ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 5

Presentation by Group 5 on Thursday 17th October

Problem 1: Difference between Schrage equation and moment method for evaporation

During the lecture, we derived the Schrage equation for evaporation mass flux across the interface:

$$j = \frac{2\sigma}{2-\sigma}\sqrt{\frac{1}{2\pi R}}\left(\frac{P_L}{\sqrt{T_L}} - \frac{P_V}{\sqrt{T_V}}\right) \tag{1}$$

where, σ is the accommodation coefficient (condensation probability), R is the specific gas constant, P_L is the saturation vapor pressure as the liquid surface, T_L is the liquid surface temperature, P_V is the far field vapor pressure, T_V is the far field vapor temperature.

A) When the vapor temperature is close to that of the liquid surface ($T_V \approx T_L = T_0$), express the Equation (1) as a linear function between the following two dimensionless quantities:

$$j^* = \frac{j}{P_L/\sqrt{2\pi R T_L}} \tag{2}$$

$$\Delta P^* = \frac{P_L - P_V}{P_L} \tag{3}$$

B) Relaxing the Schrage assumption to balance all mass momentum, and energy fluxes between the liquid surface and the far field (know as the **moment method**), the following relationship between j^* and ΔP^* can be obtained by Ytrehus, Multiphase Science and Technology, 1997:

$$j^* = \frac{\omega\sigma}{\sigma + (1-\sigma)\omega}\Delta P^* \tag{4}$$

where, $\omega = 32\pi/(32+9\pi)$.

Write your own code to plot what you obtained in A) using Schrage equation together with Ytrehus's expression, varying ΔP^* between 0 and 0.2 for i) $\sigma = 1$ and ii) $\sigma = 0.1$ — everything in one figure with proper legends.

Comment on whether the deviation between the Schrage equation and the moment method solution increases or decreases with a smaller σ.

Problem 2: Solar thermal desalination

A solar thermal desalination system converts solar radiation to thermal energy, utilizes the generated heat for vapor production (from salty water), and finally condenses the vapor into purified water.

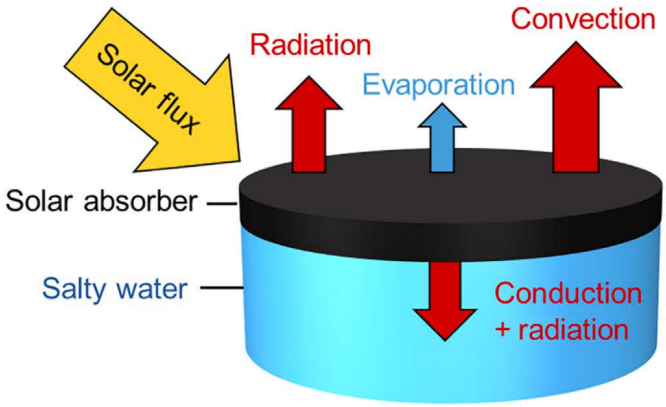

Figure 1: Wang et al., Sci. Adv. 2019; 5 : eaax0763

In Figure 1, assume we have a constant solar heat flux of $q_{sun} = 1000W/m^2$ and the solar absorber has an absorptivity of α (e.g. $\alpha = 0.9$ means 90% of the solar energy will be converted into heat). For the heat losses (all the red arrows, conduction + convection + radiation), we assign a heat loss heat transfer coefficient of 15 W/m^{2}K.

The ambient environment is at $T_\infty = 25C$ with relative humidity $\phi = 30\%$. We take the enthalpy f vaporization for water as a constant $h_{fg} = 2200kJ/kg$. The absorber has a characteristic dimension (diameter) $L = 10cm$ and evaporative mass transfer Sherwood number $Sh = 5$ between the absorber and the ambient air. Also, we know the water vapor density as a function of the temperature $\rho_{sat}(T)$ following the empirical fit:

$$\rho_{sat}(T) = 6.335 + 0.6718T - 2.0887 \cdot 10^{-2}T^2 + 7.3095 \cdot 10^{-4}T^3 \tag{5}$$

where, $\rho_{sat}(T)$ takes the unit of $[g/m^3]$ and T takes the unit of $[^\circ C]$.

The vapor mass diffusivity in the air is a function of temperature $D_{va}(T)$ Eq. 11.35 in Lienhard and Lienhard, fixing the pressure at $1atm$.

A) Write down the energy balance equation for the absorber by assuming an absorber temperature T_s and using the symbols we have defined in the problem statement.

B) Write your own code to calculate and plot the evaporation mass flux as a function of the solar absorptivity between $\alpha = 0.6$ and $\alpha = 0.9$. *Consider using the Matlab function fzero or fsolve.*

Problem 3: Homogeneous nucleation in vapor-liquid systems

In the lecture, we have derived the Gibbs free energy change from (a) to (b) (see Figure 2) for homogeneous nucleation:

$$\Delta G = \hat{N}_v(\hat{g}_v - \hat{g}_l) + (P_l - P_v)V_v + 4\pi r^2\sigma_{lv}$$

where $\hat{N}_v$ is the number of molecules in the bubble in (b), $\hat{g}_v$ and $\hat{g}_l$ are the Gibbs free energy of the vapor and liquid present in (b), V_v is the total volume of the bubble.

We also showed that:

$$\frac{d\Delta G}{dr} = \frac{d\hat{N}_v}{dr}(\hat{g}_v - \hat{g}_l) + 4\pi r^2\left(\frac{2\sigma_{lv}}{r} + P_l - P_v\right)$$

which goes to zero when chemical equilibrium and mechanical equilibrium is reached between liquid and vapor.

Derive an expression for $\frac{d^2\Delta G}{dr^2}$ at equilibrium ($r = r_e$) and comment on the implication of its sign.

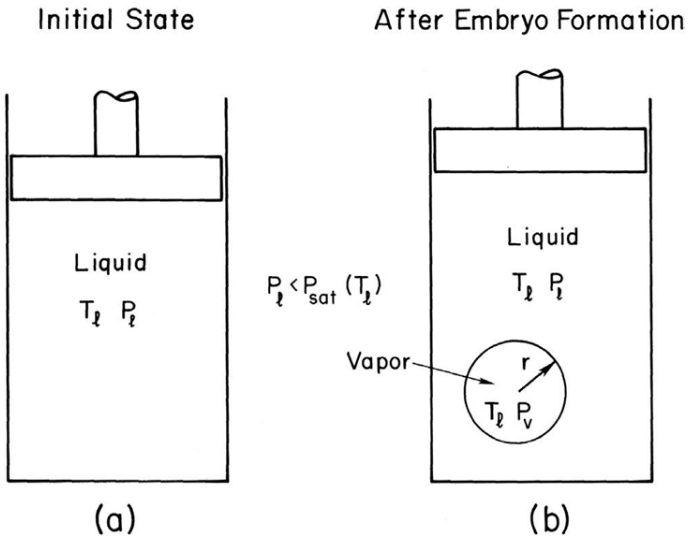

Figure 2: System considered in the thermodynamic analysis of the formation of an embryo vapor bubble (Van Carey, Chap. 5)