ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 1 - Solution

Problem 1: Sessile droplet

Let's find the radius of the spherical cap by looking at one symmetrical plane of the sessile droplet. Figure 1 helps us to quantify the relationship between r and R.

$$R = \frac{r}{\sin\theta} \tag{1}$$

Applying the equation for Laplace pressure across spherical surfaces:

$$P_{liq} - P_{atm} = \frac{2\gamma}{R} \tag{2}$$

Thus we have the relation between P_{liq} and P_{atm}:

$$P_{liq} = P_{atm} + \frac{2\gamma\sin\theta}{r} \tag{3}$$

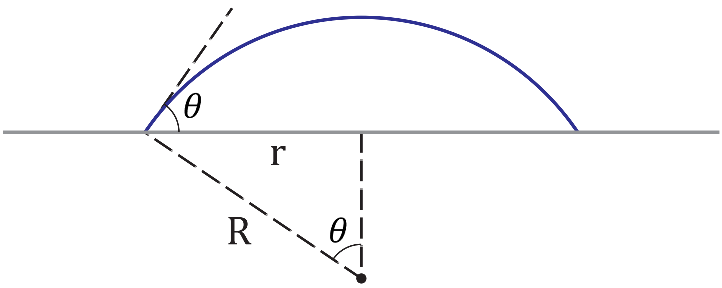

Figure 1: Sessile droplet - Geometrical consideration

Problem 2: Water jet

We need to find the curvature of the cylindrical surface. Let's find two proper cut planes at point P, with a normal vector $\vec{n}$ (Figure 2a).

The first cut plane we can take is the one formed by $\vec{n}$ and the center axis. The intersection line will be a straight line (Figure 2b). The radius of curvature for a straight line is $R_1 = \infty$.

For the second plane, which has to contain $\vec{n}$ and be perpendicular to the first one, we can take a cross-section that goes through the point P. The intersection line will be a circle with a radius r (Figure 2c). Therefore, the radius of curvature is $R_2 = r$.

The mean curvature κ is calculated as:

$$\kappa = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{1}{2r} \tag{4}$$

Applying the Young-Laplace equation, we have:

$$P_{liq} = P_{atm} + 2\kappa\gamma = P_{atm} + \frac{\gamma}{r} \tag{5}$$

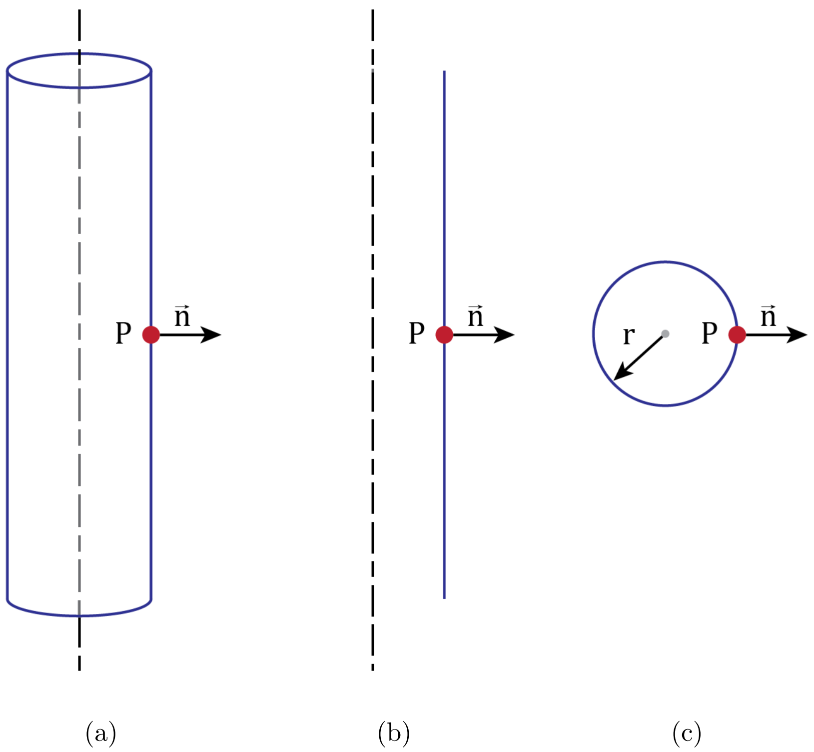

Figure 2: (a) Point P and normal $\vec{n}$ on the surface. (b) Vertical cut. (c) Horizontal cut.

Problem 3: Capillary adhesion

Considering the curvature at point P (see Figure 3), for the vertical cut we have:

$$R_1 = -\frac{H/2}{\cos\theta} \tag{6}$$

The radius is negative as the center is outside of the liquid (concave shape).

For the horizontal cut we have:

$$R_2 = R \tag{7}$$

Mean curvature:

$$\kappa = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{1}{2}\left(\frac{1}{R} - \frac{2\cos\theta_E}{H}\right) \tag{8}$$

Assuming $H << R$ we can approximate:

$$\kappa \approx -\frac{\cos\theta_E}{H} \tag{9}$$

Applying the Young-Laplace equation:

$$P_{liq} - P_{atm} = 2\gamma\kappa = -\frac{2\gamma\cos\theta_E}{H} \tag{10}$$

$P_{atm} > P_{liq}$, so the air is pressing the two plates together.

The force we need for the separation is:

$$F = |\Delta P \cdot A| = \frac{2\pi\gamma R^2\cos\theta_E}{H} \tag{11}$$

Here, we take the contact area $A = \pi R^2$ as $H << R$.

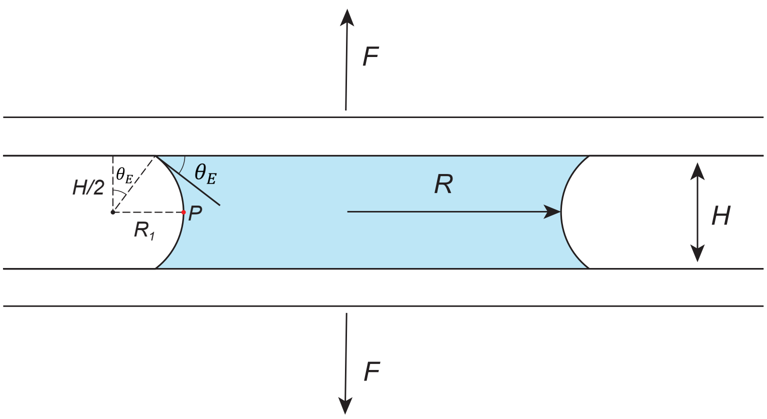

Figure 3: Capillary adhesion - Geometrical considerations

Problem 4: Pendant drop

Using the Young-Laplace equation at points 1 and 2 and assuming the pressure in the air is the same at both points we get the following equation:

$$P_{l,1} - P_{atm} = 2\kappa_1\gamma \tag{12}$$

$$P_{l,2} - P_{atm} = 2\kappa_2\gamma \tag{13}$$

Within the droplet we consider the hydrostatic pressure caused by gravity:

$$P_{l,1} - P_{l,2} = \rho g H \tag{14}$$

We have three equations and three variables, $P_{l,1}$, $P_{l,2}$ and γ. Solving for γ, we obtain:

$$\gamma = \frac{\rho g H}{2(\kappa_1 - \kappa_2)} \tag{15}$$