ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 3 - Solution

Problem 1: Natural mass convection

A) SI unit of Ra_T

$$\frac{[kg/m^3] \cdot [m^3] \cdot [N/kg]}{[Pa.s] \cdot [m^2/s]} = 1$$

Ra_T is non-dimensional.

B) Mass transfer Rayleigh number

$$Ra_M = \frac{\Delta \rho_M L^3 g}{\mu D}$$

where D is the diffusion coefficient of water vapor in air-vapor mixture.

C) The mass transfer Sherwood number in the case of natural mass convection should be

$$Sh = C Ra_M^{0.2} = \frac{g_m L}{D}$$

where g_m is the mass transfer coefficient.

Vapor mass flux across the water surface can be expressed as

$$\begin{aligned}
j_v &= g_m(\rho_{v0} - \rho_{vf}) \\
&= \frac{ShD}{L}(\rho_{v0} - \rho_{vf}) \\
&= \frac{C Ra_M^{0.2} D}{L}(\rho_{v0} - \rho_{vf}) \\
&= \frac{C(\rho_{mf} - \rho_{m0})^{0.2} L^{-0.4} g^{0.2} D^{0.8}}{\mu^{0.2}}(\rho_{v0} - \rho_{vf})
\end{aligned}$$

Problem 2: Setting up numerical simulation

Assumptions:

- Steady-state
- Isothermal
- No fluid motion
- No mass generation/destruction in the bulk mixture

The governing equation for the domain is

$$-D_{va}\nabla^2\rho_v = 0$$

Boundary conditions

- Edge 1 $\rightarrow$ $\rho_v = \rho_{v,sat}(T_s)$
- Edge 2 $\rightarrow$ No flux: $\nabla\rho_v \cdot \vec{n_2} = 0$
- Edge 3 $\rightarrow$ $\rho_v = \rho_{v,sat}(T_{dew})$
- Edge 4 $\rightarrow$ $\rho_v = \rho_{v,sat}(T_{dew})$
- Edge 5 $\rightarrow$ No flux: $\nabla\rho_v \cdot \vec{n_5} = 0$

with $\vec{n_i}$ being the normal vection on the edge i.

The evaporative flux on the sessile droplet interface is

$$J_s = -D_{va}\nabla\rho_v \cdot \vec{n_1}$$

Note that since ρ_v is constant along the edge 1, $\nabla\rho_v$ is indeed in the $\vec{n_1}$ direction.

The simulation setting assumes the vapor domain reaches a steady state much faster than the liquid-gas interface motion.

Problem 3: Symmetry of diffusion coefficients in an air-vapor mixture

In a binary mixture case, we have by definition

$$v_m = \omega_v v_v + \omega_a v_a \tag{1}$$

and

$$\omega_a + \omega_v = 1 \tag{2}$$

The diffusion fluxes relative to the center of mass of the mixture can be written as

$$j_{vd} = \rho(\omega_v v_v - \omega_v v_m) \tag{3}$$

and

$$j_{ad} = \rho(\omega_a v_a - \omega_a v_m) \tag{4}$$

Taking Equation (3) + Equation (4) we obtain

$$j_{vd} + j_{ad} = \rho(\omega_v v_v - \omega_v v_m + \omega_a v_a - \omega_a v_m) \tag{5}$$

Combining with Equation (1) we have

$$j_{vd} + j_{ad} = \rho v_m(1 - \omega_v - \omega_a) \tag{6}$$

From Equation (2) the right-hand side is null and we have

$$j_{vd} = -j_{ad} \tag{7}$$

Thus, we can write

$$D_{va}\frac{\partial \omega_v}{\partial x} = -D_{av}\frac{\partial \omega_a}{\partial x} \tag{8}$$

Using Equation (2) we have

$$D_{va}\frac{\partial \omega_a}{\partial x} = D_{av}\frac{\partial \omega_a}{\partial x} \tag{9}$$

Therefore

$$D_{va} = D_{av} \tag{10}$$