ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 6 - Solution

Problem 1: Clausius-Clapeyron relation

A) Consider the phase change occurring at constant temperature and constant pressure

$$h_{lv} = \int_{L}^{V} dh = \int_{L}^{V} T ds + v dP = \int_{L}^{V} T ds = T \Delta s \tag{1}$$

B) Take the difference between the expression of the differential Gibbs equation calculated from the liquid phase and from the vapor phase

$$(v_l - v_v)dP - \Delta s dT = 0 (2)$$

Combining Equation (1) and Equation (2) we obtain the Clausius-Clayperon relation:

$$\frac{dP}{dT} = \frac{\Delta s}{v_l - v_v} = \frac{h_{lv}}{T(v_v - v_l)}$$

Problem 2: Hsu's model

- A) As the boundary layer thins, the maximum bubble size decreases. The top of a large bubble will be surrounded by cooler water, further lowering the upper limit. Consequently, the range of cavity sizes decreases (Figure 1a).
- B) Heating the substrate increases the upper limit and decreases the lower limit of the active cavity size, as evident from the graph (shifting toward the right).
- C) Lowering the surface tension will reduce the lower limit as the Laplace pressure is smaller at the same bubble size. The range of cavity sizes increases (Figure 1b).

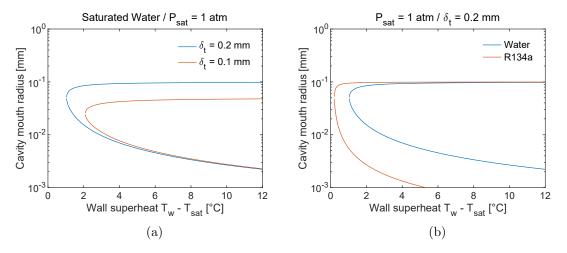


Figure 1: Prediction of the range of cavity sizes using Hsu's model for different boundary layers thicknesses (a) and fluids with different surface tension (b).

Problem 3: Equilibrium vapor pressure for meniscus in a cylindrical pore

A) At the saturation point, we have

$$\hat{g}_{sat,l}(T_l, P_{sat}) = \hat{g}_{sat,v}(T_l, P_{sat}) = \hat{g}_{sat}$$

The deviation of vapor Gibbs free energy:

$$\hat{g}_v - \hat{g}_{sat} = \int_{P_{sat}}^{P_v} v_v dP = \int_{P_{sat}}^{P_v} \frac{RT_l}{P} dP = RT_l \ln\left(\frac{P_v}{P_{sat}}\right)$$

The deviation of liquid Gibbs free energy:

$$\hat{g}_l - \hat{g}_{sat} = \int_{P_{sat}}^{P_l} v_l dP = v_l (P_l - P_{sat})$$

In equilibrium $\hat{g}_v = \hat{g}_l$, and therefore

$$v_l(P_l - P_{sat}) = RT_l \ln \left(\frac{P_v}{P_{sat}}\right)$$

With Young-Laplace equation:

$$P_v - P_l = \frac{2\sigma}{r}$$

Using the two equations above allows you to solve for P_v and P_l .

B) The code has been uploaded separately.

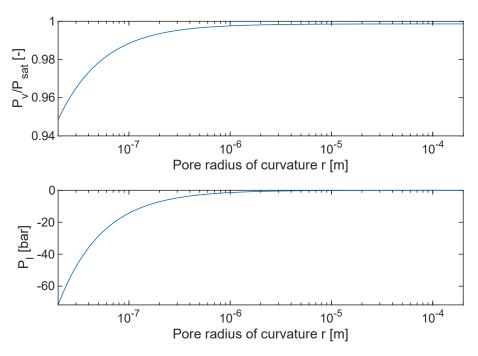


Figure 2: P_v/P_{sat} and P_l as a function of the pore radius of curvature r.