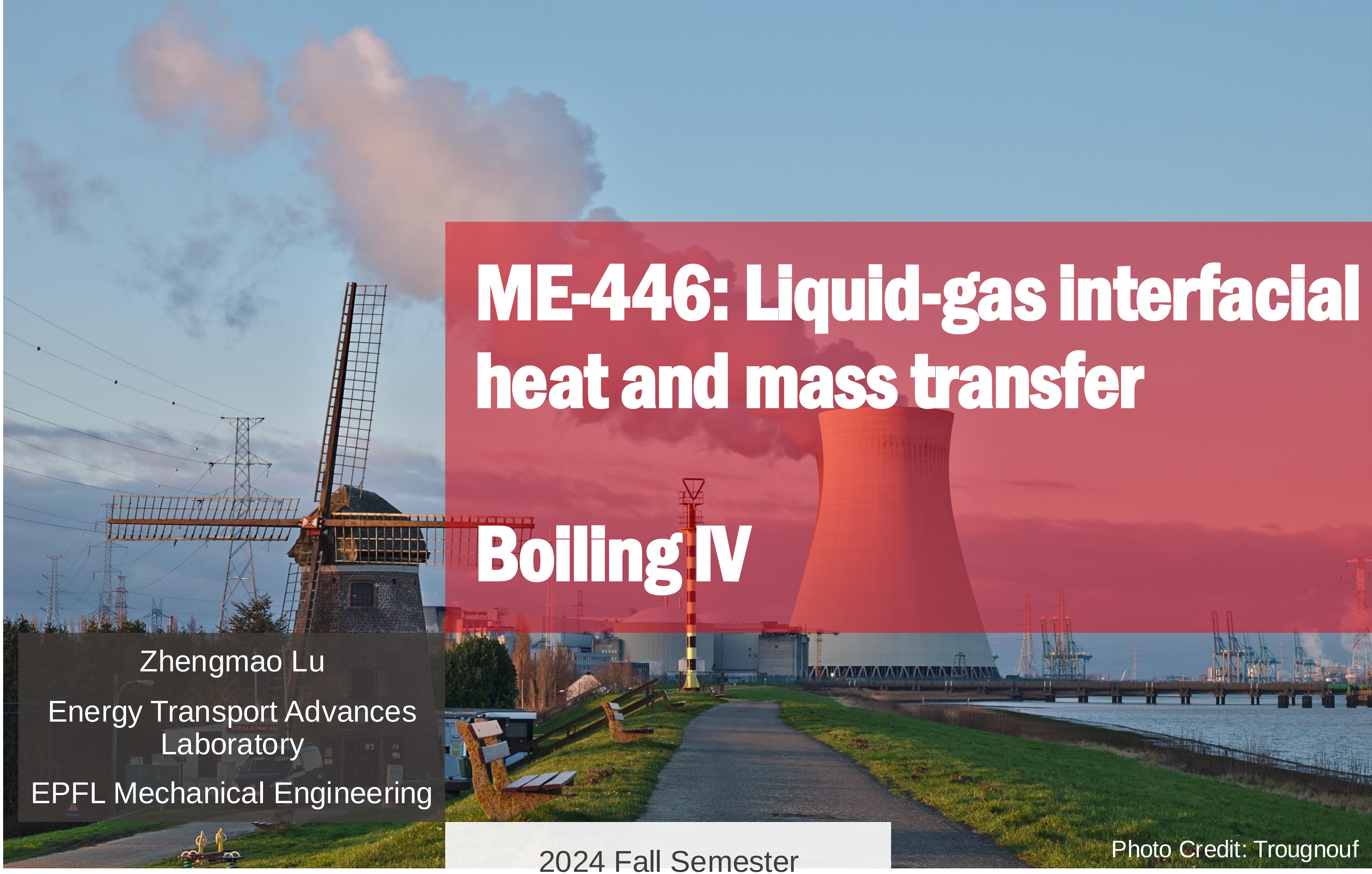


ME-446: Liquid-gas interfacial heat and mass transfer

Boiling IV

Zhengmao Lu

Energy Transport Advances Laboratory

EPFL Mechanical Engineering

2024 Fall Semester

Photo Credit: Trougnouf

EPFL

ETA-Lab
Energy Transport Advances

Last Time

- Bubble departure frequency and diameter
- Different regimes in pool boiling
- Rohsenow’s microconvection model for nucleate boiling

ILOs

- Zuber’s CHF model based on Helmholtz and Taylor instabilities
- Force balance model for CHF
- Statistical approach for CHF

Helmholtz Instability of Vapor Columns

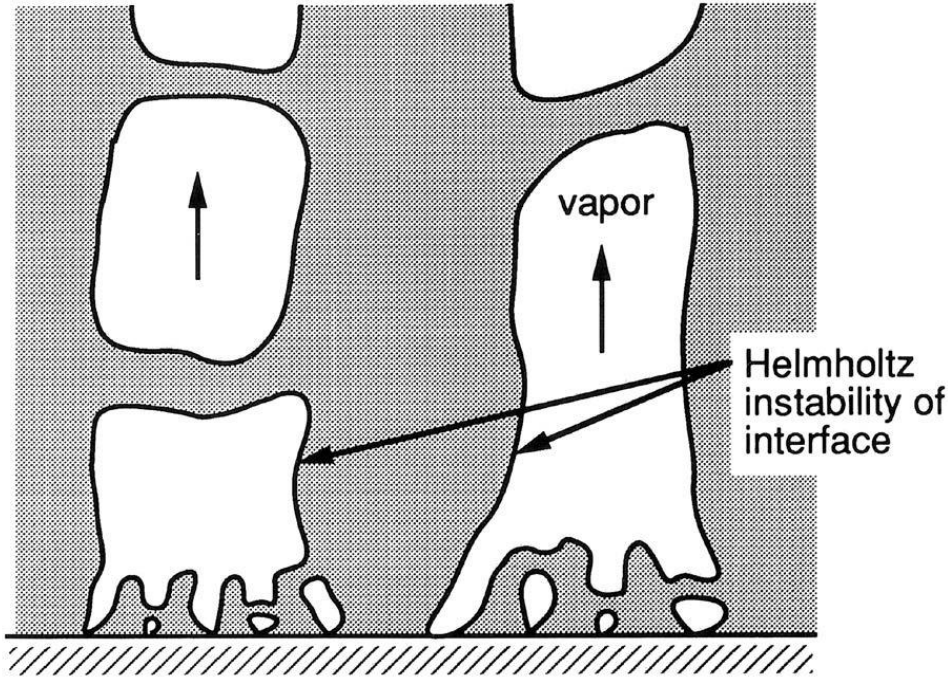


Figure 7.16 Carey

Video credit: Dr. Rameez Iqbal

Vapor columns form at high heat fluxes

Helmholtz Instability

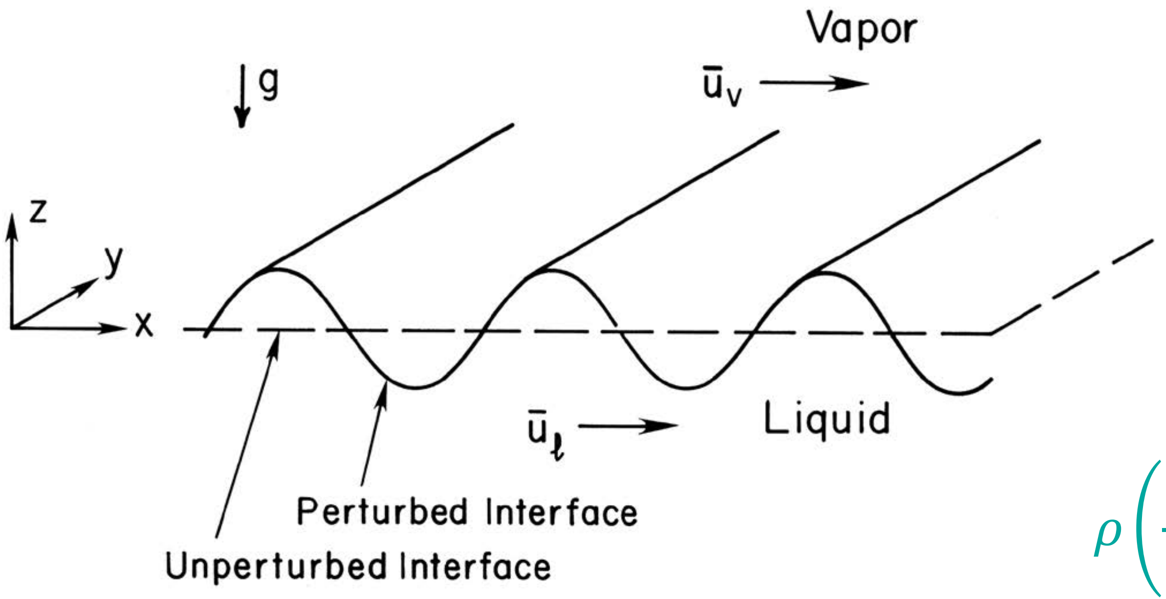


Figure 4.4 in Carey

Continuity

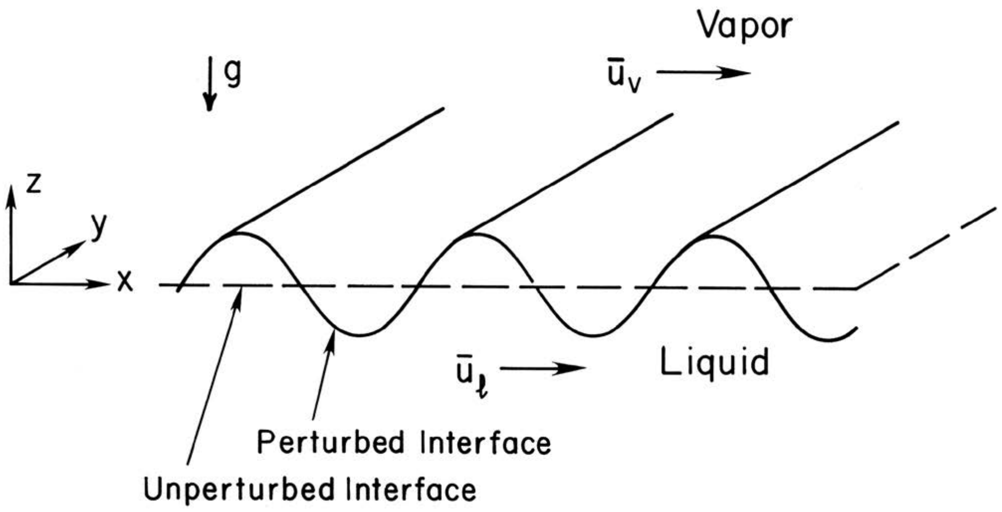
$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

Momentum balance
(neglecting viscosity)

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial z} - \rho g$$

Helmholtz Instability



Consider after a perturbation $\delta(x, t = 0) = Ae^{i\alpha x}$

$$u: \bar{u} \to \bar{u} + u', \qquad w: 0 \to w', \qquad P: \bar{P} \to \bar{P} + P'$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0$$

$$\rho\left(\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial x}\right) = -\frac{\partial P'}{\partial x}$$

$$\rho\left(\frac{\partial w'}{\partial t} + \bar{u}\frac{\partial w'}{\partial x}\right) = -\frac{\partial P'}{\partial z}$$

$$\Rightarrow \frac{\partial^2 P'}{\partial x^2} + \frac{\partial^2 P'}{\partial z^2} = 0$$

Postulate the form of the response function:

$$\delta = Ae^{i\alpha x + \beta t} \qquad w' = \hat{w}(z)e^{i\alpha x + \beta t} \qquad P' = \hat{P}(z)e^{i\alpha x + \beta t}$$

We are interested in β

Helmholtz Instability

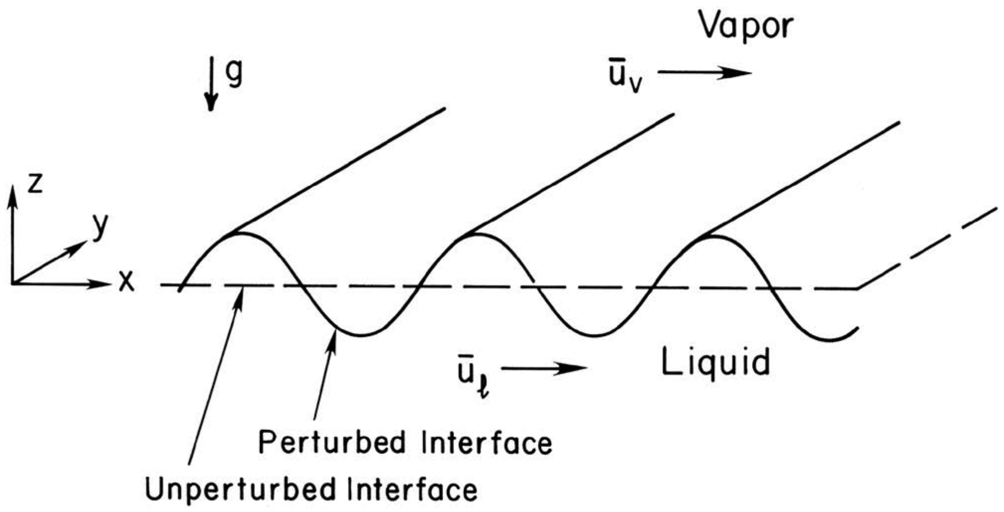
$$\frac{\partial^2 P'}{\partial x^2} + \frac{\partial^2 P'}{\partial z^2} = 0 \qquad P' = \hat{P}(z) e^{i\alpha x + \beta t}$$

$$\frac{d^2 \hat{P}}{dz^2} = \alpha^2 \hat{P}$$

$\hat{P} \to 0$ far from interface

$$\hat{P}_v = a_v e^{-\alpha z} \qquad \hat{P}_l = a_l e^{\alpha z}$$

Typo in 4.29



$$\hat{w}(z) = -\frac{1}{\rho(\beta + i\alpha\bar{u})} \frac{d\hat{P}}{dz}$$

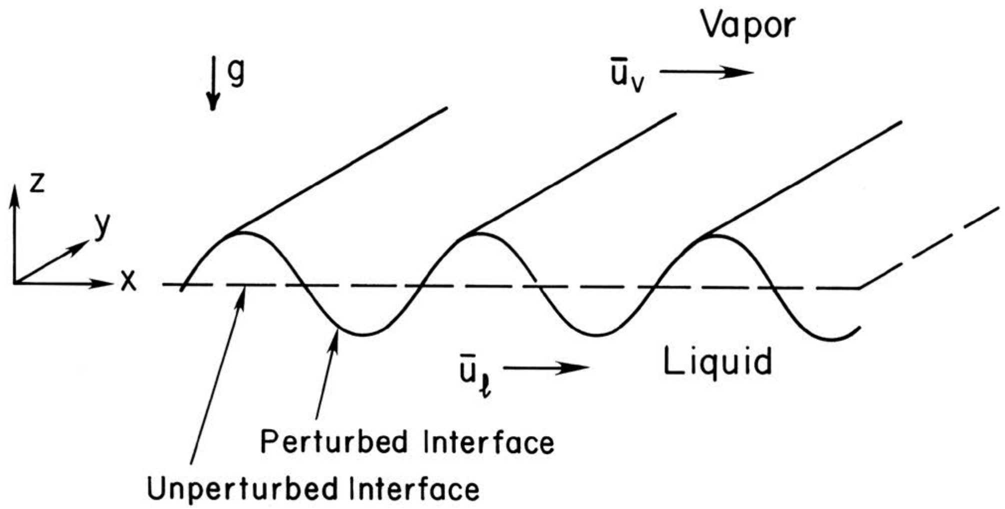
$$\hat{w}_v(z) = \frac{a_v \alpha}{\rho_v(\beta + i\alpha\bar{u})} e^{-\alpha z}$$

$$\hat{w}_l(z) = -\frac{a_l \alpha}{\rho_l(\beta + i\alpha\bar{u})} e^{\alpha z}$$

$$w' = \hat{w}(z) e^{i\alpha x + \beta t} \qquad \rho\left(\frac{\partial w'}{\partial t} + \bar{u}\frac{\partial w'}{\partial x}\right) = -\frac{\partial P'}{\partial z}$$

Helmholtz Instability

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0 \qquad w' = \hat{w}(z)e^{i\alpha x+\beta t}$$



$$\delta = Ae^{i\alpha x+\beta t}$$

$$\hat{w}_v(z) = \frac{a_v\alpha}{\rho_v(\beta + i\alpha\bar{u})}e^{-\alpha z}$$

$$\hat{w}_l(z) = -\frac{a_l\alpha}{\rho_l(\beta + i\alpha\bar{u})}e^{\alpha z}$$

$u' \rightarrow 0$ far from interface

$$u' = \frac{i}{\alpha}\frac{d\hat{w}}{dz}e^{i\alpha x+\beta t}$$

Interface vertical motion is due to the time evolution of the vibration and the traveling of the perturbation wave

$$w'_{z\rightarrow 0} = \frac{\partial\delta}{\partial t} + \bar{u}\frac{\partial\delta}{\partial x}$$

True for both two phases

$$a_v = \frac{\rho_v}{\alpha}(\beta + i\alpha\bar{u}_v)^2 A$$

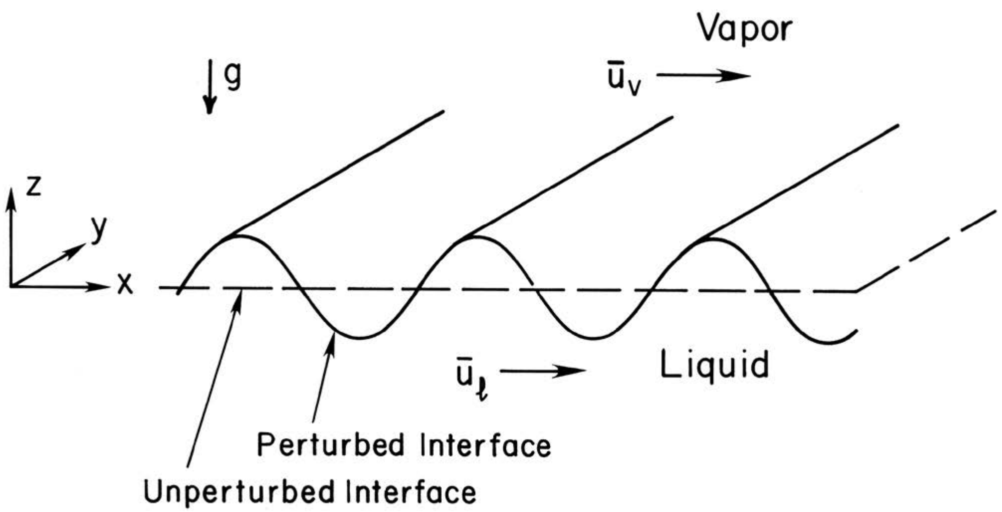
$$a_l = -\frac{\rho_l}{\alpha}(\beta + i\alpha\bar{u}_l)^2 A$$

Helmholtz Instability

$$a_v = \frac{\rho_v}{\alpha}(\beta + i\alpha\bar{u}_v)^2 A$$

$$a_l = -\frac{\rho_l}{\alpha}(\beta + i\alpha\bar{u}_l)^2 A$$

$$\hat{P}_v = a_v e^{-\alpha z} \qquad \hat{P}_l = a_l e^{\alpha z}$$



$$P_{v,in} - P_{l,in} = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

r_2: interface radius of curvature in y-z plane

r_1: interface radius of curvature in x-z plane

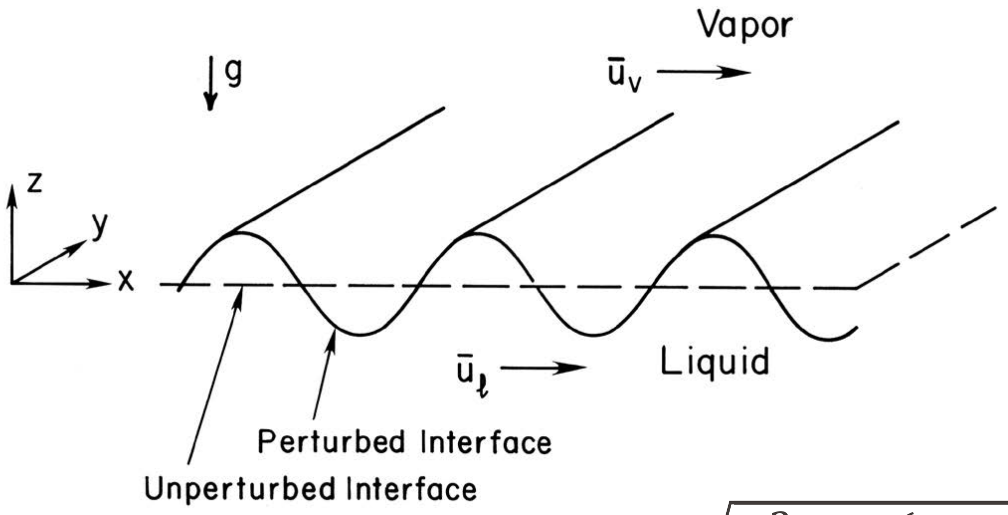
$$\frac{1}{r_1} \approx -\frac{\partial^2 \delta}{\partial x^2}$$

$$P_{v,in} = P_0 - \rho_v g\delta + \hat{P}_v e^{i\alpha x + \beta t}$$

$$P_{l,in} = P_0 - \rho_l g\delta + \hat{P}_l e^{i\alpha x + \beta t}$$

$$a_v - a_l = -(\Delta\rho g + \sigma\alpha^2)A$$

Helmholtz Instability



Perturbation $\delta(x, t = 0) = Ae^{i\alpha x}$

$$\delta = Ae^{i\alpha x + \beta t}$$

$$w' = \hat{w}(z)e^{i\alpha x + \beta t}$$

$$P' = \hat{P}(z)e^{i\alpha x + \beta t}$$

$$\beta = \pm \frac{\sqrt{\alpha^2 \rho_v \rho_l (\bar{u}_v - \bar{u}_l)^2 - (\sigma\alpha^3 + \Delta\rho g \alpha)}}{\rho_v + \rho_l} - i\alpha \frac{\rho_l \bar{u}_l + \rho_v \bar{u}_v}{\rho_v + \rho_l}$$

The perturbation will cause a growing response if and only if β has a positive real part

Instability condition:

$$|\bar{u}_v - \bar{u}_l| > \sqrt{\frac{\left(\sigma\alpha + \frac{\Delta\rho g}{\alpha}\right)(\rho_l + \rho_v)}{\rho_l \rho_v}}$$

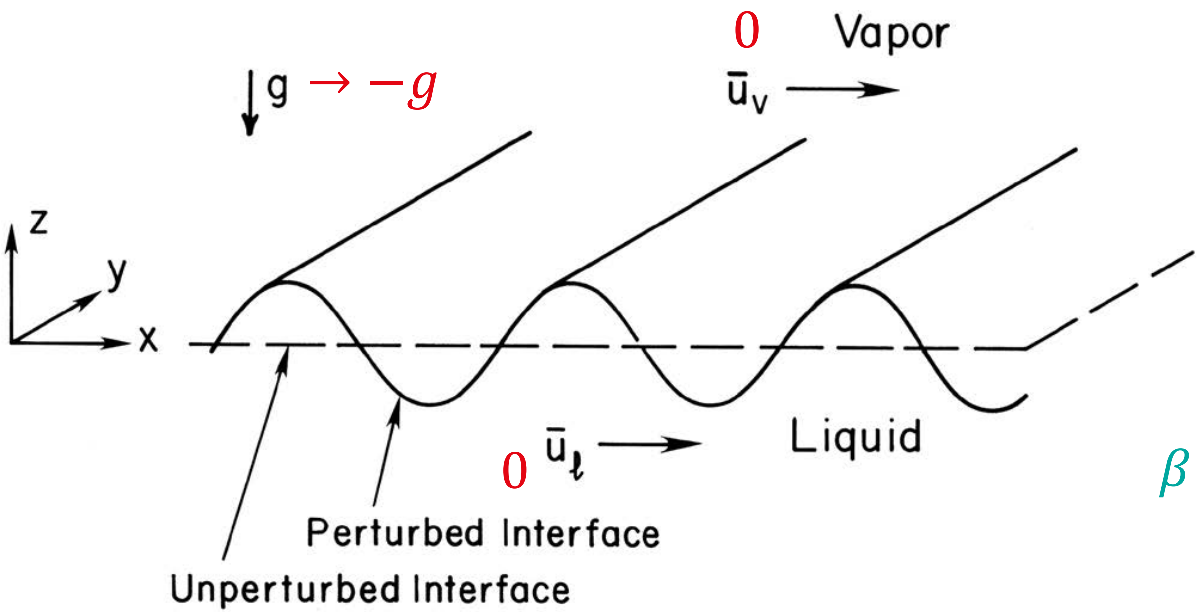
$\bar{u}_v - \bar{u}_l$ promotes instability while gravity and surface tension suppressing instability, we can adjust the value of g based on the orientation of the system.

Helmholtz Instability

https://www.youtube.com/watch?v=mf_143gkKSQ

Helmholtz Instability

Taylor Instability



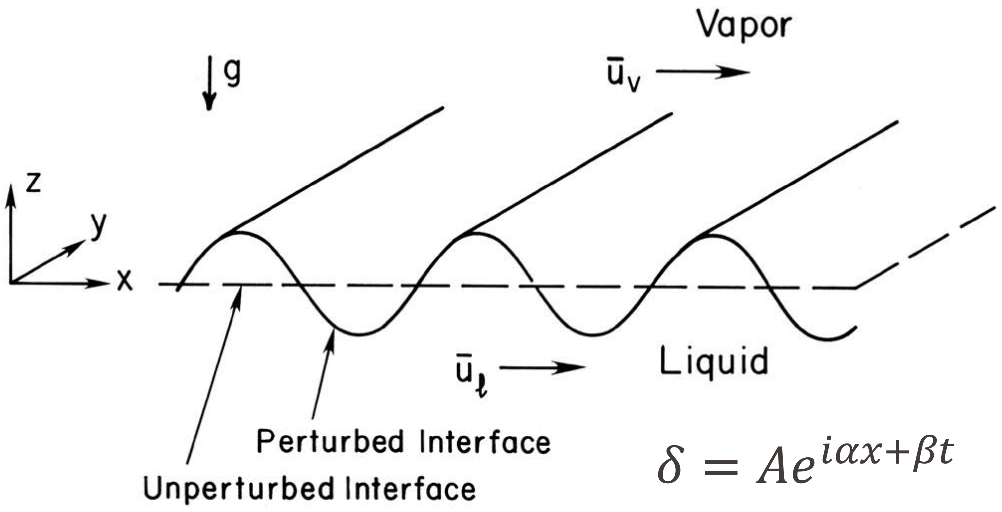
$$\delta = Ae^{i\alpha x + \beta t}$$

$$\beta = \pm\sqrt{\frac{\Delta\rho g\alpha - \sigma\alpha^3}{\rho_l + \rho_v}}$$

The fastest growing perturbation (α_{max}) in this case can be found by setting $\frac{d\beta}{d\alpha} = 0$

The corresponding most dangerous wavelength $\lambda_D = \frac{2\pi}{\alpha_{max}} = 2\pi\sqrt{\frac{3\sigma}{\Delta\rho g}}$

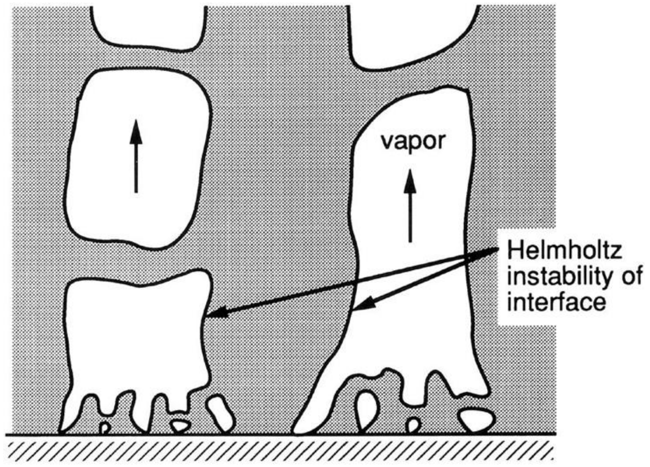
How It's Related to Boiling



$$\delta = Ae^{i\alpha x+\beta t}$$

Helmholtz Instability

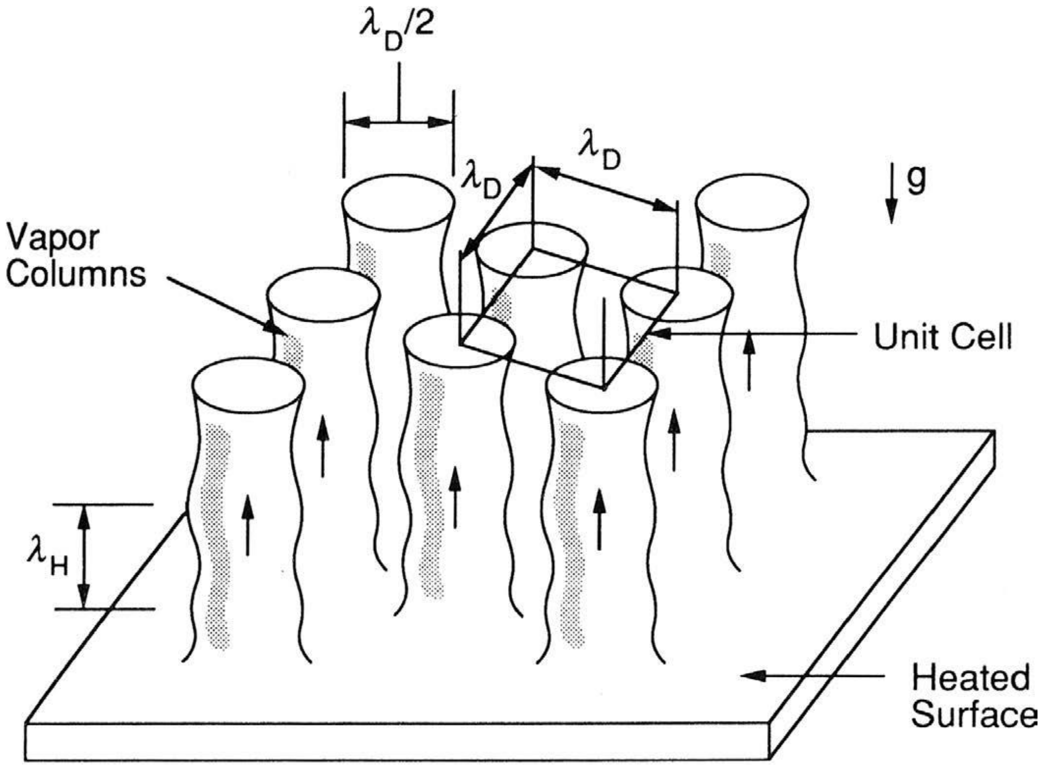
$$|\bar{u}_v - \bar{u}_l| > \sqrt{\frac{\left(\sigma\alpha + \frac{\Delta\rho g}{\alpha}\right)(\rho_l + \rho_v)}{\rho_l \rho_v}}$$



Setting g = 0 for vertical interfaces

$$|\bar{u}_v - \bar{u}_l| > \sqrt{\frac{\sigma\alpha\,(\rho_l + \rho_v)}{\rho_l \rho_v}} = \sqrt{\frac{2\pi\sigma\,(\rho_l + \rho_v)}{\rho_l \rho_v \lambda_H}} = u_c$$

Zuber's Model



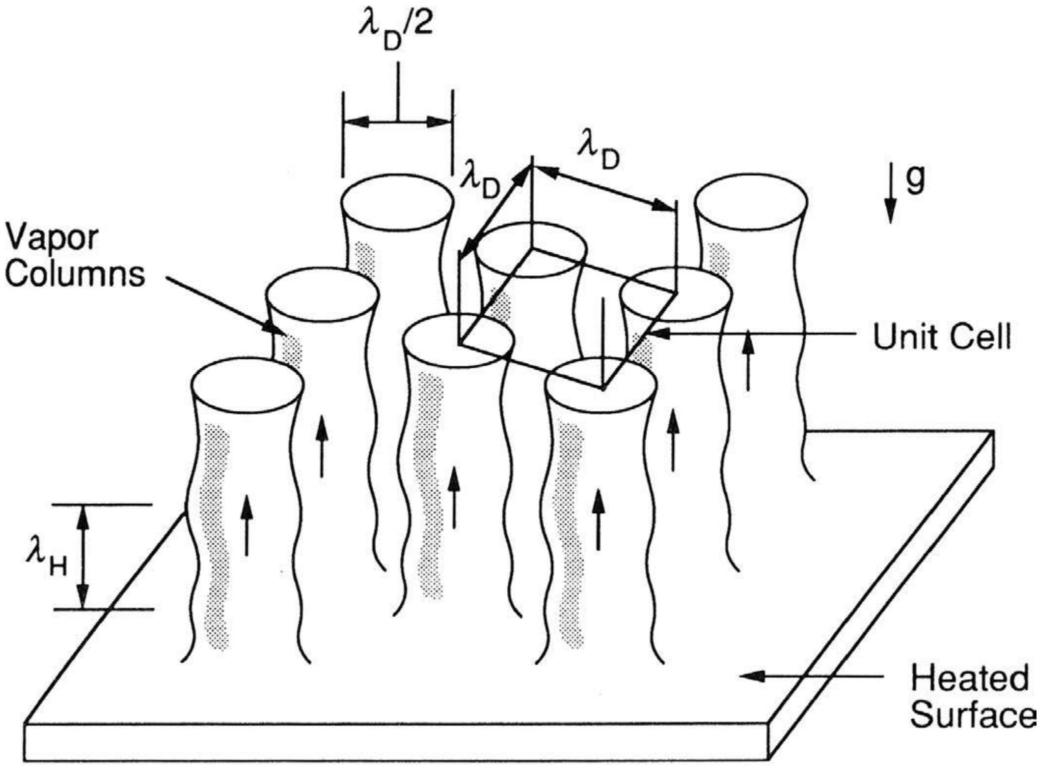
- CHF is reached when interface of vapor columns becomes Helmholtz unstable (λ_H)
- The pitch of the vapor columns coincides with the most dangerous wavelength in Taylor instability

$$\lambda_D = 2\pi\sqrt{3\sigma/\Delta\rho g}$$

- The diameter of vapor column is $\lambda_D/2$
- $\lambda_H = \lambda_D \Rightarrow u_c = \sqrt{\frac{2\pi\sigma\,(\rho_l+\rho_v)}{\rho_l\rho_v\lambda_H}} \approx \sqrt{\frac{2\pi\sigma}{\rho_v\lambda_D}}$

Zuber's Model

$$u_c = \sqrt{\frac{2\pi\sigma}{\rho_v \lambda_D}} \qquad \lambda_D = 2\pi\sqrt{\frac{3\sigma}{\Delta\rho g}}$$



$$u_c = \frac{q''_{max}}{\rho_v h_{lv}}\left(\frac{A_{surf}}{A_{col}}\right) = \frac{16}{\pi}\frac{q''_{max}}{\rho_v h_{lv}}$$

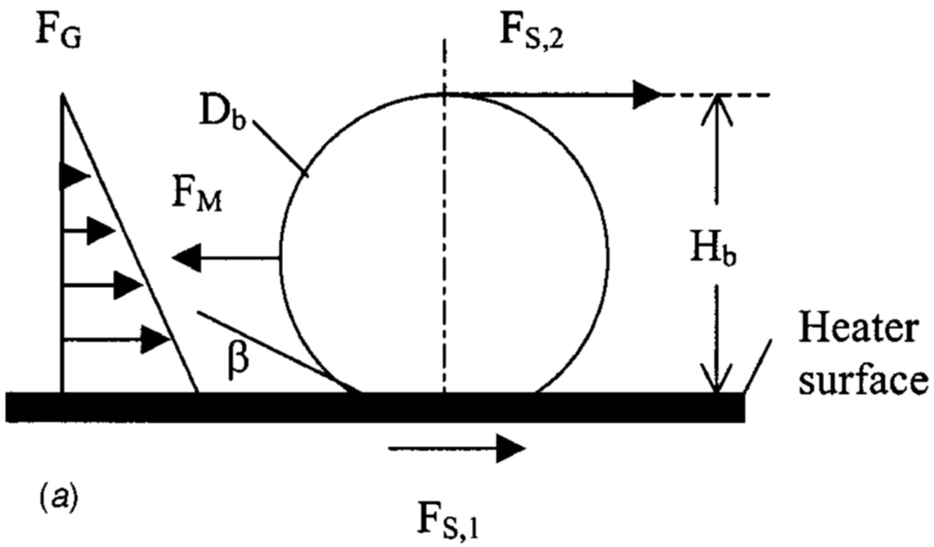
$$q''_{max} = 0.149\rho_v h_{lv}\left(\frac{\sigma\Delta\rho g}{\rho_v^2}\right)^{1/4}$$

Comments on Zuber's Model

- No way to accommodate effects from geometry and surface wettability
- No clear justification for the choice of vapor column diameter as $\lambda_D/2$
- No visual observation of Helmholtz instability during boiling to date
- Still widely used as a reference model for all subsequence CHF models

Lateral Force Balance Model

https://doi.org/10.1115/1.1409265



Considering the lateral direction

Surface tension force : $F_{s,1}$, $F_{s,2}$

Hydrostatic force: F_G

Momentum force: F_M

Kandlikar suggested momentum balance requires $F_{s,1} + F_{s,2} + F_G = F_M$

chose a bubble diameter at CHF $D_b = \lambda_D/2$

λ_D: the most dangerous wavelength in Taylor instability

and set a bubble influence area πD_b^2

Contact Angle Dependence

$$q''_K = \rho_v h_{fg} \left(\frac{1+\cos\beta}{16}\right)\left[\frac{2}{\pi}+\frac{\pi}{4}(1+\cos\beta)\right]^{\frac{1}{2}}\left(\frac{\sigma\Delta\rho g}{\rho_v^2}\right)^{1/4}$$ (Horizontal surface)

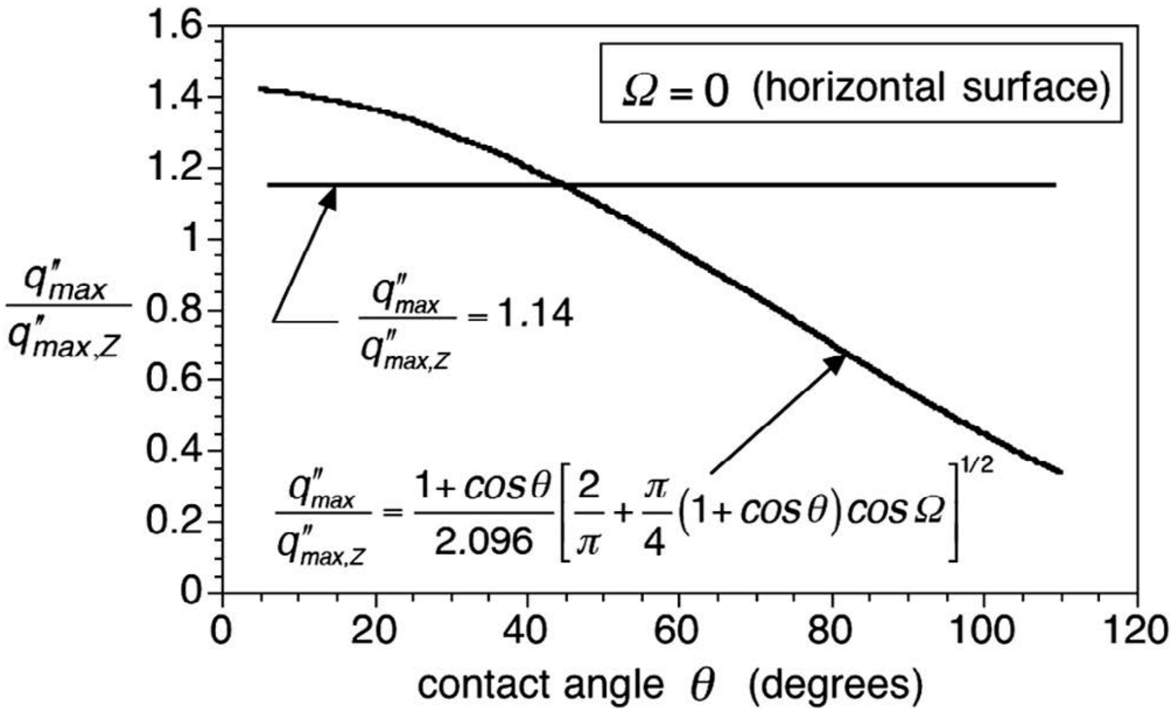
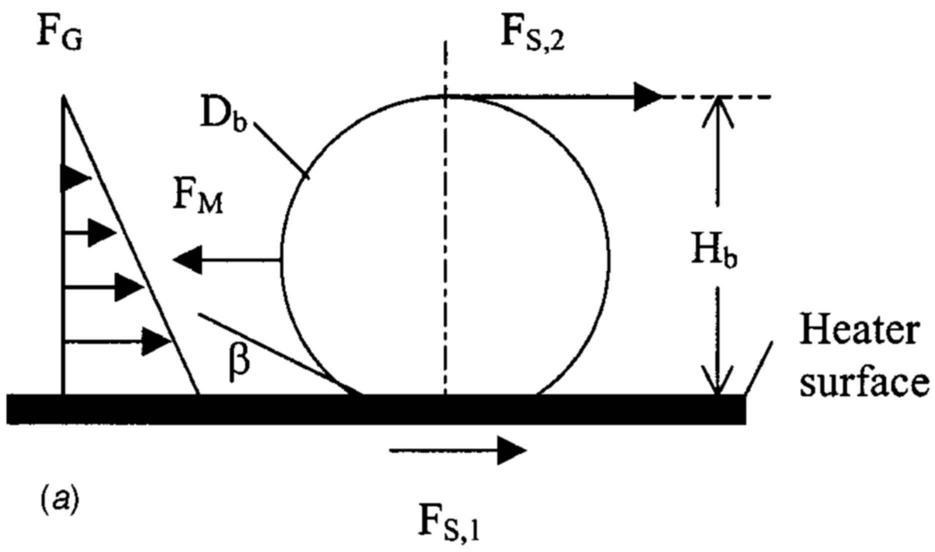


FIGURE 7.19 in Carey

$$q''_{max,Z} = 0.149\rho_v h_{lv}\left(\frac{\sigma\Delta\rho g}{\rho_v^2}\right)^{1/4}$$

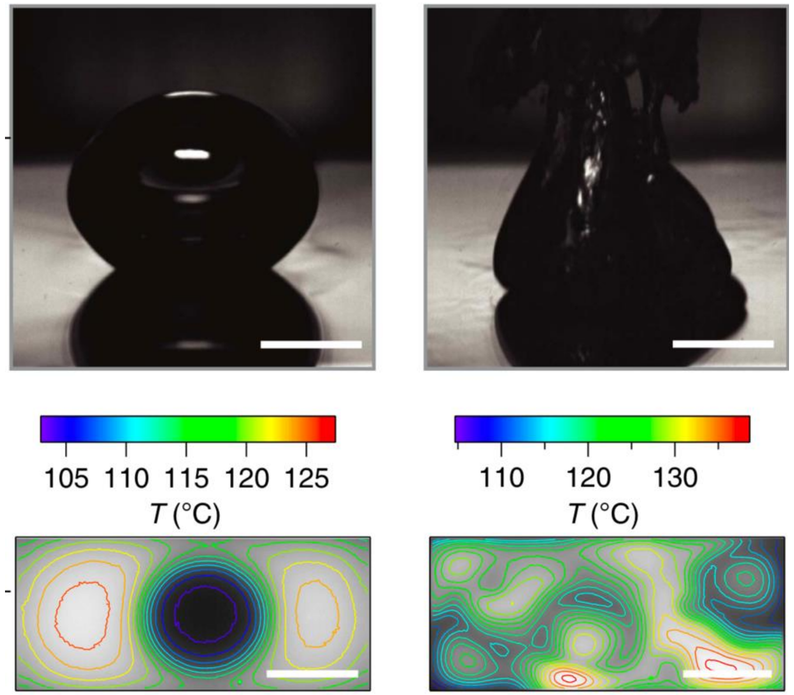
$$\frac{q''_K}{q''_{max,Z}} = \frac{1+\cos\theta}{2.096}\left[\frac{2}{\pi}+\frac{\pi}{4}(1+\cos\beta)\right]^{\frac{1}{2}}$$

Comments on Kandlikar's Model



$$F_{s,1} + F_{s,2} + F_G = F_M$$

Dhillon *et al.*, *Nat Commun* 2015

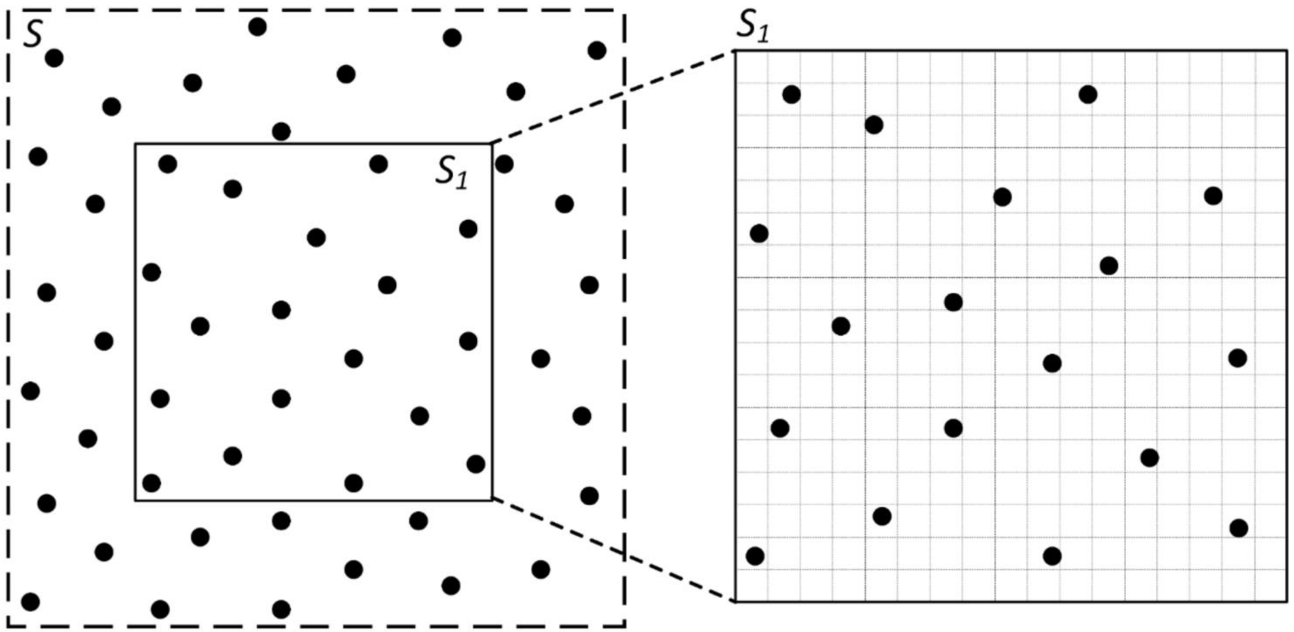


Liquid-vapor pressure difference not accounted for in momentum balance

Not clear how geometric parameters are chosen at CHF

Statistical Approach for Flat Surface Boiling

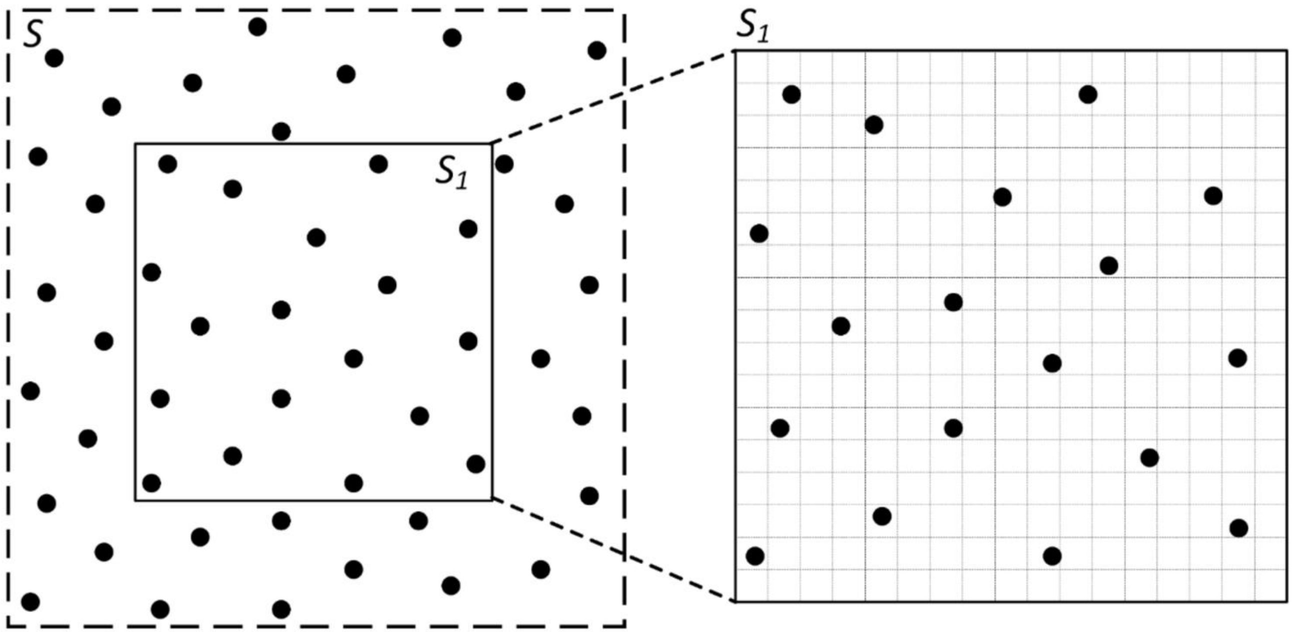
Population Distribution of Intrinsic Nucleation Sites



https://doi.org/10.1016/j.ijheatmasstransfer.2021.121904

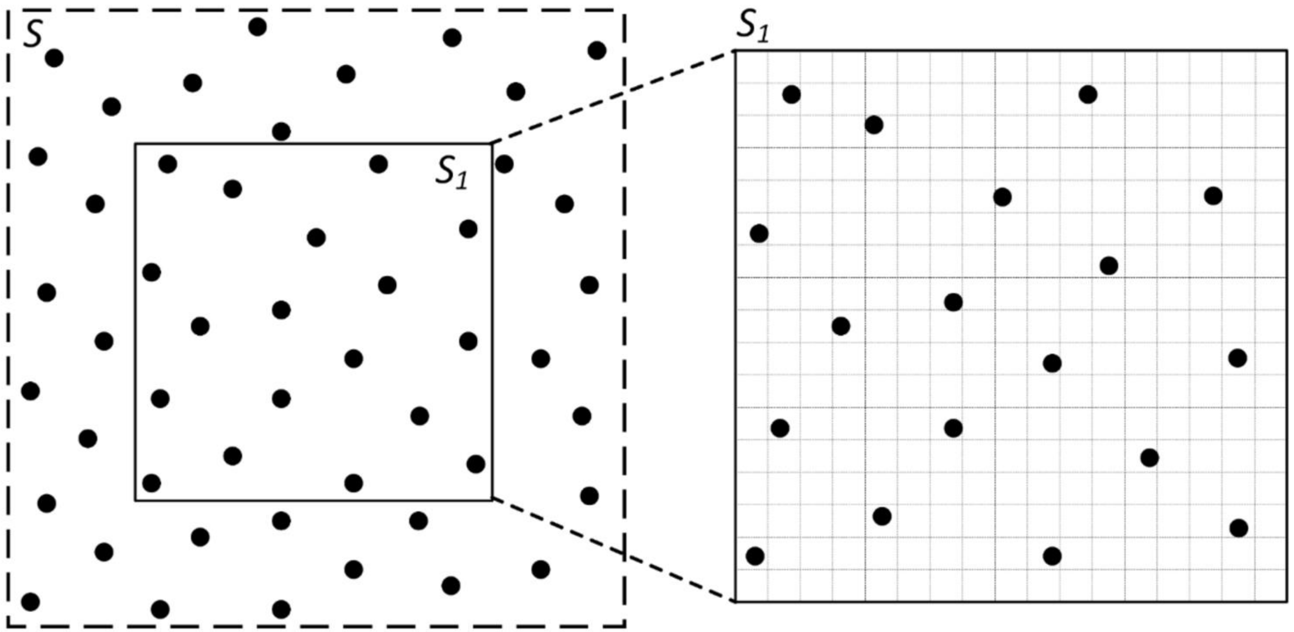
- Consider a large surface S (large enough to ignore edge effects)
- Probability of each point on the surface becoming an active nucleation sites is equal

Population Distribution of Intrinsic Nucleation Sites



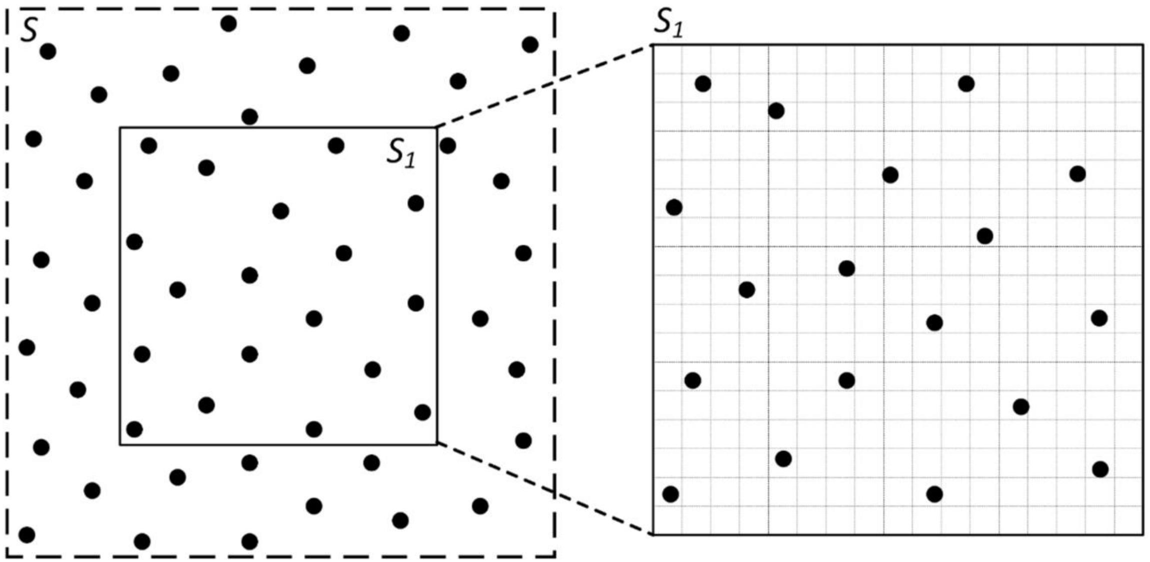
- Average nucleation density n_0 [m^{-2}]
- For an arbitrary segment of the surface S_1 of area *A*, the average number of nucleation sites is $N_0 = n_0 A$
- The actual number of nucleation sites in S_1, ***N***, is a random variable with an expectation value N_0

Population Distribution of Intrinsic Nucleation Sites



- Divide S_1 into *M* subsegments with the same area *A/M*
- Make *M* large enough such that the number of nucleation sites in each square is 0 or 1.
- Probability of finding a nucleation site in one square $p=N_0/M$

Population Distribution of Intrinsic Nucleation Sites

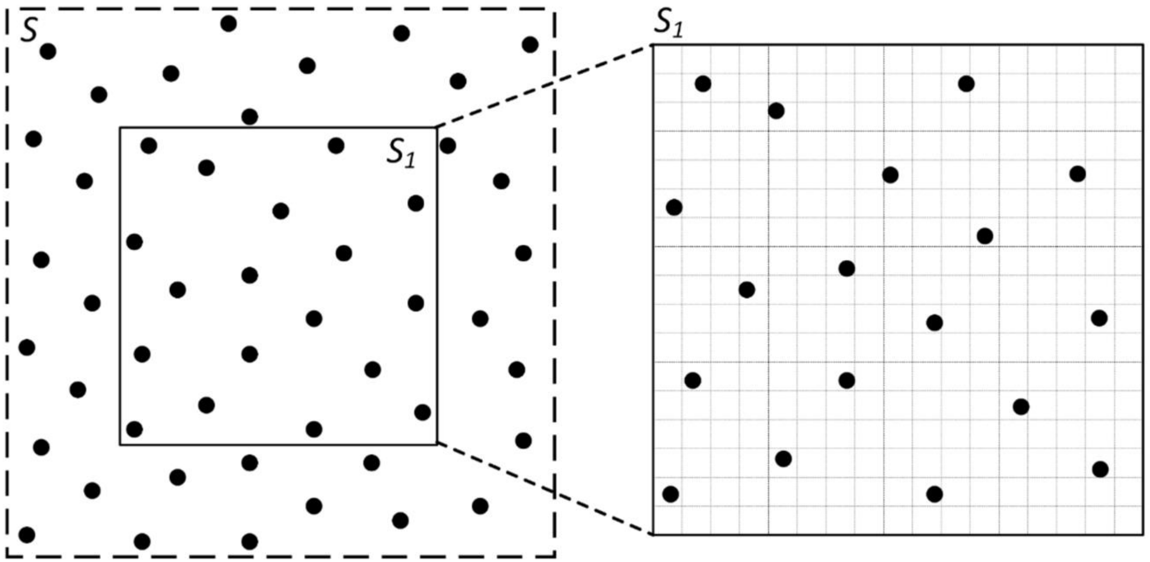


- Probability of find N squares that contain nucleation sites is a binomial distribution

$$P(N, N_0, M) = \frac{M!}{N!\,(M-N)!} p^N (1-p)^{M-N}$$

$$= \frac{1}{N!} \cdot \frac{M!}{(M-N)!\,M^N} \cdot N_0^N \left(1 - \frac{N_0}{M}\right)^{M-N}$$

Population Distribution of Intrinsic Nucleation Sites



$$P(N, N_0, M) = \frac{1}{N!} \cdot \frac{M!}{(M-N)!\,M^N} \cdot N_0^N \left(1 - \frac{N_0}{M}\right)^{M-N}$$

Stirling's approximation: $\ln(n!) = n \ln n - n + O(\ln n) \quad \text{for } n \to \infty$

$$\lim_{M\to\infty} \ln\left[\frac{M!}{(M-N)!\,M^N}\right] = 0 \Rightarrow \lim_{M\to\infty} \frac{M!}{(M-N)!\,M^N} = 1$$

$$\lim_{M\to\infty} P(N, N_0, M) = \frac{N_0^N}{N!} e^{-N_0}$$

Poisson Distribution

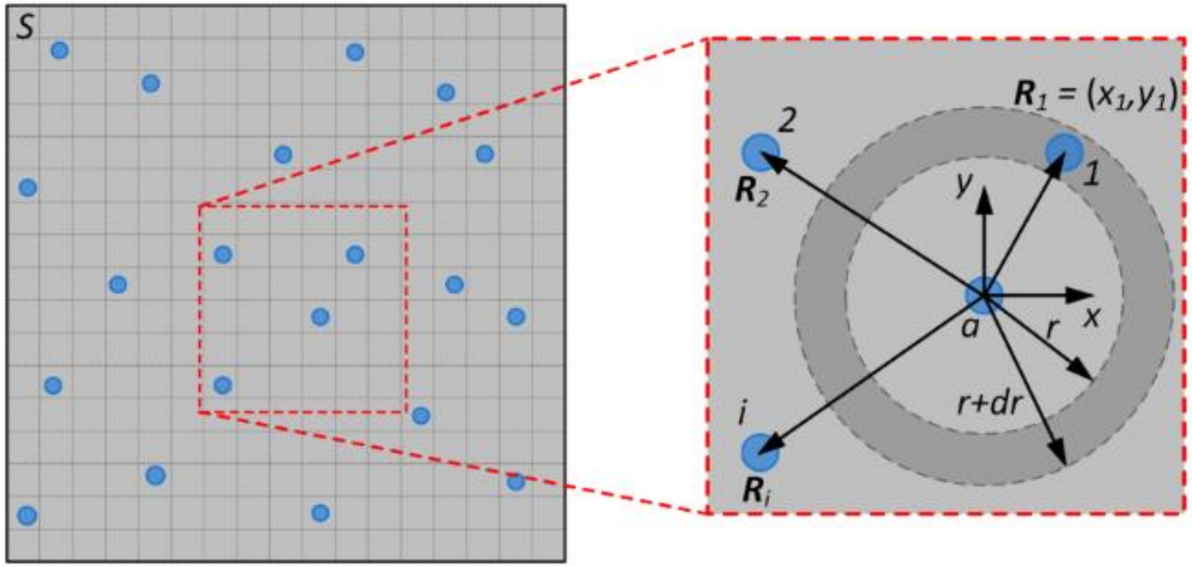
$$\mathrm{Po}(N, N_0) = \frac{N_0^N}{N!} e^{-N_0}$$

$$\sum_{N=0}^{\infty} P(N, N_0) = \sum_{N=0}^{\infty} \frac{N_0^N}{N!} e^{-N_0} = e^{N_0} \cdot e^{-N_0} = 1$$

What Marks the CHF

- Isolated bubbles dissipate heat better than merged bubbles
- CHF is reached when you have the maximum number of isolated bubbles
- With elevated temperature, more nucleation sites become activated while more bubbles are likely to merge into each other.
- It is important to consider the distance between bubbles

Nearest Neighbor Distance Between Nucleation Sites

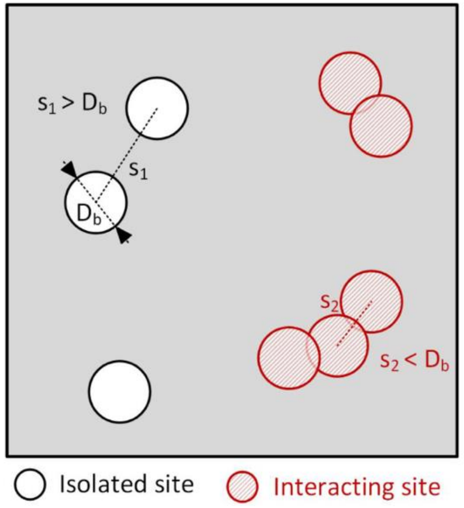


The probability distribution function for distance between nearest neighbors if there are N points randomly distributed on a surface of area A

$$f(s) = \frac{2\pi N s}{A} e^{-\frac{\pi N s^2}{A}}$$

Rayleigh distribution

Number of Isolated Bubbles



$$P_{iso} = P(s > D_b) = \int_{D_b}^{\infty} f(s)ds = \int_{D_b}^{\infty} \frac{2\pi N s}{A} e^{-\frac{\pi N s^2}{A}} ds = e^{-\frac{\pi N D_b^2}{A}}$$

$$N_{iso} = \sum_{N=1}^{\infty} N P_{iso} \mathrm{Po}(N, N_0) = \sum_{N=1}^{\infty} \frac{N_0^N}{(N-1)!} \exp\left(-N_0 - \frac{\pi N D_b^2}{A}\right)$$

CHF Criteria

$$\frac{\partial N_{iso}}{\partial T} = 0$$

$$\frac{\partial}{\partial T}\left[\sum_{N=1}^{\infty} \frac{N_0^N}{(N-1)!} \exp\left(-N_0 - \frac{\pi N D_b^2}{A}\right)\right] = 0$$

$$\Rightarrow n_0 \pi D_b^2 = 1$$

A unified relationship between the nucleation density at CHF and bubble diameter