
I_j	Total Capital Investment in year j $_{[CHF/year]}$
b	The years of investment before startup $_{[year]}$
n	Life time of the plant $_{[year]}$
i	Interest rate

With the constraint: $NPV > 0$

In fact the net present value is the value of money at the time zero. Now, the above equation can be simplified considering some assumptions:

- If the amount of benefit for all year is equal to the constant value of B. $\Rightarrow \{\forall j; B_j = B\}$
- If the amount of investment would be equal to the constant value of I for all years, and if the investment has been started at the startup time. $\Rightarrow \{\forall j; I_j = I\}$ and $b = 0$

Then we will have:

$$NPV = B\sum_{j=1}^{n}\frac{1}{(1+i)^j} - I\sum_{j=0}^{n}\frac{1}{(1+i)^j}$$

Therefore, with the help of series:

$$\sum_{j=0}^{n}\frac{1}{(1+i)^j} = \frac{(1+i)^{n+1}-1}{i(1+i)^n} = 1 + \sum_{j=1}^{n}\frac{1}{(1+i)^j} \quad \Rightarrow \quad \sum_{j=1}^{n}\frac{1}{(1+i)^j} = \frac{(1+i)^{n+1}-1}{i(1+i)^n} - 1 = \frac{(1+i)^n-1}{i(1+i)^n}$$

The present value of B [CHF/year] benefit during the life time of n years with the interest rate of i $\rightarrow \quad B\left(\frac{(1+i)^n-1}{i(1+i)^n}\right)$ $_{[CHF]}$

The net present value of B [CHF/year] benefit and I [CHF/year] capital investment during the life time of n years with the interest rate of i: $\rightarrow \quad NPV = B\left(\frac{(1+i)^n-1}{i(1+i)^n}\right) - I\left(\frac{(1+i)^{n+1}-1}{i(1+i)^n}\right)$ $_{[CHF]}$

Now, if we also add the assumption of investing only once at the time zero, then the related term to the investment could be equal to I:

$$I\left(\frac{(1+i)^{0+1}-1}{i(1+i)^0}\right) = I$$

The net present value of B [CHF/year] benefit and I [CHF] capital investment at the startup time, during the life time of n years with the interest rate of i: $\rightarrow \quad NPV = B\left(\frac{(1+i)^n-1}{i(1+i)^n}\right) - I$ $_{[CHF]}$

By considering the same assumptions and using the result of the example 3, if we invest I CHF at time zero, we have to obtain $I\left(\frac{(1+i)^n i}{(1+i)^n-1}\right)$ CHF per year (during the life time of n years) to reimburse it.

The Net Benefit per year of B [CHF/year] benefit and I [CHF] capital investment at the startup time, during the life time of n years with the interest rate of i: $\rightarrow \quad NB = B - I\left(\frac{(1+i)^n i}{(1+i)^n-1}\right)$ $_{[CHF]}$