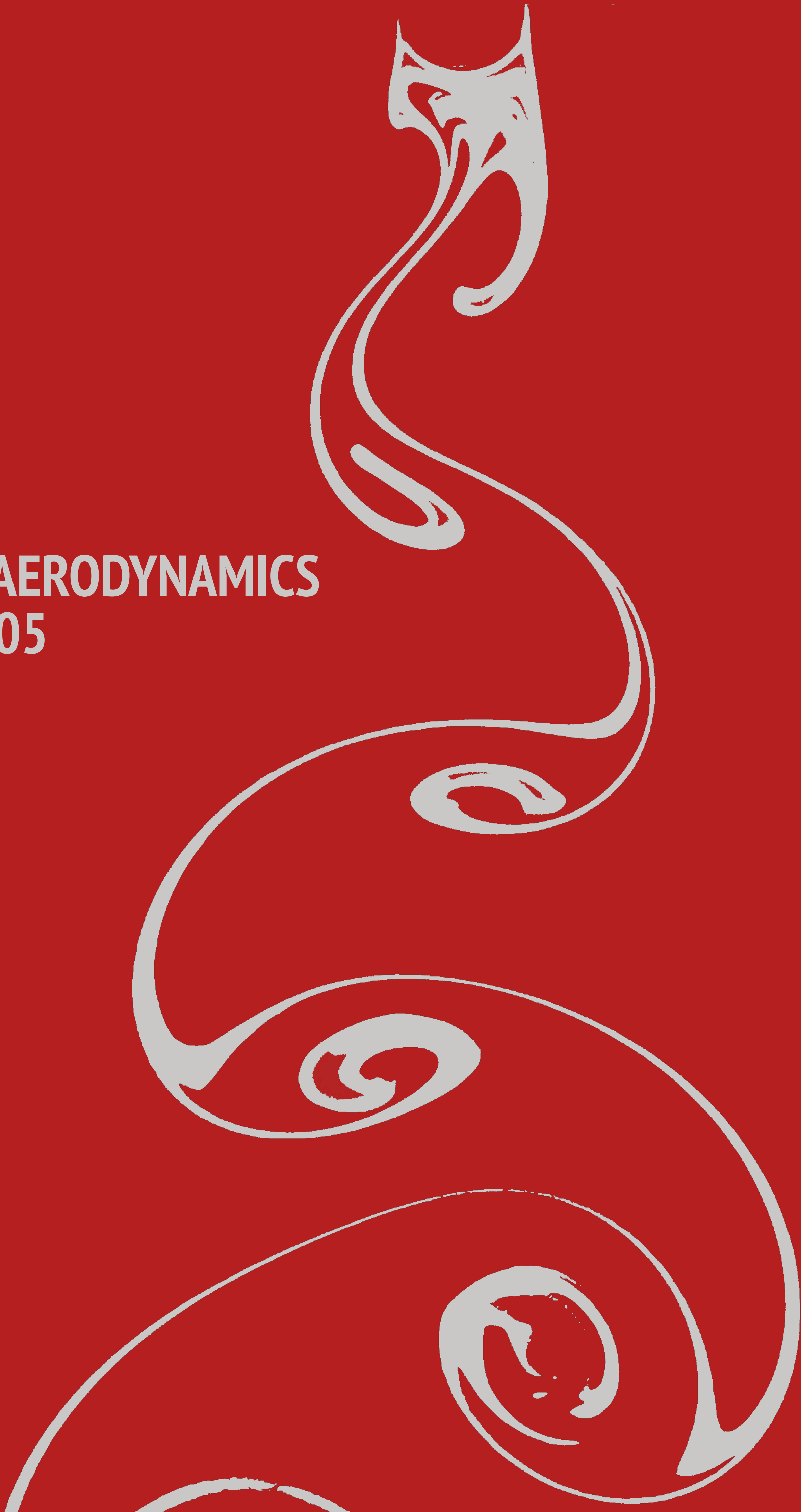
EPFL



ME-445 AERODYNAMICS
Exercise 05
Week 4

Formula sheet

Cylindrical coordinates

$$\nabla \vec{u} = \left(\frac{\partial v_r}{\partial r}, \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}, 0\right)$$

$$\nabla \cdot \vec{u} = \frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}$$

$$\nabla \times \vec{u} = \left(0, 0, \frac{1}{r}\left[\frac{\partial (r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta}\right]\right)$$

Potential flow

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

Uniform parallel flow $\quad w = \phi + i\psi = \mathrm{U}_\infty e^{-i\alpha} z$

Potential vortex in z_0 $\quad w = -\dfrac{i\gamma}{2\pi}\ln(z - z_0)$

Point source or sink in z_0 $\quad w = \dfrac{Q}{2\pi}\ln(z - z_0)$

Source-sink doublet in z_0 $\quad w = \dfrac{\mu}{2\pi(z - z_0)}$

$$\frac{\mathrm{d}w}{\mathrm{d}z} = u - iv$$

Milne-Thomson circle theorem:

$$g(z) = w(z) + \overline{w\left(\frac{a^2}{\bar{z}}\right)}$$

Thin airfoil theory

For a camber line with:

$$\frac{\mathrm{d}y_c}{\mathrm{d}x} = A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta$$

$$\frac{x}{c} = \frac{(1 - \cos\theta)}{2}$$

we know:

$$k = 2\mathrm{U}_\infty \left[(\alpha - A_0)\frac{\cos\theta + 1}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right]$$

$$A_0 = \frac{1}{\pi}\int_0^\pi \frac{\mathrm{d}y_c}{\mathrm{d}x}\mathrm{d}\theta$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{\mathrm{d}y_c}{\mathrm{d}x}\cos n\theta \mathrm{d}\theta$$

$$C_l = 2\pi\alpha + \pi(A_1 - 2A_0)$$

$$C_{m,1/4} = -\frac{\pi}{4}(A_1 - A_2)$$

$$x_{cp} = \frac{1}{4} + \frac{\pi}{4C_l}(A_1 - A_2)$$

Finite wings with AR=b^2/S

Sign convention:

if induced velocity points downward: $w(y) > 0$, $\alpha_i(y) > 0$

if induced velocity points upward: $w < 0$, $\alpha_i < 0$

Prandtl's lifting-line theory

$$\mathrm{U}_\infty \alpha_i(y_0) = w(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2} \frac{(\mathrm{d}\Gamma/\mathrm{d}y)}{y - y_0}\mathrm{d}y$$

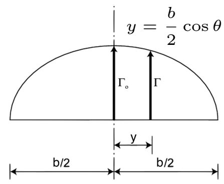
$$\alpha(y_0) = \alpha_{\text{eff}}(y_0) + \alpha_i(y_0)$$

Elliptical loading $\Gamma(y) = \Gamma_0\sqrt{1 - \left(\dfrac{2y}{b}\right)^2}$

$$w = \frac{\Gamma_0}{2b}$$

$$\alpha_i = \frac{C_L}{\pi \mathrm{AR}}$$

$$C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}$$



General loading $\Gamma(\theta) = 2b\mathrm{U}_\infty \sum_{n=1}^{\infty} A_n \sin n\theta$

$$w(\theta) = \mathrm{U}_\infty \sum_{n=1}^{\infty} nA_n \frac{\sin n\theta}{\sin\theta}$$

$$C_L = \pi A_1 \,\mathrm{AR}$$

$$C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}(1 + \delta) \text{ with } \delta = \sum_{n=2}^{\infty} n\,(A_n/A_1)^2$$

Boundary Layer

Flat plate **laminar** boundary layer

$\dfrac{\delta}{x} = \dfrac{5}{\sqrt{Re_x}}$ boundary layer growth

$C_f = \dfrac{1.328}{\sqrt{Re_x}}$ skin friction drag coefficient

Flat plate **turbulent** boundary layer

$\dfrac{\delta}{x} = \dfrac{0.37}{Re_x^{1/5}}$ boundary layer growth

$C_f = \dfrac{0.074}{Re_x^{1/5}}$ skin friction drag coefficient

Miscellanous

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

water

kinematic viscosity $\nu = 1 \times 10^{-6}\,\mathrm{m^2\,s^{-1}}$

density $\rho = 1000\,\mathrm{kg\,m^{-3}}$

air

kinematic viscosity $\nu = 1.5 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$

density $\rho = 1.2\,\mathrm{kg\,m^{-3}}$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$
$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\int_0^\pi \cos\theta \mathrm{d}\theta = 0$$
$$\int_0^\pi \sin\theta \mathrm{d}\theta = 2$$
$$\int_0^\pi \cos^2\theta \mathrm{d}\theta = \int_0^\pi \sin^2\theta \mathrm{d}\theta = \frac{\pi}{2}$$

$$\int_0^\pi \frac{\cos n\theta}{\cos\theta - \cos\theta_1}\mathrm{d}\theta = \pi\frac{\sin n\theta_1}{\sin\theta_1} \qquad n = 0, 1, 2, \ldots$$
$$\int_0^\pi \frac{\sin n\theta \sin\theta}{\cos\theta - \cos\theta_1}\mathrm{d}\theta = -\pi\cos n\theta_1 \qquad n = 1, 2, 3, \ldots$$

1. The velocity components of a two-dimensional inviscid incompressible flow are given by

$$u = 2y + \frac{y}{\sqrt{x^2 + y^2}}$$

$$v = -2x - \frac{x}{\sqrt{x^2 + y^2}}$$

(a) Find the stream function ψ that satisfies the boundary condition ψ (0,0) = 0 in cartesian and polar coordinates.

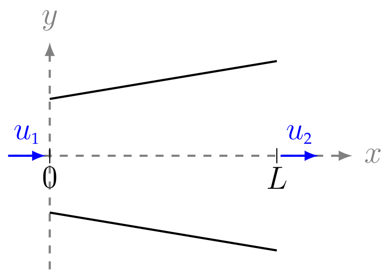
(b) Is this flow irrotational? Hint:

$$\nabla \times \vec{U} = \left(0, 0, \frac{1}{r}\left(\frac{\partial r v_\theta}{\partial r} - \frac{\partial v_r}{\partial \theta}\right)\right)$$

(c) Sketch the streamlines.

(d) What is the circulation Γ in the contour given by ψ = 1?

2. Consider the steady 2D potential flow in a diverging channel. The velocity field is given by $\vec{U} = (u, v)$; the x-component of $\vec{U}$ is given by $u = \alpha x + \beta$, with α and β constant. The velocity at $x = 0$ is equal to u_1 and the x-component of the velocity at $x =$ L is u_2.



(a) What are the assumptions of potential flow?

(b) Use the fact that the flow is incompressible to derive an expression for the y-component of the velocity field given that $v(y = 0) = 0$.

(c) Is this flow irrotational?

(d) Derive the expression for the complex potential $w(z)$, with $z = x + iy$ and the boundary condition $w(0) = 0$.

(e) A solid circular cylinder with radius R is mounted on the central axis of the diverging channel. Assume that the diameter of the cylinder is small compared to the local width of the channel. Determine the complex potential $w(z)$ of the diverging flow in which this cylinder is placed.
Hint: Use the Milne-Thomson circle theorem.

(f) (i) What is the stream function ψ for the flow over the circular cylinder?

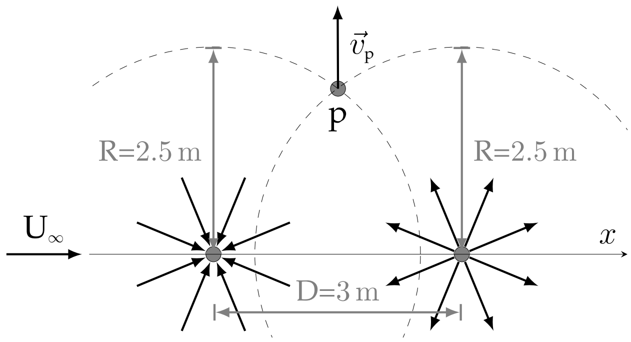
(ii) Find the velocity on the surface of the cylinder given that

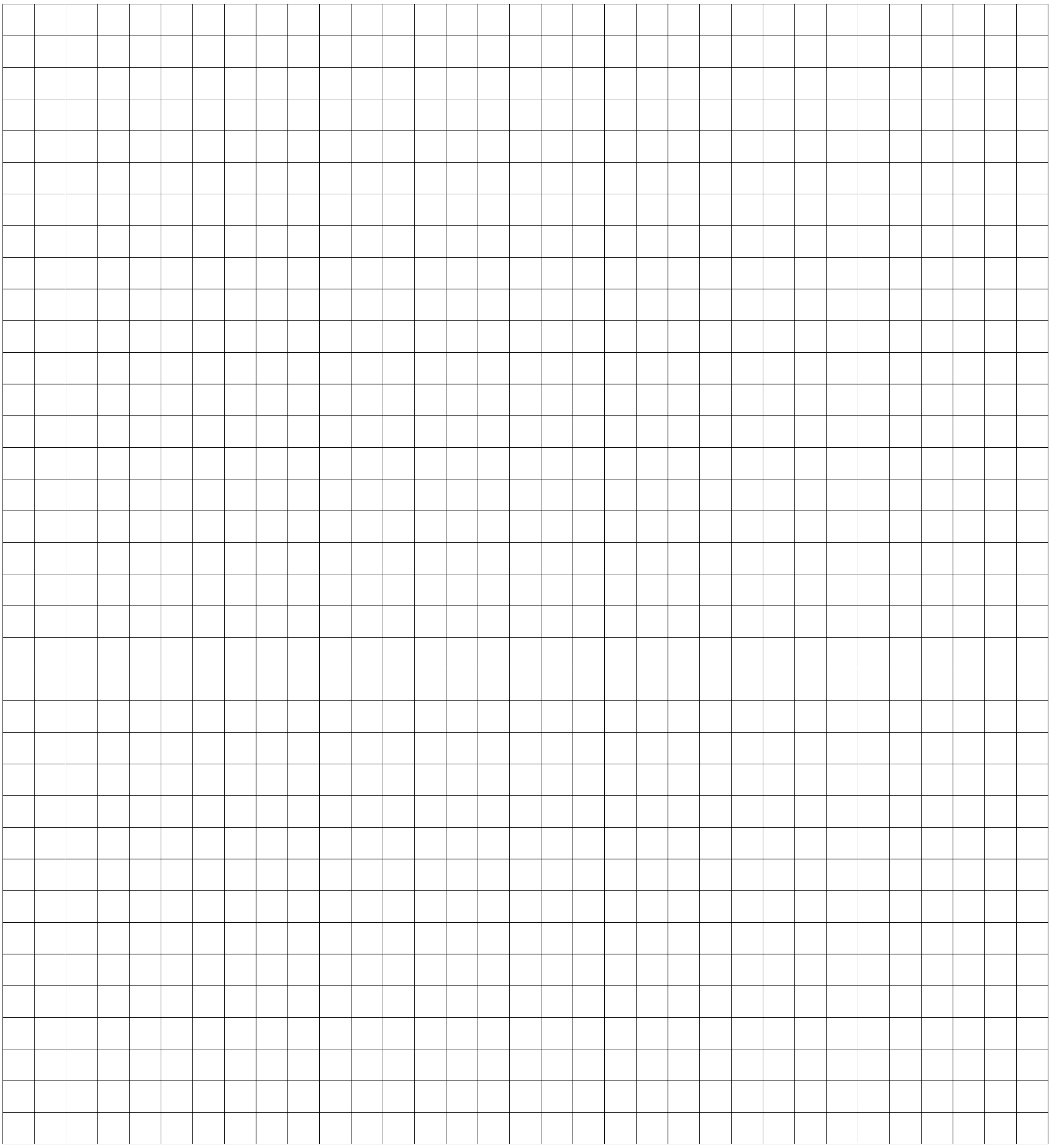
$$v_\theta = -\frac{\partial \psi}{\partial r}.$$

(iii) Find the lift force exerted on the cylinder. Assume that $U_\infty = u(L/2)$.

(iv) In what direction should the cylinder be translated in order for a non-zero resultant force to be exerted on it?

3. A sink of strength $20\,\mathrm{m^2\,s^{-1}}$ is located $3\,\mathrm{m}$ upstream of a source of $40\,\mathrm{m^2\,s^{-1}}$ in a horizontal uniform irrotational flow that goes from left to right. At a point p located $2.5\,\mathrm{m}$ from both the source and the sink. Find the velocity at point p and the velocity of the uniform flow U_∞ that satisfy the condition that the resulting local velocity at p is vertical.





4. A two-dimensional source is placed in a uniform flow of $U_\infty = 2\,\mathrm{m\,s^{-1}}$ from left to right along the x-direction. The volume flow rate coming from the source is $4\,\mathrm{m^2\,s^{-1}}$.

   (a) Find the location of the stagnation point.

   (b) Sketch the body shape passing through the stagnation point. Find the maximum width of the body.

   (c) Find the maximum and minimum pressure coefficients on the body.