

Homework 9 - Solution

Problem 1: Wicking model and flow boiling

The wicking model part of the paper includes the following typos:

1. The velocity profile $u(y)$ in the porous micropillars (Eq. 9) should be written as:

$$u = Ae^{\alpha\sqrt{\varepsilon}y} + Be^{-\alpha\sqrt{\varepsilon}y} - \frac{1}{\alpha^2\mu} \frac{dP}{dx}$$

2. The pressure gradient should be:

$$\frac{dP}{dx} = -\frac{P_{cap}}{L_w}$$

With that in mind, we can follow the implementation according to the paper. Notice that since the water is boiling, we should consider the water properties at saturation $\approx 100^\circ\text{C}$ at 1 atm.

A Matlab code capable of reproducing the Fig. 10 in the paper is provided separately.

The relationship between the wicking velocity (u_{ave}) and the flow boiling is governed by the contact angle (θ) on the micropillars surface. In the simplified wicking model, the critical condition for dryout is assumed, where the contact line begins to recede ($\theta = \theta_r$, the receding contact angle). At this point the liquid pressure at the interfaces reaches its minimum, creating the largest capillary pressure gradient in the channel and driving the maximum wicking velocity.

The flow boiling, or in this particular case, the thin film evaporation influences this process as a function of the applied heat flux q'' . Increasing q'' will enhance the evaporation, reducing the liquid film thickness and dynamically decreasing the local contact angle. This effect increases the surface area (further increasing the evaporation rate) and enables a higher wicking velocity to replenish the thin liquid film. However, once the receding contact angle (θ_r) is reached, any further increase in heat flux results in a complete dryout as no liquid remains to sustain the capillary pressure gradient.

In summary, the wicking velocity increases with the applied heat flux up to the critical heat flux (CHF) where the system can no longer maintain the liquid film, leading to dryout.

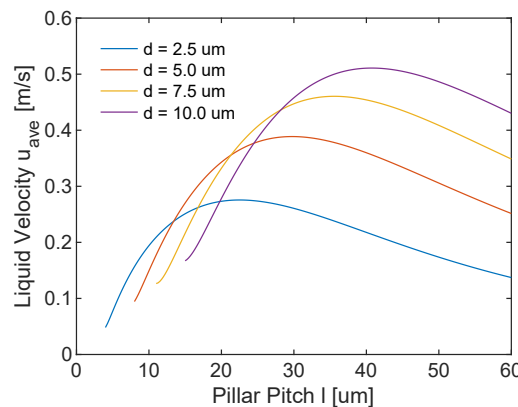


Figure 1: Liquid velocity u_{ave} as a function of the micropillars diameter d and pitches l . Reproduction of the Fig.10 in Zhu et al.

Problem 2: Bubble coalescence

The sum of the two bubbles volume and surface area in the initial case are:

$$V_i = \frac{8}{3}\pi r^3$$

$$A_i = 8\pi r^2$$

Assuming the volume is conserved, we can write the radius of the final bubble as a function of r :

$$r_f = 2^{1/3}r$$

and,

$$A_f = 4\pi 2^{2/3}r^2$$

The surface energy change between the final and initial case is expressed as:

$$\Delta E = \gamma(A_f - A_i) = 4\pi r^2 \gamma(2^{2/3} - 2) < 0$$

Problem 3: Air bubble in oversaturated liquid

From geometric consideration (see Figure 2) we have:

$$\sin \theta = \frac{d}{2R}$$

The only unknown is the radius of curvature R . At the surface, we can write the Young-Laplace equation:

$$P_{in} = P_0 + \frac{2\gamma}{R}$$

At equilibrium, and assuming there is no gas concentration gradient in the liquid, the concentration of the dissolved gas is related to the pressure inside the bubble:

$$c_\infty = K_H P_{in} = K_H \left[P_0 + \frac{2\gamma}{R} \right]$$

:

$$R = \frac{2\gamma}{\left[\frac{c_\infty}{K_H} - P_0 \right]}$$

Rearranging:

$$\sin \theta = \frac{d}{4\gamma} \left[\frac{c_\infty}{K_H} - P_0 \right]$$

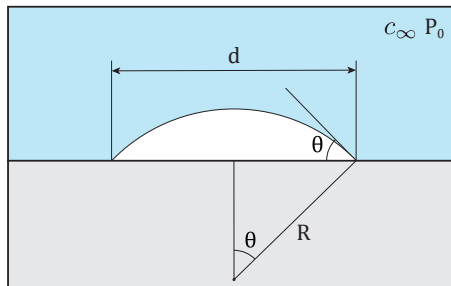


Figure 2: Geometrical consideration for the radius of curvature.