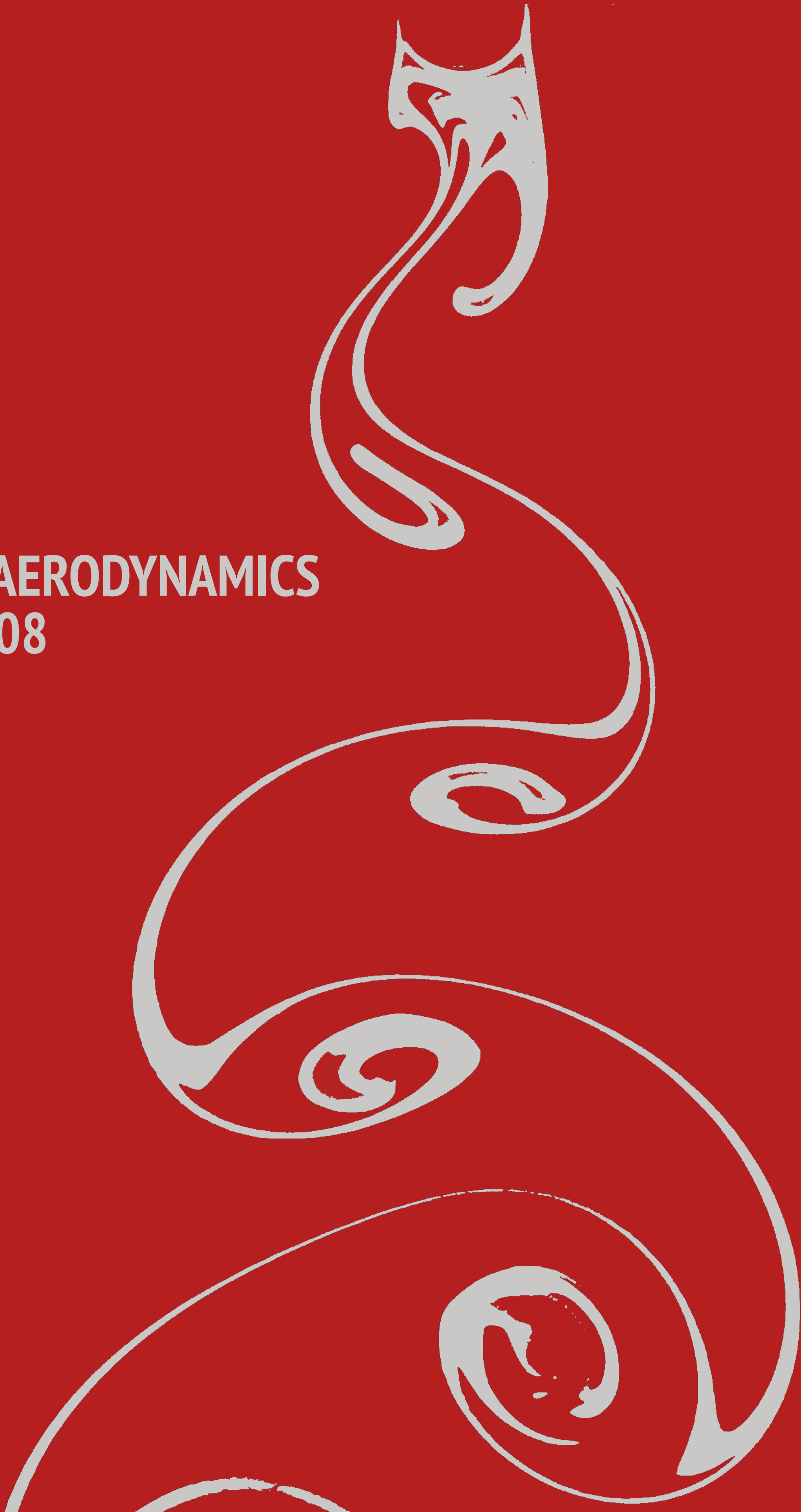
EPFL

ME-445 AERODYNAMICS
Exercise 08
Week 7



Formula sheet

Cylindrical coordinates

$$\nabla\vec{u} = \left(\frac{\partial v_r}{\partial r}, \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}, 0\right)$$

$$\nabla\cdot\vec{u} = \frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}$$

$$\nabla\times\vec{u} = \left(0, 0, \frac{1}{r}\left[\frac{\partial (r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta}\right]\right)$$

Potential flow

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

Uniform parallel flow $\quad w = \phi + i\psi = \mathrm{U}_\infty e^{-i\alpha}\, z$

Potential vortex in z_0 $\quad w = -\dfrac{i\gamma}{2\pi}\ln(z - z_0)$

Point source or sink in z_0 $\quad w = \dfrac{Q}{2\pi}\ln(z - z_0)$

Source-sink doublet in z_0 $\quad w = \dfrac{\mu}{2\pi(z - z_0)}$

$$\frac{\mathrm{d}w}{\mathrm{d}z} = u - iv$$

Milne-Thomson circle theorem:

$$g(z) = w(z) + \overline{w\left(\frac{a^2}{\bar{z}}\right)}$$

Thin airfoil theory

For a camber line with:

$$\frac{\mathrm{d}y_c}{\mathrm{d}x} = A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta$$

$$\frac{x}{c} = \frac{(1 - \cos\theta)}{2}$$

we know:

$$k = 2\mathrm{U}_\infty\left[(\alpha - A_0)\frac{\cos\theta + 1}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right]$$

$$A_0 = \frac{1}{\pi}\int_0^\pi \frac{\mathrm{d}y_c}{\mathrm{d}x}\mathrm{d}\theta$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{\mathrm{d}y_c}{\mathrm{d}x}\cos n\theta\,\mathrm{d}\theta$$

$$C_l = 2\pi\alpha + \pi(A_1 - 2A_0)$$

$$C_{m,1/4} = -\frac{\pi}{4}(A_1 - A_2)$$

$$x_{cp} = \frac{1}{4} + \frac{\pi}{4C_l}(A_1 - A_2)$$

Finite wings with AR=b^2/S

Sign convention:

if induced velocity points downward: $w(y) > 0$, $\alpha_i(y) > 0$

if induced velocity points upward: $w < 0$, $\alpha_i < 0$

Prandtl's lifting-line theory

$$\mathrm{U}_\infty \alpha_i(y_0) = w(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2}\frac{(\mathrm{d}\Gamma/\mathrm{d}y)}{y - y_0}\mathrm{d}y$$

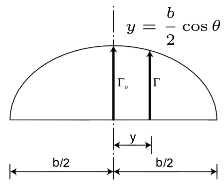
$$\alpha(y_0) = \alpha_{\text{eff}}(y_0) + \alpha_i(y_0)$$

Elliptical loading $\Gamma(y) = \Gamma_0\sqrt{1 - \left(\dfrac{2y}{b}\right)^2}$

$$w = \frac{\Gamma_0}{2b}$$

$$\alpha_i = \frac{C_L}{\pi \mathrm{AR}}$$

$$C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}$$



General loading $\Gamma(\theta) = 2b\mathrm{U}_\infty \displaystyle\sum_{n=1}^{\infty} A_n \sin n\theta$

$$w(\theta) = \mathrm{U}_\infty \sum_{n=1}^{\infty} n A_n \frac{\sin n\theta}{\sin\theta}$$

$$C_L = \pi A_1\,\mathrm{AR}$$

$$C_{D,i} = \frac{C_L^2}{\pi\mathrm{AR}}(1 + \delta) \text{ with } \delta = \sum_{n=2}^{\infty} n\,(A_n/A_1)^2$$

Boundary Layer

Flat plate **laminar** boundary layer

$\dfrac{\delta}{x} = \dfrac{5}{\sqrt{Re_x}}$ boundary layer growth

$C_f = \dfrac{1.328}{\sqrt{Re_x}}$ skin friction drag coefficient

Flat plate **turbulent** boundary layer

$\dfrac{\delta}{x} = \dfrac{0.37}{Re_x^{1/5}}$ boundary layer growth

$C_f = \dfrac{0.074}{Re_x^{1/5}}$ skin friction drag coefficient

Miscellanous

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

water

kinematic viscosity $\nu = 1 \times 10^{-6}\,\mathrm{m}^2\,\mathrm{s}^{-1}$

density $\rho = 1000\,\mathrm{kg}\,\mathrm{m}^{-3}$

air

kinematic viscosity $\nu = 1.5 \times 10^{-5}\,\mathrm{m}^2\,\mathrm{s}^{-1}$

density $\rho = 1.2\,\mathrm{kg}\,\mathrm{m}^{-3}$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

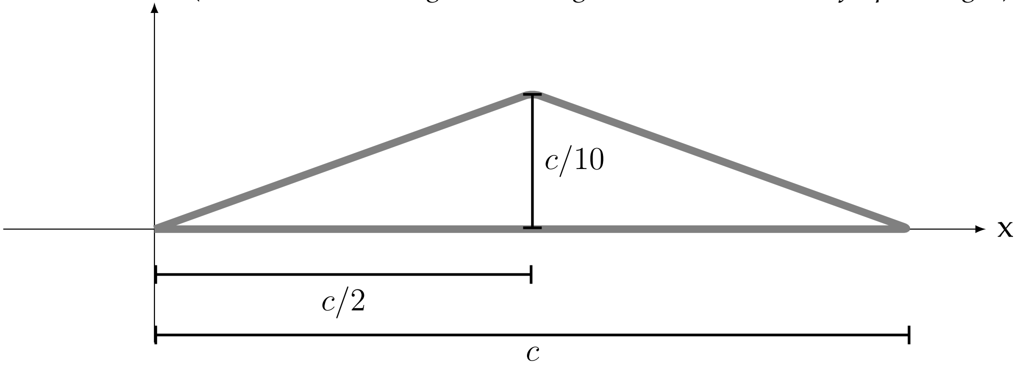
$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$
$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\int_0^\pi \cos\theta \mathrm{d}\theta = 0$$
$$\int_0^\pi \sin\theta \mathrm{d}\theta = 2$$
$$\int_0^\pi \cos^2\theta \mathrm{d}\theta = \int_0^\pi \sin^2\theta \mathrm{d}\theta = \frac{\pi}{2}$$

$$\int_0^\pi \frac{\cos n\theta}{\cos\theta - \cos\theta_1}\mathrm{d}\theta = \pi\frac{\sin n\theta_1}{\sin\theta_1} \qquad n = 0, 1, 2, \ldots$$
$$\int_0^\pi \frac{\sin n\theta \sin\theta}{\cos\theta - \cos\theta_1}\mathrm{d}\theta = -\pi\cos n\theta_1 \qquad n = 1, 2, 3, \ldots$$

1. Consider the isosceles* triangle airfoil with a thickness of $10\,\%$ drawn in the figure below.

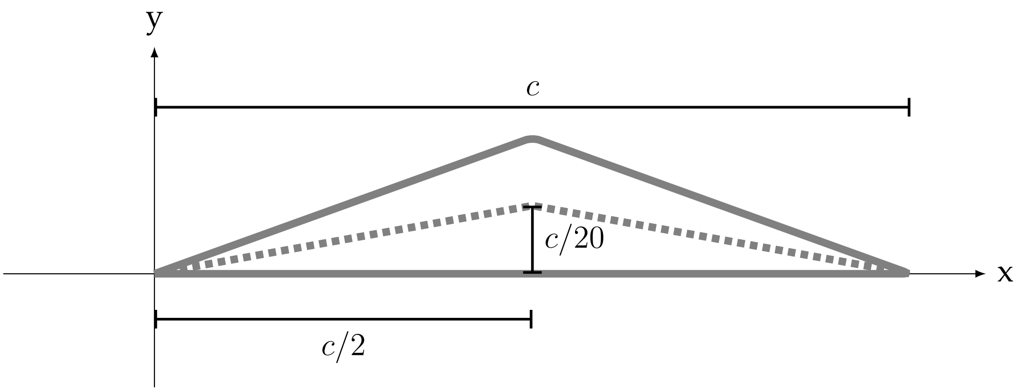
(* *An isosceles triangle is a triangle that has two sides of equal length*)



(a) Give an expression for the camber-line y_c of the isosceles triangle airfoil.

Solution:

The camber-line is indicated by the dotted line and is a two-part linear curve:



For $x \leq c/2$: $y_c = p_1 x + p_2$
For $x \geq c/2$: $y_c = p_3 x + p_4$

We find the coefficients p_1, ..., p_4 by using the boundary conditions:

$$\begin{aligned} y_c(0) &= 0 && \Rightarrow p_2 = 0 \\ y_c(c/2) &= c/20 && \Rightarrow p_1 = 0.1 = -p_3 \\ y_c(c) &= 0 && \Rightarrow p_4 = 0.1c \end{aligned}$$

For $x \leq c/2$: $y_c = 0.1x$
For $x \geq c/2$: $y_c = 0.1(c - x)$

(b) What would be the 4-digit airfoil profile that matches the geometric properties of our triangle aifoil?

Solution:

- thickness = 0.1 c
- maximum camber = 0.05 c

- location of maximum camber = 0.5 c

The NACA profile is NACA5510.

(c) Find the first four Fourier coefficients (A_0, A_1, A_2, A_3) of the cosine series that describes the camber-line gradient of the triangle airfoil based in the general **thin airfoil theory**.

Solution:

$$\text{For } x \leq c/2, \theta \leq \pi/2 : \frac{\mathrm{d}y_c}{\mathrm{d}x} = 0.1$$

$$\text{For } x \geq c/2, \theta \geq \pi/2 : \frac{\mathrm{d}y_c}{\mathrm{d}x} = -0.1$$

$$A_0 = \frac{1}{\pi}\int_0^{\pi} \frac{\mathrm{d}y_c}{\mathrm{d}x}\mathrm{d}\theta = \frac{1}{\pi}\int_0^{\pi/2} 0.1 \ \mathrm{d}\theta + \frac{1}{\pi}\int_{\pi/2}^{\pi} -0.1 \ \mathrm{d}\theta = 0$$

$$A_1 = \frac{2}{\pi}\int_0^{\pi} \frac{\mathrm{d}y_c}{\mathrm{d}x}\cos\theta \mathrm{d}\theta = \frac{0.2}{\pi}\left(\int_0^{\pi/2} \cos\theta \ \mathrm{d}\theta - \int_{\pi/2}^{\pi} \cos\theta \ \mathrm{d}\theta\right) = \frac{0.2}{\pi}(1+1) = \frac{0.4}{\pi}$$

$$A_2 = \frac{2}{\pi}\int_0^{\pi} \frac{\mathrm{d}y_c}{\mathrm{d}x}\cos 2\theta \mathrm{d}\theta = \frac{0.2}{\pi}\left(\int_0^{\pi/2} 0.1\cos 2\theta \ \mathrm{d}\theta - \int_{\pi/2}^{\pi} \cos 2\theta \ \mathrm{d}\theta\right) = \frac{0.2}{\pi}(0+0) = 0$$

$$A_3 = \frac{2}{\pi}\int_0^{\pi} \frac{\mathrm{d}y_c}{\mathrm{d}x}\cos 3\theta \mathrm{d}\theta = \frac{0.2}{\pi}\left(\int_0^{\pi/2} 0.1\cos 3\theta \ \mathrm{d}\theta - \int_{\pi/2}^{\pi} \cos 3\theta \ \mathrm{d}\theta\right) = \frac{0.2}{\pi}\left(-\frac{1}{3} - \frac{1}{3}\right) = -\frac{2}{15\pi}$$

$$A_0 = 0$$

$$A_1 = \frac{2}{5\pi} = 0.127$$

$$A_2 = 0$$

$$A_3 = -\frac{2}{15\pi} = -0.042$$

(d) Write an expression for the **lift coefficient** and **moment coefficient about the leading edge** as a function of the angle of attack. Consider only small angles of attack such that $\cos\alpha \approx 1$.

Solution:

$$C_l = 2\pi\alpha + \pi(A_1 - 2A_0) = 2\pi\alpha + \pi\frac{2}{5\pi}$$

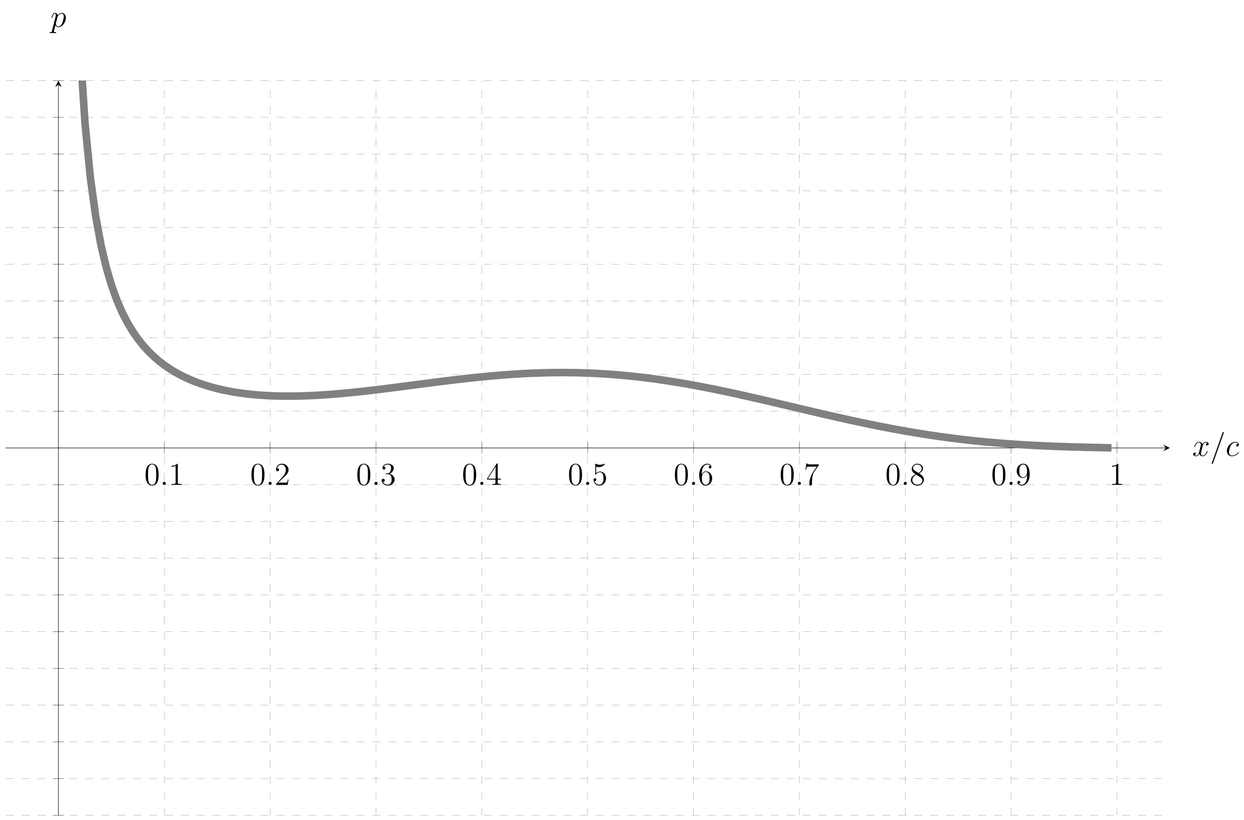
$$C_l = 2\pi\alpha + \frac{2}{5}$$

$$C_{m,1/4} = -\frac{\pi}{4}(A_1 - A_2) = -\frac{\pi}{4}\frac{2}{5\pi} = -0.1$$

$$C_{m,LE} = C_{m,1/4} - \frac{1}{4}c_l\cos\alpha \approx -\frac{1}{10} - \frac{1}{4}\left(2\pi\alpha + \frac{2}{5}\right) = -\frac{1}{10} - \frac{\pi}{2}\alpha - \frac{1}{10}$$
$$C_{m,LE} = -\frac{1}{5} - \frac{\pi}{2}\alpha$$

(e) Sketch the pressure difference along the chord for an angle of attack $\alpha = 2°$.

Solution:



$$\begin{aligned} p &= \rho k U_\infty \\ &= 2\rho U_\infty^2\left[(\alpha - A_0)\frac{\cos\theta + 1}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right] \\ &= 2\rho U_\infty^2\left[(\alpha - A_0)\frac{\cos\theta + 1}{\sin\theta} + A_1\sin\theta + A_2\sin 2\theta + A_3\sin 3\theta\right] \end{aligned}$$

$$p = 2\rho U_\infty^2\left[\alpha\frac{\cos\theta + 1}{\sin\theta} + A_1\sin\theta + A_3\sin 3\theta\right]$$

(f) Calculate the location of the centre of pressure for an angle of attack $\alpha = 2°$.

Solution:

$$
\begin{aligned}
x_{\text{cp}} &= \frac{1}{4} + \frac{\pi}{4C_1}\left(A_1 - A_2\right) \\
&= \frac{1}{4} + \frac{A_1 - A_2}{8\alpha + 4\left(A_1 - 2A_0\right)} \\
&= \frac{1}{4} + \frac{2/(5\pi)}{8\alpha + 8/(5\pi)} \\
&= \frac{1}{4} + \frac{1}{4}\left(\frac{1}{5\pi\alpha + 1}\right)
\end{aligned}
$$

$$x_{\text{cp}} = 0.411$$