ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 3

Presentation by Group 3 on Thursday 3rd October

Problem 1: Natural mass convection

For natural convection heat transfer, an important concept is the Rayleigh number. For thermal transport, we define

$$Ra_T = \frac{\Delta\rho_T L^3 g}{\mu\alpha}$$

where, μ is the dynamic viscosity, α is the thermal diffusion coefficient, $\Delta\rho_T$ is the density difference between the surface and the far field caused by temperature difference (thermal expansion), L is the characteristic length, and g is the gravitational constant.

A) Derive the SI unit for Ra_T.

B) The buoyancy induced by $\Delta\rho_T$ drives natural convection. For the mass transfer of water vapor in air, different relative humidity can also cause a change in the total air-vapor mixture density. Accordingly, we can define $\Delta\rho_M$ as the density difference between the surface and the far field caused by humidity difference.

Define a mass transfer Rayleigh Ra_M using $\Delta\rho_M$ to characterize natural convection mass transfer.

C) For natural heat convection on a heated element surrounded by thermal insulation (Figure 2a), we have the correlation for the Nusselt number

$$Nu = C Ra_T^{0.2}$$

with C being a known constant.

Now consider natural convection occurring above a contained water surface (Figure 2b). You are given the thermophysical properties of the air-vapor mixture.

At the water surface:

- The water vapor density is denoted by ρ_{v0}
- The air-vapor mixture density is denoted by ρ_{m0}

In the far field:

- The water vapor density is denoted by ρ_{vf}
- The air-vapor mixture density is denoted by ρ_{mf}

Using this information, derive an expression for the vapor mass flux j_v across the water surface.

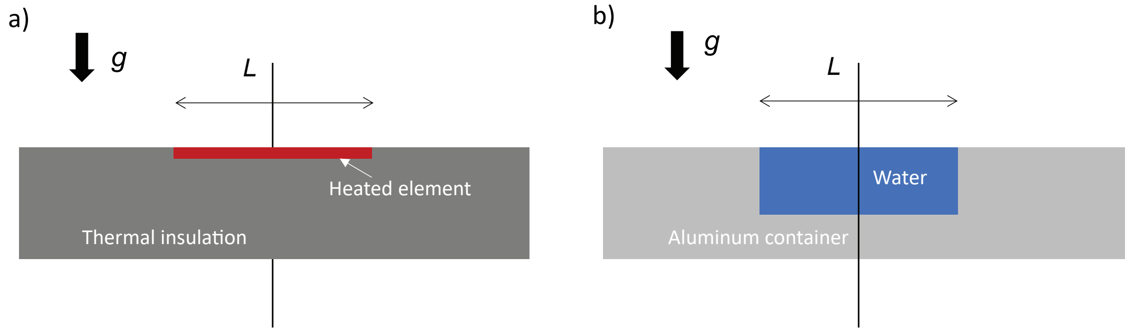


Figure 1: Natural heat (a) and mass (b) convection on horizontal plates.

Problem 2: Setting up numerical simulation

You are tasked with setting up a numerical simulation for vapor mass transfer in air from an evaporating sessile droplet (see Figure 2a). To model this, we define an axisymmetric simulation domain (see Figure 2b) with the following boundaries:

- **Edge 1:** The droplet surface, which is at temperature T_s.
- **Edge 2:** The axisymmetry centerline.
- **Edges 3 and 4:** The air far from the droplet, where the dew point is T_{dew}.
- **Edge 5:** The solid substrate on which the droplet rests.

(Try visualizing the simulation domain by imagining Figure 2b revolving around edge 2.)

Governing Equation

Assume there is no bulk motion of the air-vapor mixture. You are given the diffusion coefficient D, and the saturation water vapor density $\rho_{v,\text{sat}}(T)$, which is a function of temperature.

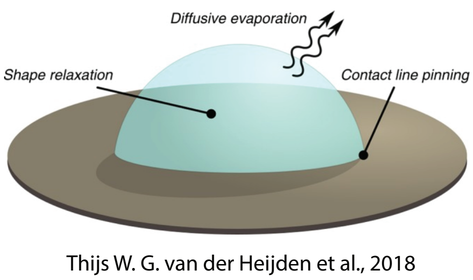
A) Write down the governing equation for vapor mass transfer in the simulation domain, using the appropriate **divergence** and/or **gradient** operators.

B) Specify the boundary conditions on edges 1-5.

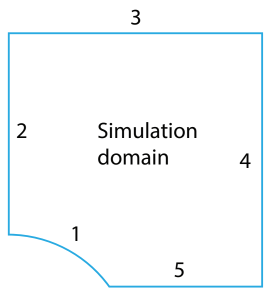
(These are the key inputs required by the numerical solver to compute the solution.)

Evaporative Mass Flux Calculation

Once the governing equation is solved numerically with the correct boundary conditions and meshing, describe how to calculate the evaporative mass flux J_s, at the droplet surface. Provide an explanation using equations and/or words.



(a)



(b)

Figure 2: (a) Evaporation from a sessile droplet. (b) Simulation domain.

Problem 3: Symmetry of diffusion coefficients in an air-vapor mixture

On Slide 16 of today's lecture (September 26, 2024), we have expressed the 1D Fick's law of the diffusion for air in an air-vapor mixture as

$$j_{ad} = -\rho D_{av} \frac{d\omega_a}{dx}$$

where, j_{ad} is the diffusion mass flux of air, ρ is the mixture total mass density, ω_a is the mass fraction of the air, and D_{av} is the diffusion coefficient for air moving in air-vapor mixture.

On the other hand, for vapor, we have written

$$j_{vd} = -\rho D_{va} \frac{d\omega_v}{dx}$$

with j_{vd} being the diffusion mass flux of vapor, ω_v being the mass fraction of the vapor, and D_{va} being the diffusion coefficient for vapor moving in air-vapor mixture.

Prove that $D_{av} = D_{va}$.

Note that diffusion flux can be understood as the relative flux to the center of mass of the mixture.

$$j_{vd} = \rho\omega_v(v_v - v_m)$$
$$j_{ad} = \rho\omega_a(v_a - v_m)$$

where v_a, v_v, and v_m represent the velocity of air, vapor, and the mixture (mass-averaged), respectively.