ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 8

Presentation by Group 8 on Thursday 14th November

Problem 1: Hydrodynamic instabiltiy

Carefully read the pages 113 to 119 of the Van Carey book and answer the following questions with words and/or equations.

A) In Eq. 4.23, we assume and initial perturbation $\delta(x, 0) = Ae^{i\alpha x}$. And then, we consider subsequent oscillation of the interface with the functional form $\delta(x, t) = Ae^{i\alpha x} + \beta t$ with β being the unknown. Assume you now have found the expression for β as a complex number $a + ib$. Describe the condition under which the amplitude of the perturbation will grow with time.

B) If the relative velocity is zero and the liquid is on top of the vapor. We have the following expression for β (Eq. 4.53a in Van Carey).

$$\beta = \pm \left\{ \frac{(\rho_l - \rho_v)g\alpha - \sigma\alpha^3}{\rho_l + \rho_v} \right\}^{\frac{1}{2}}$$

Derive an expression for the perturbation wavelength with the fastest growth.

Problem 2: Maxima of number of isolated bubbles

In *International Journal of Heat and Mass Transfer 182 (2022) 121904*, Zhang et al gave an expression for the number of isolated bubbles on a surface:

$$N_{iso} = \sum_{N=1}^{\infty} \frac{N_0^N}{(N-1)!} e^{-\left(N_0 + \frac{\pi N D_b^2}{A}\right)}$$

where N_0 is the expectation value of the number of active nucleation sites on the surface, A is the surface area, and D_b is the diameter when departing from the surface. Assume the critical heat flux (CHF) is reached when $\frac{\partial N_{iso}}{\partial T} = 0$ and N_0 is the only parameter on the right-hand side that is a function of temperature T.

A) Prove that the following is true at CHF,

$$n_0 \pi D_b^2 = 1$$

where n_0 is the expected active nucleation site density at CHF or $n_0 = N_0/A$ at CHF.

B) Show that at CHF,

$$N_{iso} = \frac{N_0}{e}$$

Problem 3: Kandlikar Model

Kandlikar expression for the critical heat flux of pool boiling at an upward-facing horizontal heated large surface is given by:

$$q_c'' = h_{fg}\rho_g^{\frac{1}{2}} \left(\frac{1 + \cos\beta}{16}\right) \left[\frac{2}{\pi} + \frac{\pi}{4}(1 + \cos\beta)\right]^{\frac{1}{2}} [\sigma g(\rho_l - \rho_v)]^{\frac{1}{4}}$$

Plot the CHF prediction for contact angles β between 5° and 50° using water properties at atmospheric pressure.