Fall 2024

ME-445 AERODYNAMICS
06 - Viscous flows

K. Mulleners

Viscous flow effects

Drag

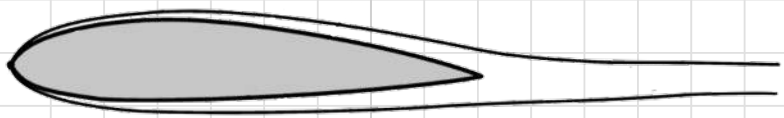
Boundary layers

Boundary layer separation

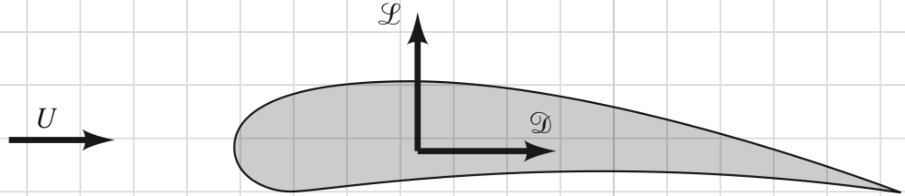
Boundary layer separation on airfoils

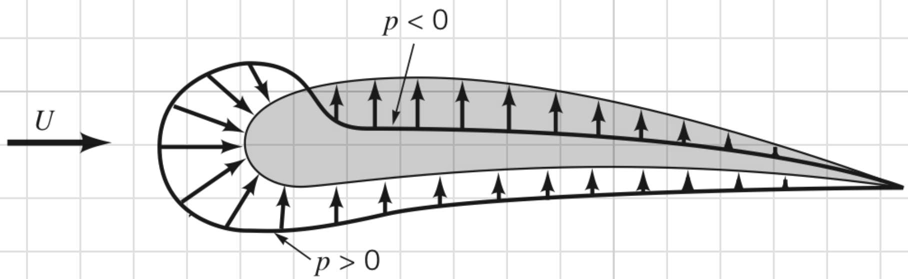
where are viscous effects important? ☐

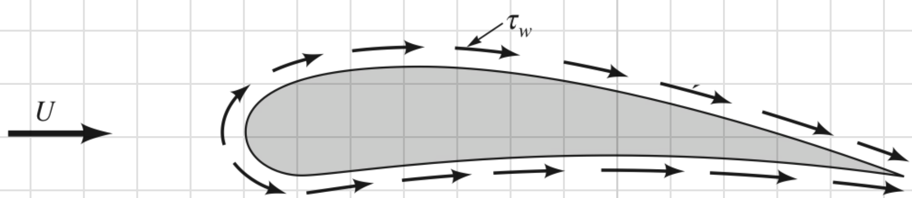
Viscous flows



Drag



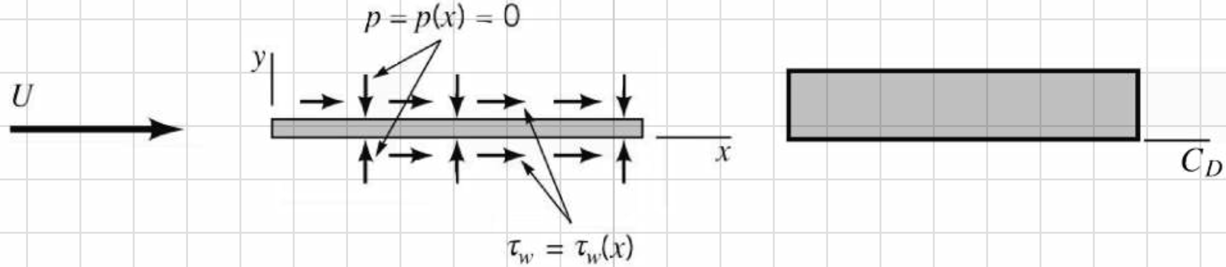


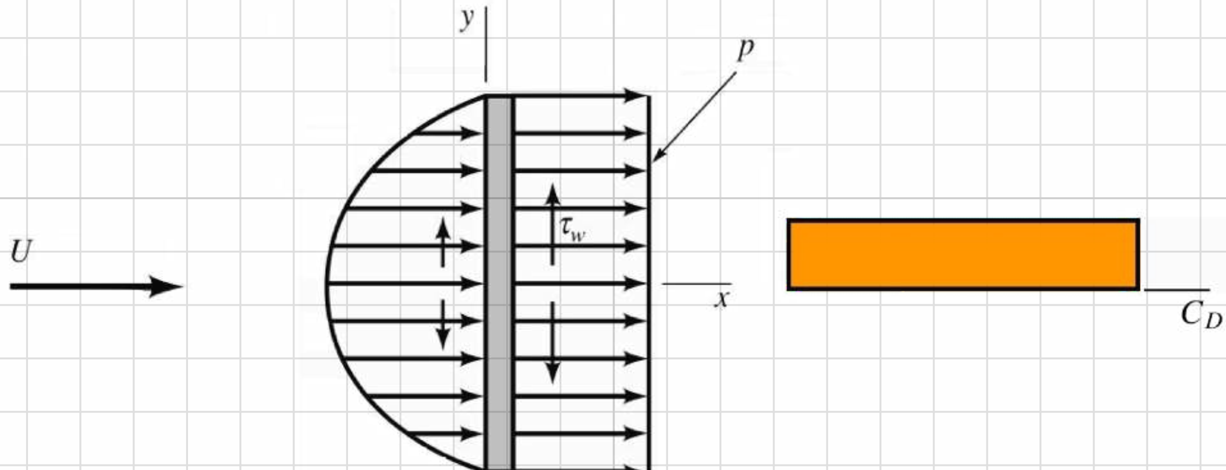


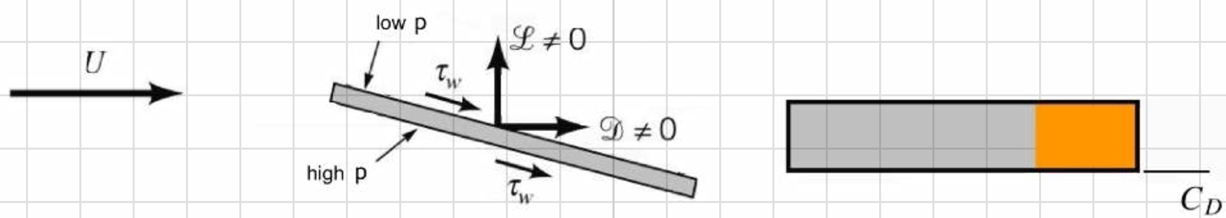
Drag

skin friction drag

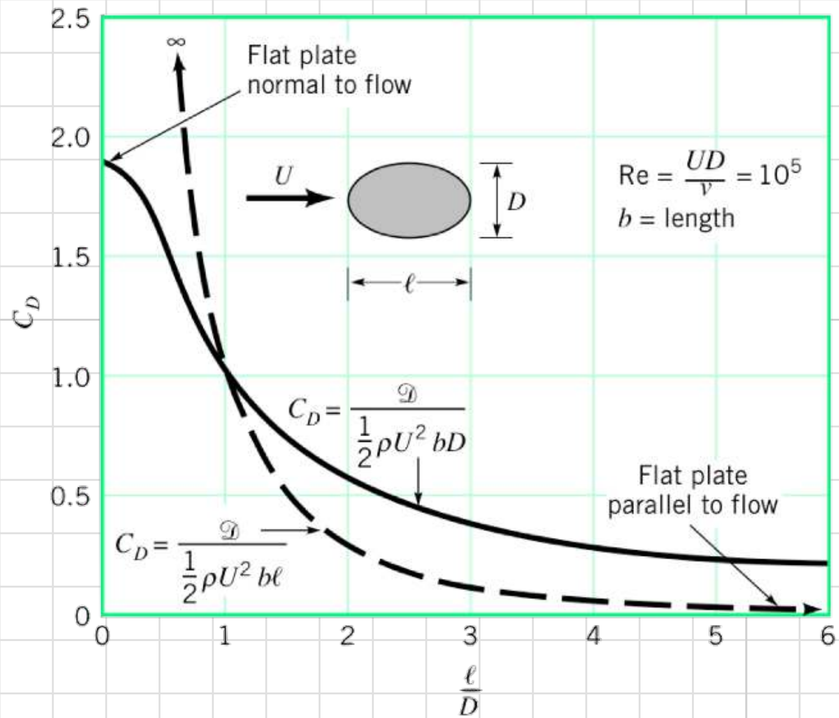
pressure drag

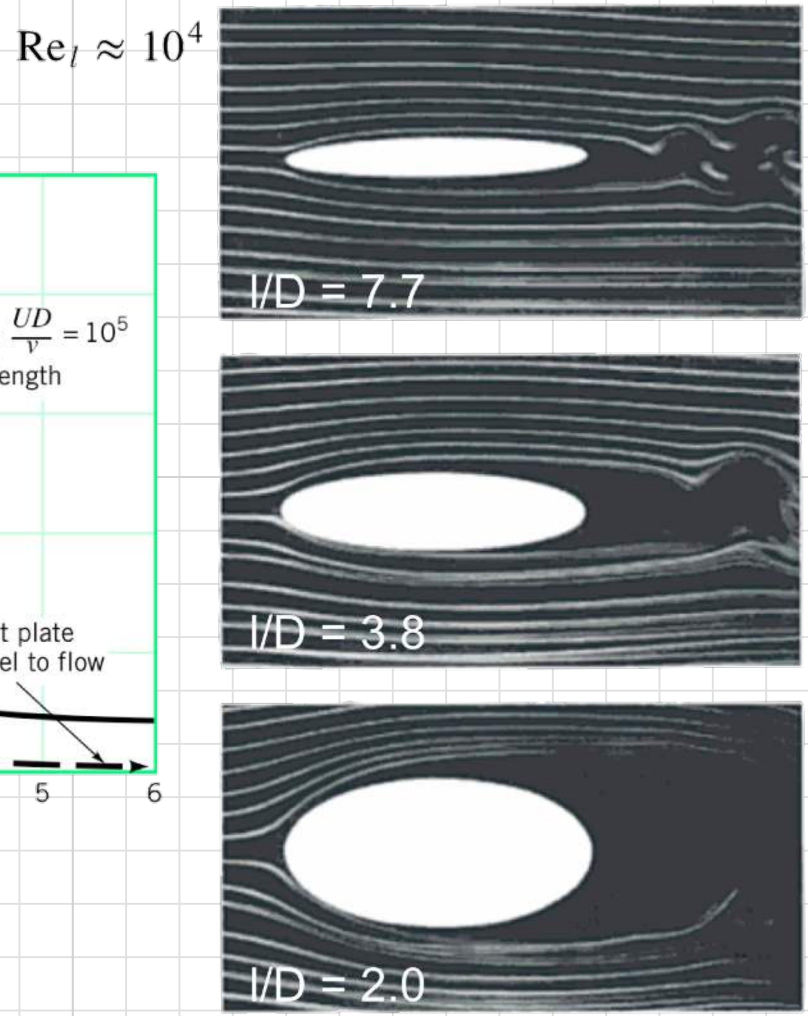




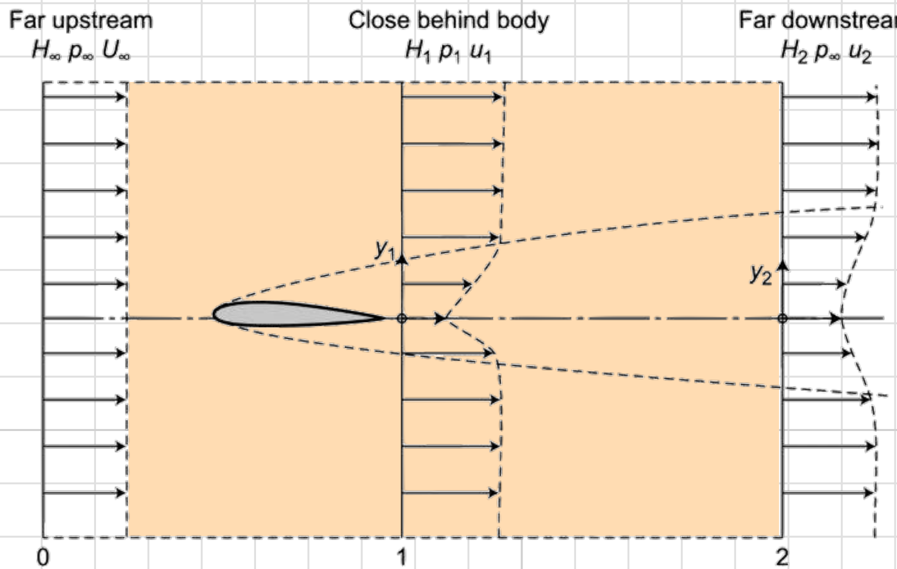


Influence of form





Estimate profile drag from the velocity profile in the wake

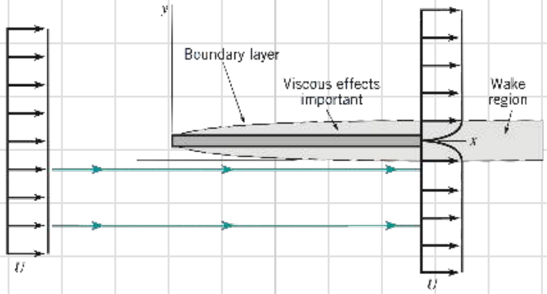


$$D = \int_2 \rho u_2 (U_\infty - u_2)\, dy_2$$

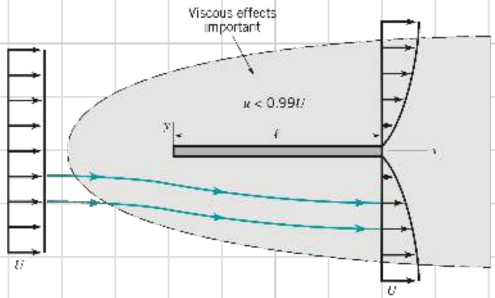
Influence of Re

Which situation corresponds to the highest Re?

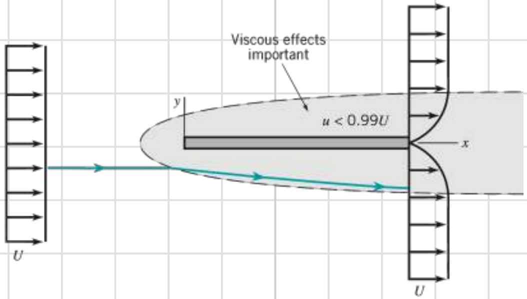
(A)



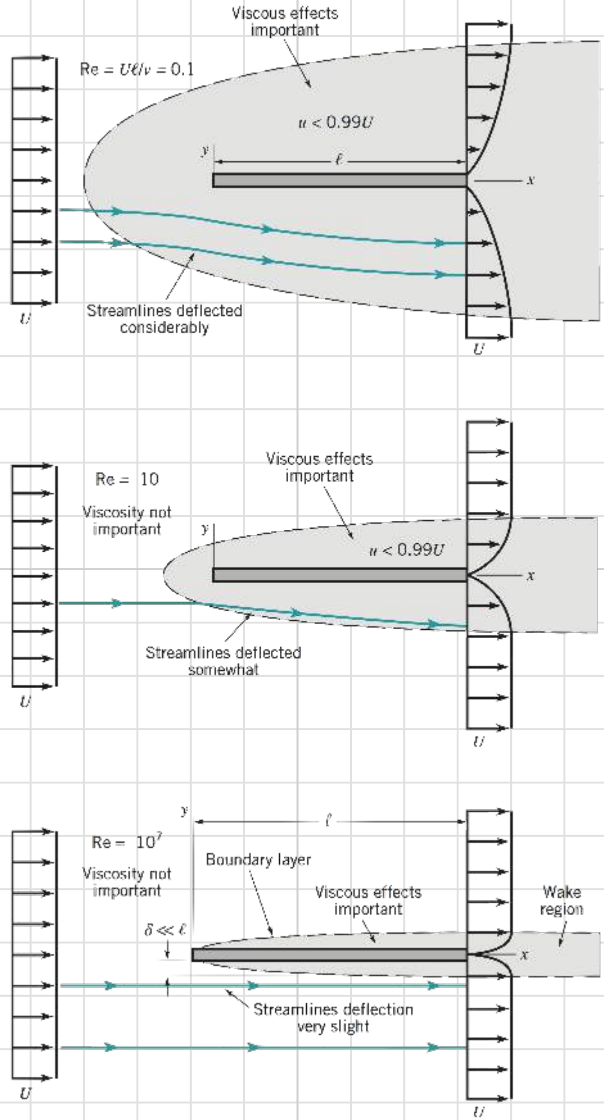
(B)



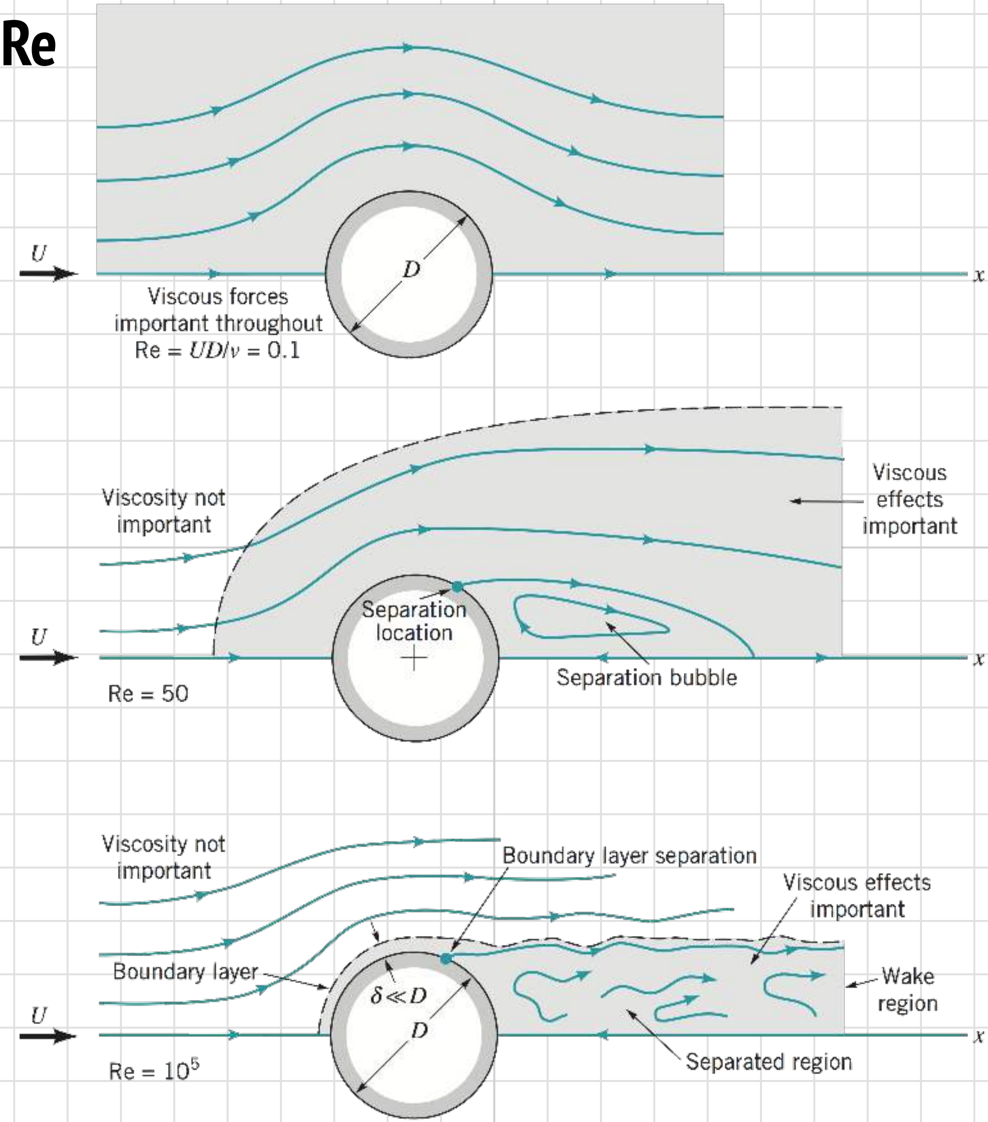
(C)



Influence of Re



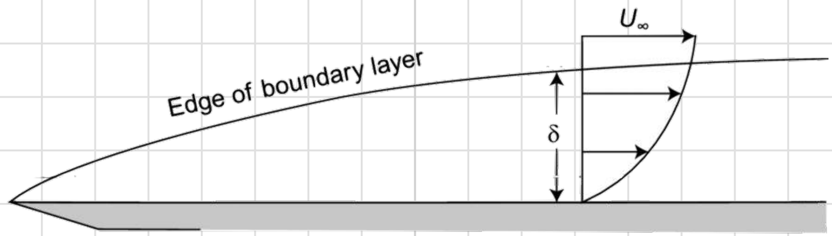
Influence of Re

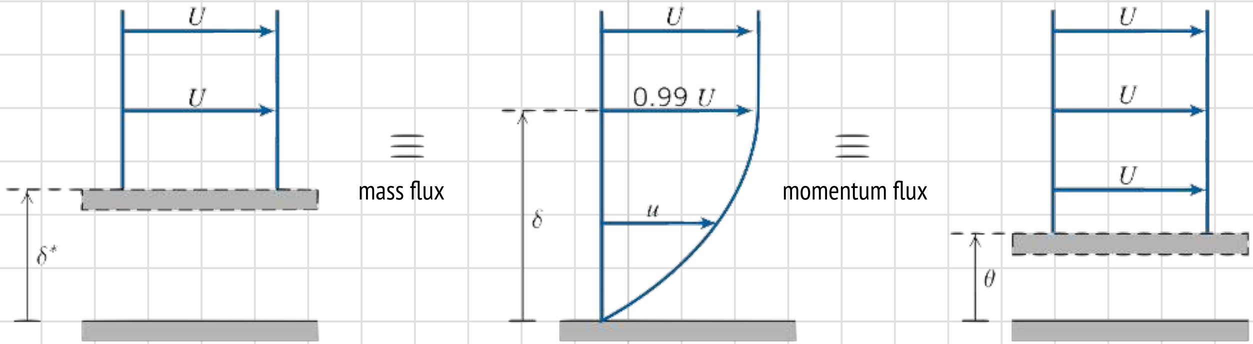


boundary layer height ☐

Re_x ☐

Boundary layers



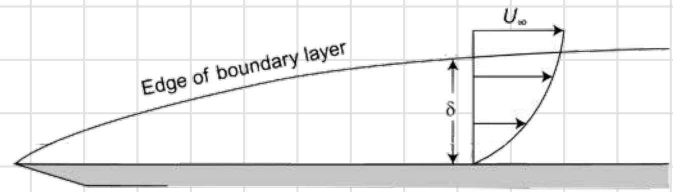


$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Boundary layer theory

Consider a 2D, steady, incompressible boundary layer:

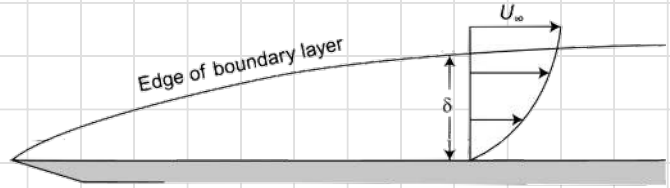


$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$x^* = x/L = \mathcal{O}(1)$$
$$y^* = y/L = \mathcal{O}(\delta/L)$$
$$u^* = u/\mathrm{U}_\infty = \mathcal{O}(1)$$
$$v^* = v/\mathrm{U}_\infty$$

Boundary layer theory

Consider a 2D, steady, incompressible boundary layer:

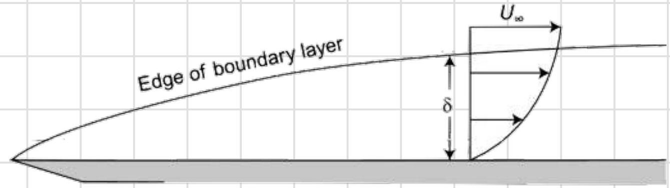


$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$x^* = x/L = \mathcal{O}(1)$$
$$y^* = y/L = \mathcal{O}(\delta/L)$$
$$u^* = u/\mathrm{U}_\infty = \mathcal{O}(1)$$
$$v^* = v/\mathrm{U}_\infty = \mathcal{O}(\delta/L)$$
$$Re = \mathrm{U}_\infty L/\nu$$

Boundary layer theory

Consider a 2D, steady, incompressible boundary layer:

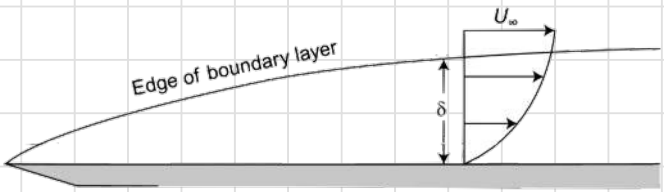


$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$x^*=x/L=\mathcal{O}(1)$$
$$y^*=y/L=\mathcal{O}(\delta/L)$$
$$u^*=u/\mathrm{U}_\infty=\mathcal{O}(1)$$
$$v^*=v/\mathrm{U}_\infty=\mathcal{O}(\delta/L)$$
$$Re=\mathrm{U}_\infty L/\nu=\mathcal{O}\left(L^2/\delta^2\right)$$

Boundary layer theory

Prandtl's thin boundary layer equations



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial p}{\partial y} \approx 0$$

Boundary conditions:

$$u|_{y=0} = 0$$

$$v|_{y=0} = 0$$

$$u|_{y=\infty} = \mathrm{U}_\infty$$

$$\frac{\mathrm{d}p}{\mathrm{d}x} = ?$$

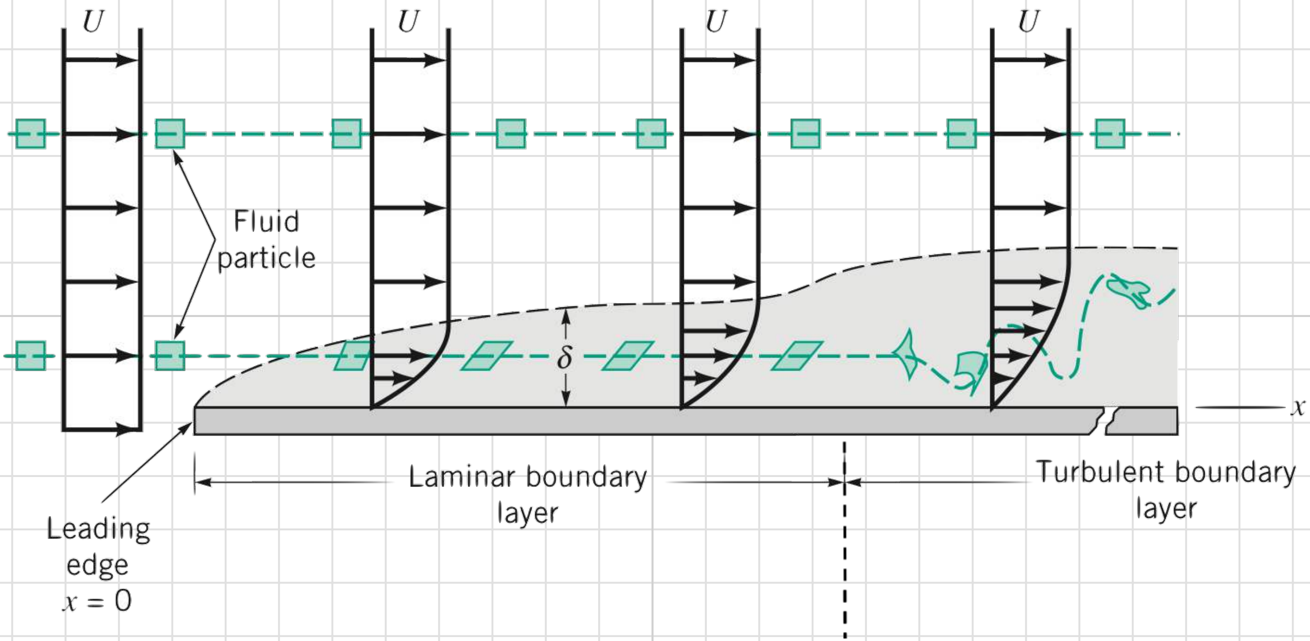
Boundary layer theory

Prandtl's thin boundary layer equations

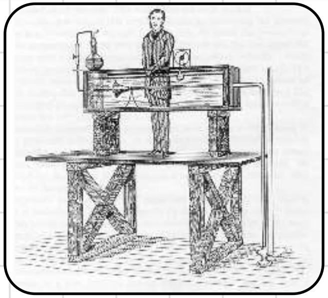
For $y \to \infty$:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

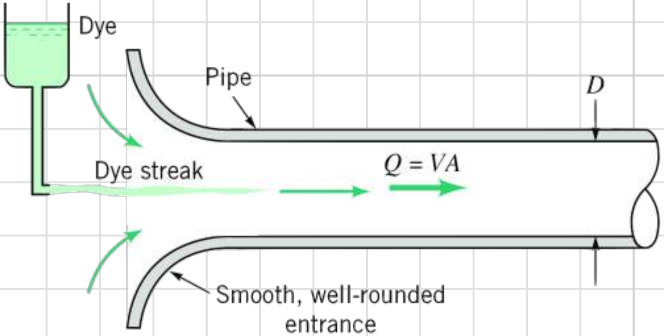
Flat plate boundary layer

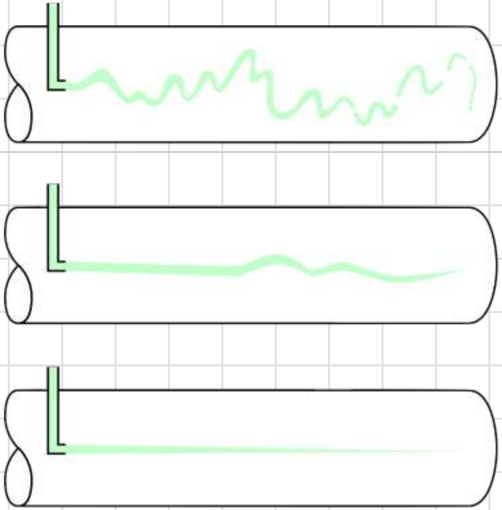


Laminar versus turbulent flows

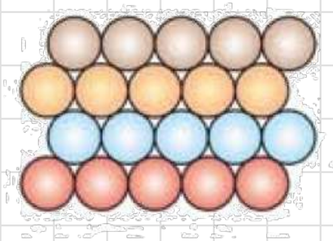


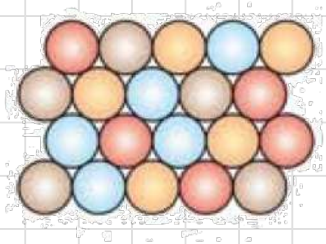
Osborne Reynolds (1842-1912)

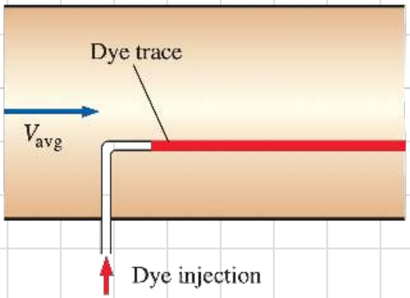


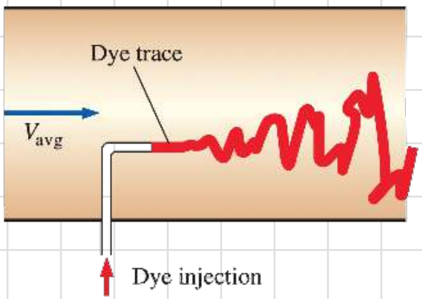


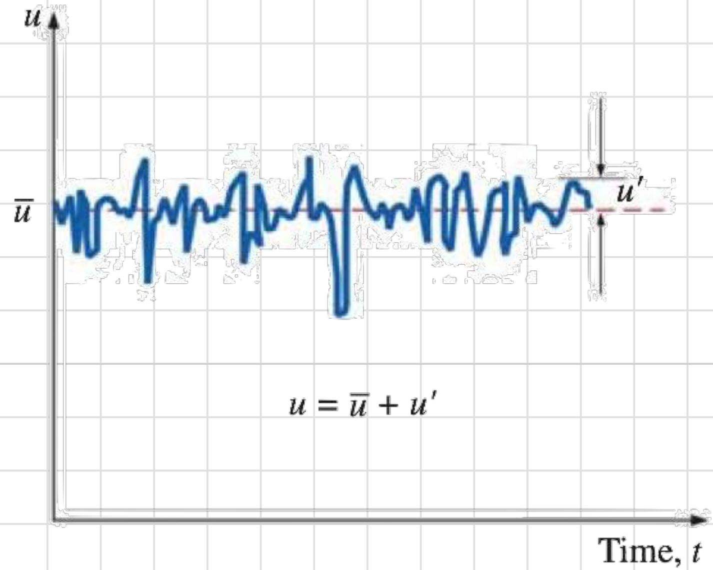
Laminar versus turbulent flows







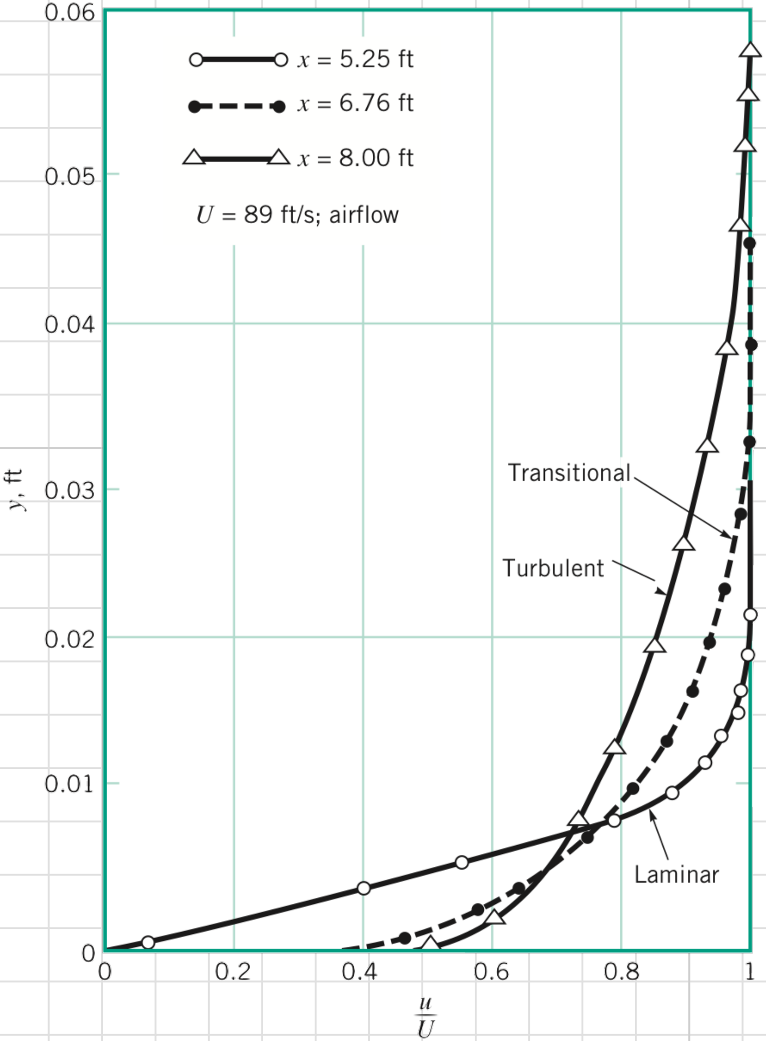


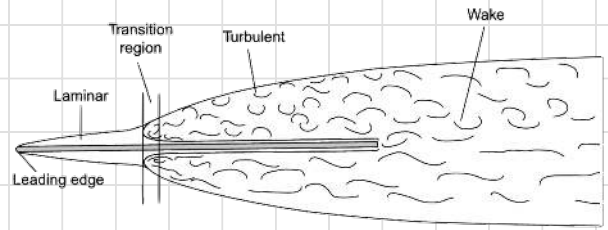


laminar vs turbulent boundary layers ☐

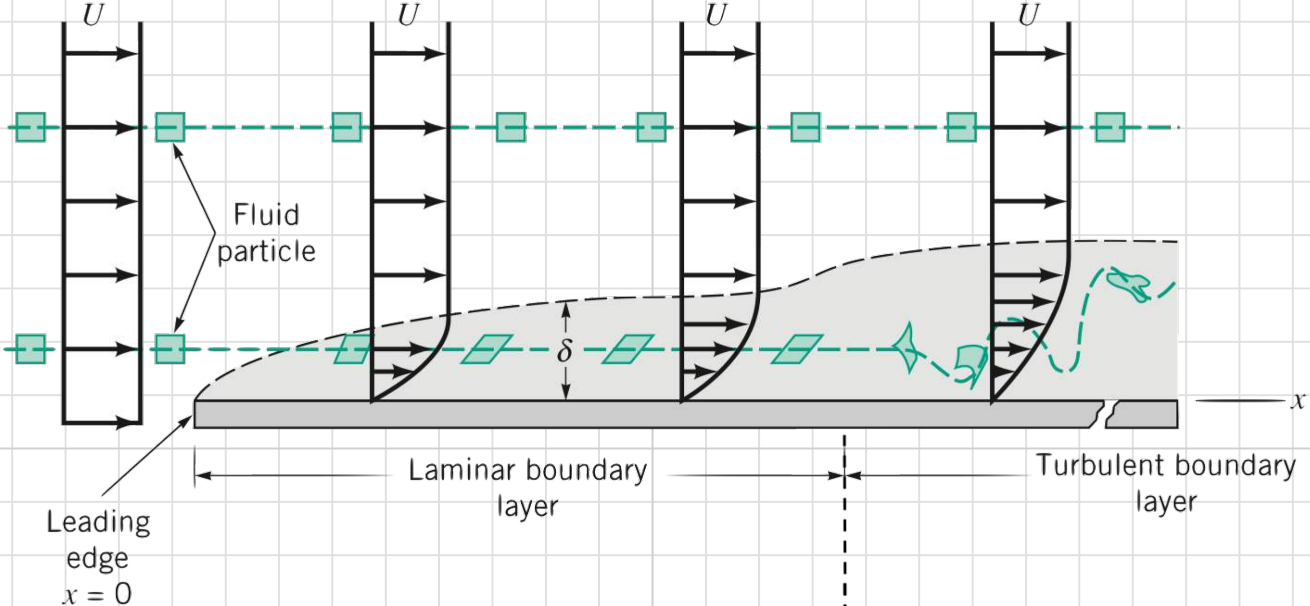
$\left.\frac{\partial u}{\partial y}\right|_{y=0}$ ☐

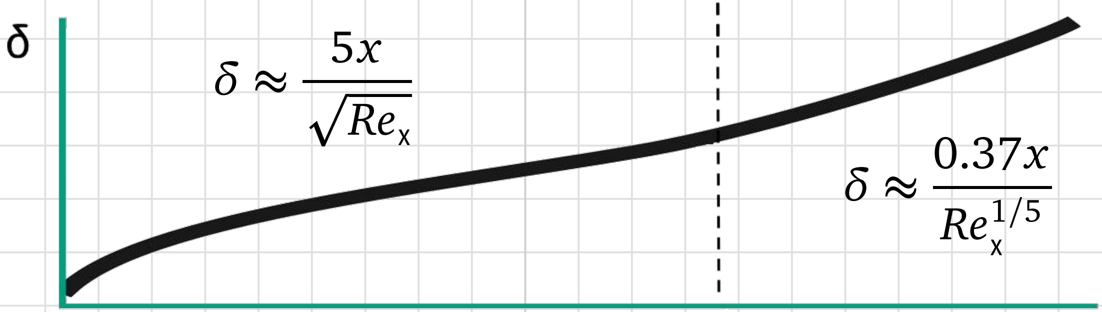
Laminar and turbulent boundary layers



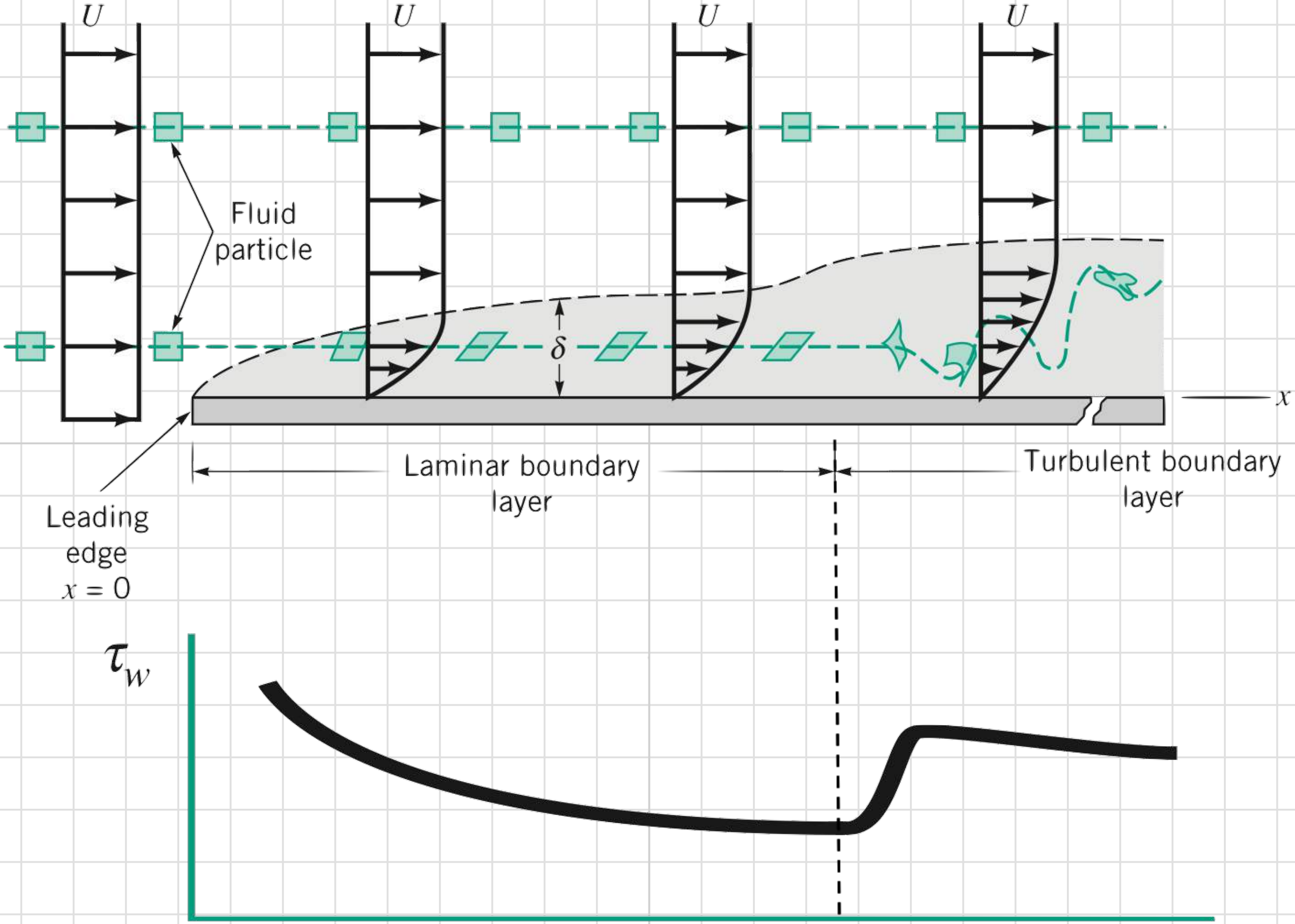


Boundary layer growth



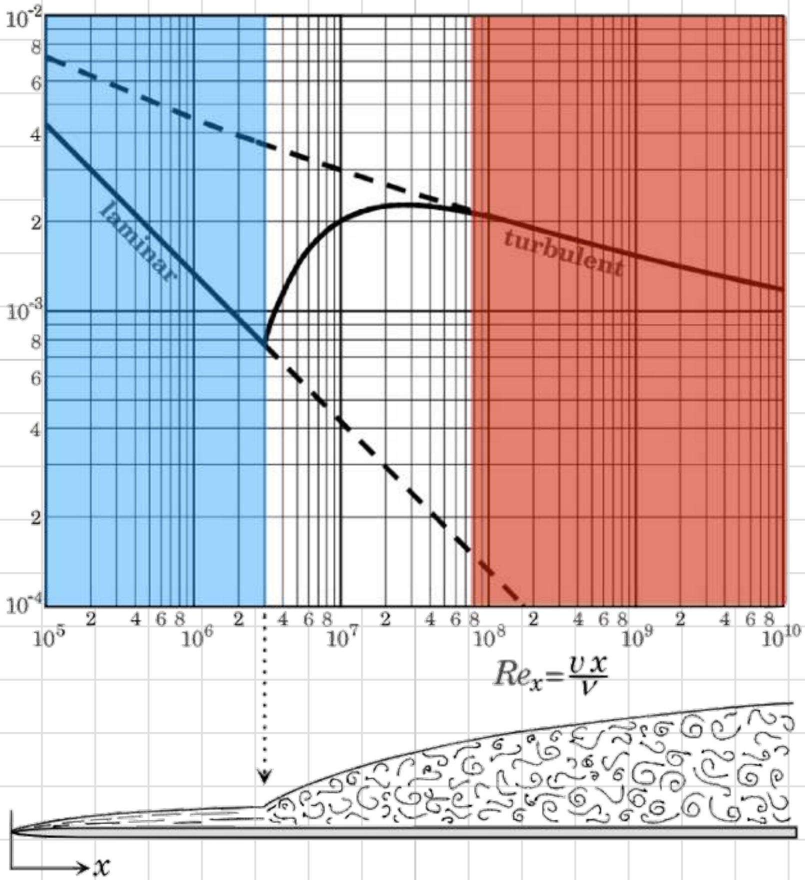


Wall shear stress



Skin friction drag

$$C_F = \frac{1.328}{\sqrt{Re_x}}$$

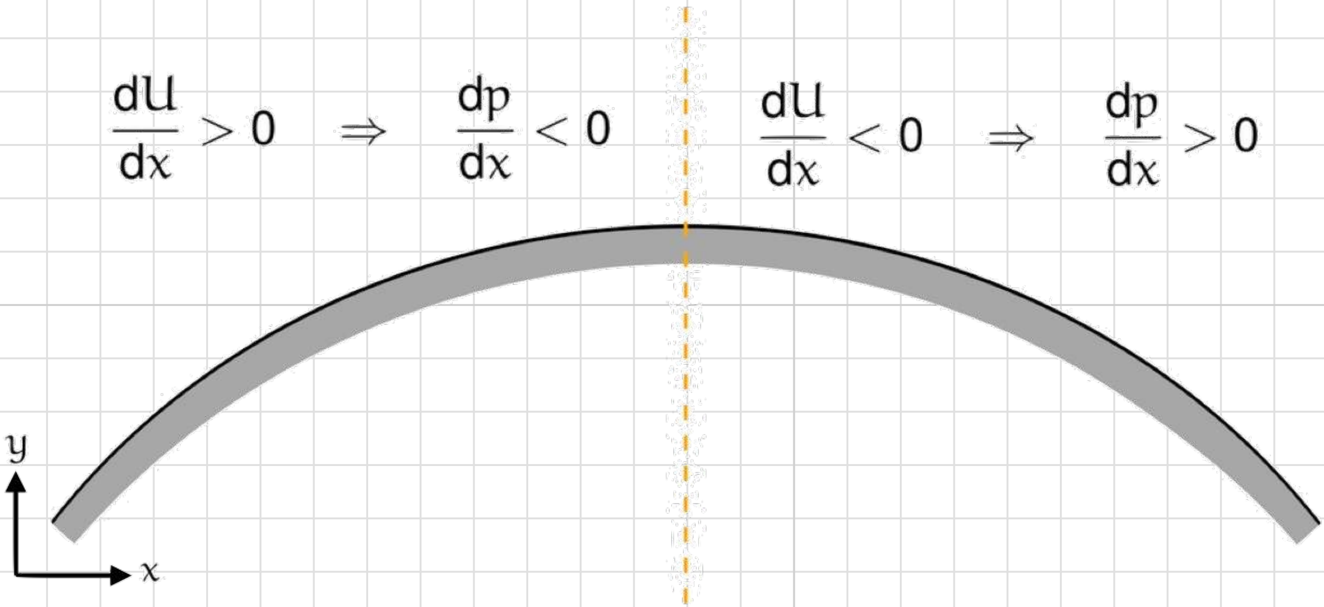


$$C_F = \frac{0.074}{Re_x^{1/5}}$$

Boundary layers on curved surfaces

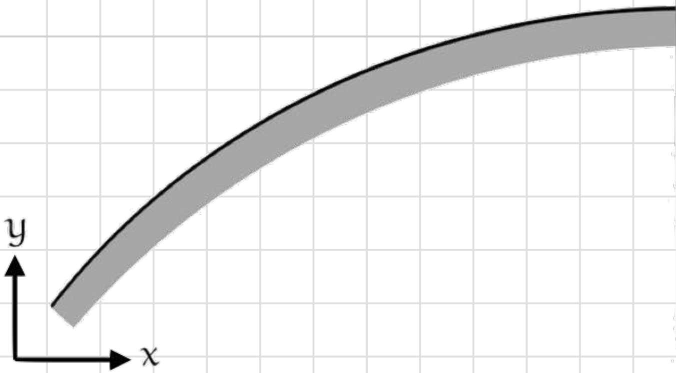
$$\frac{dp}{dx} = -\rho U_\infty \frac{dU_\infty}{dx}$$

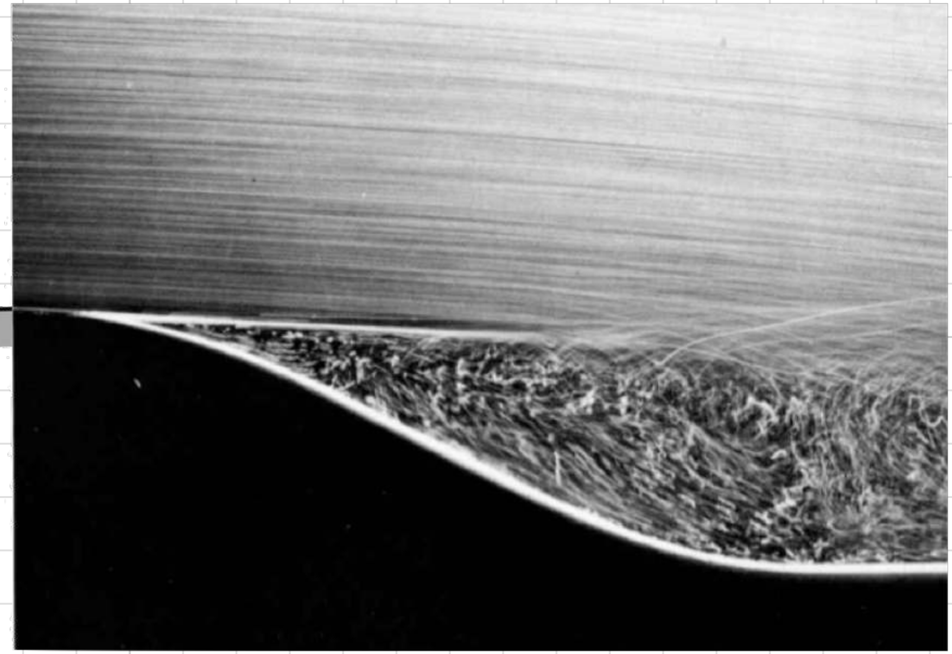
$$\frac{dU}{dx} > 0 \quad \Rightarrow \quad \frac{dp}{dx} < 0 \qquad \frac{dU}{dx} < 0 \quad \Rightarrow \quad \frac{dp}{dx} > 0$$



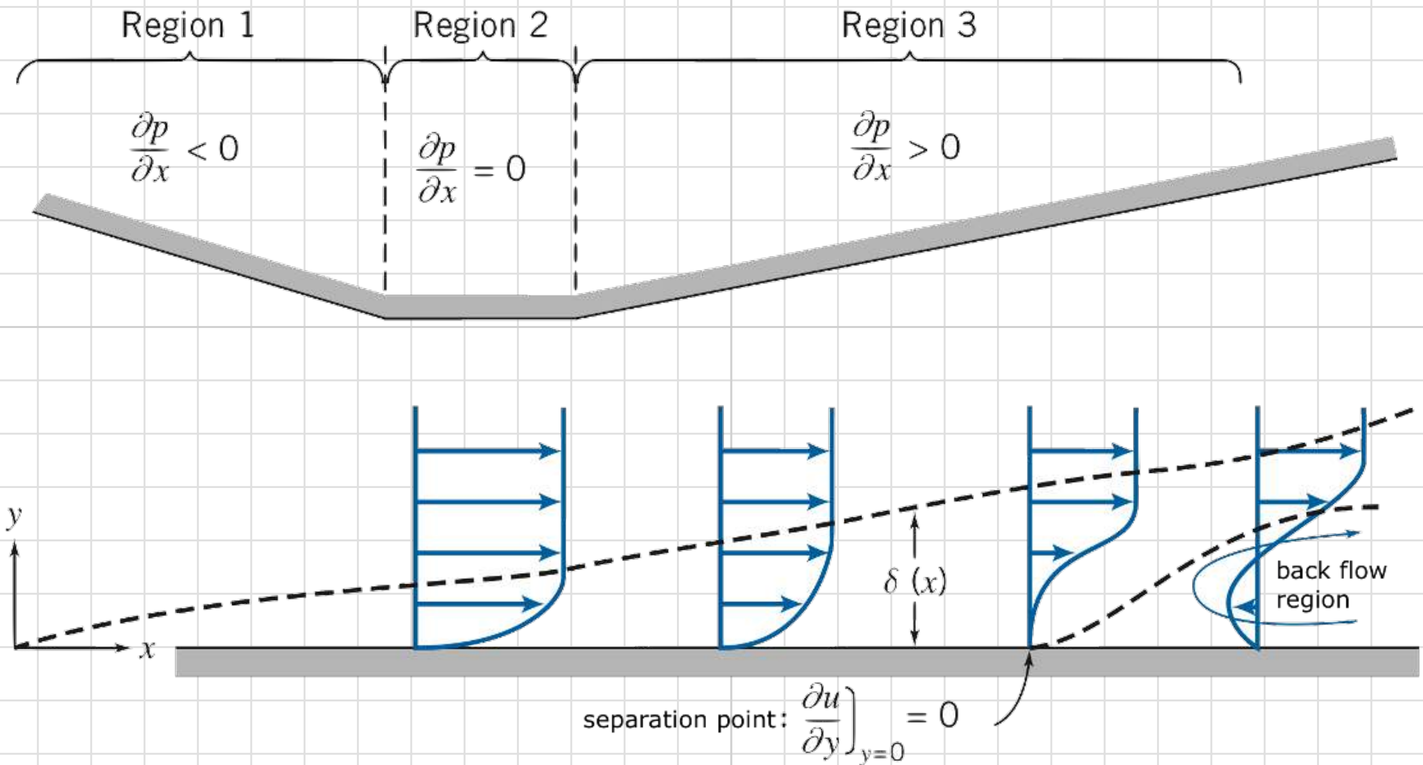
Boundary layers on curved surfaces

$$\frac{dU}{dx} > 0 \quad \Rightarrow \quad \frac{dp}{dx} < 0$$

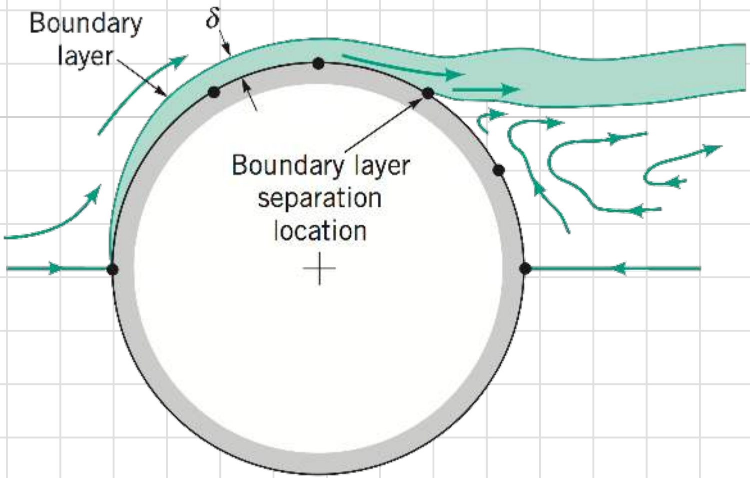


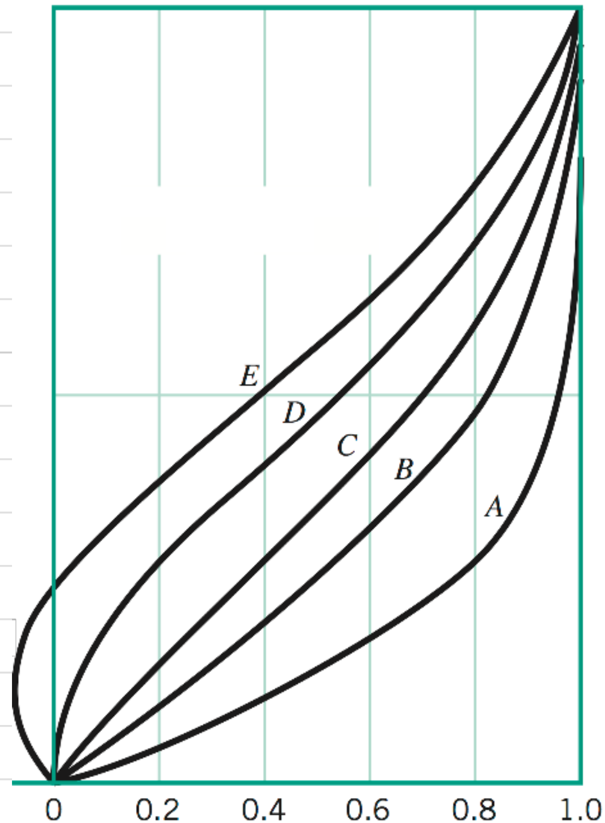


Boundary layer separation

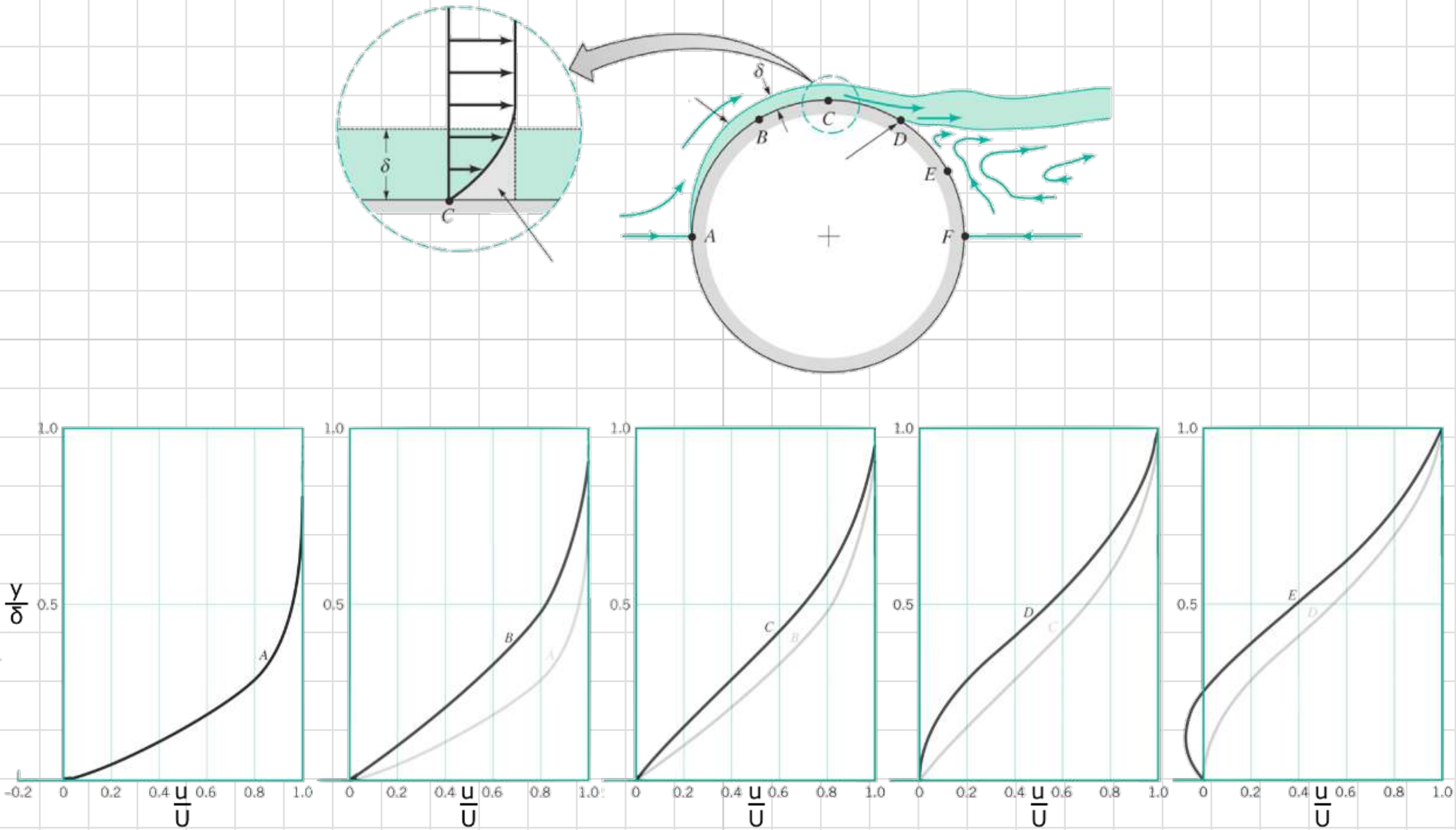


Boundary layer separation

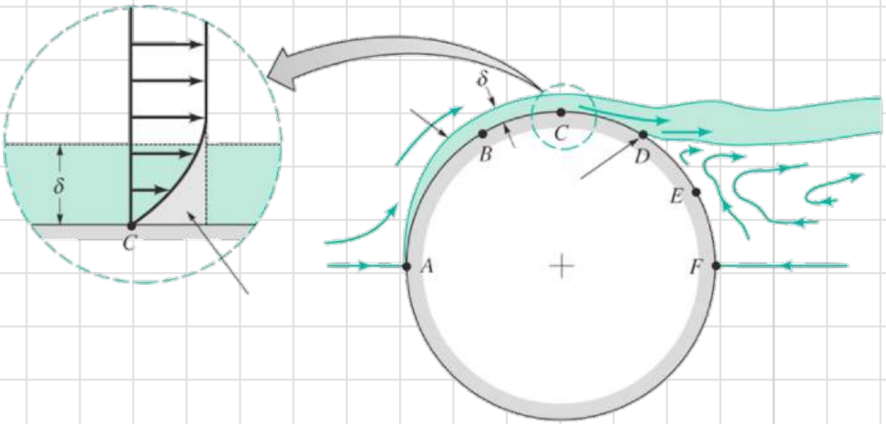


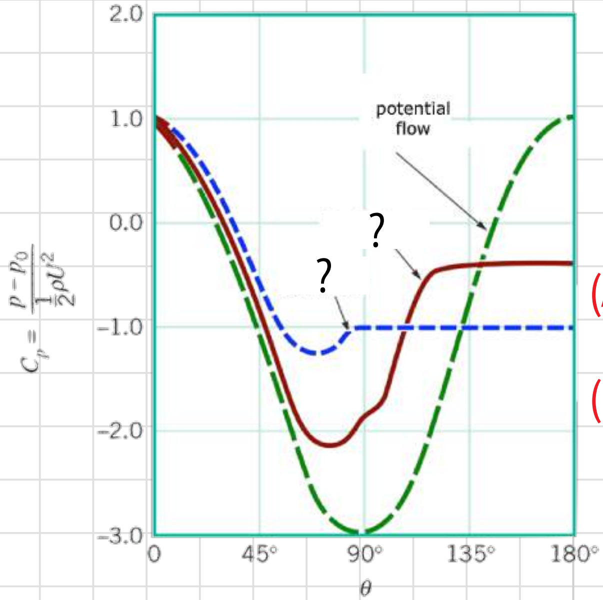


Boundary layer separation



Boundary layer separation



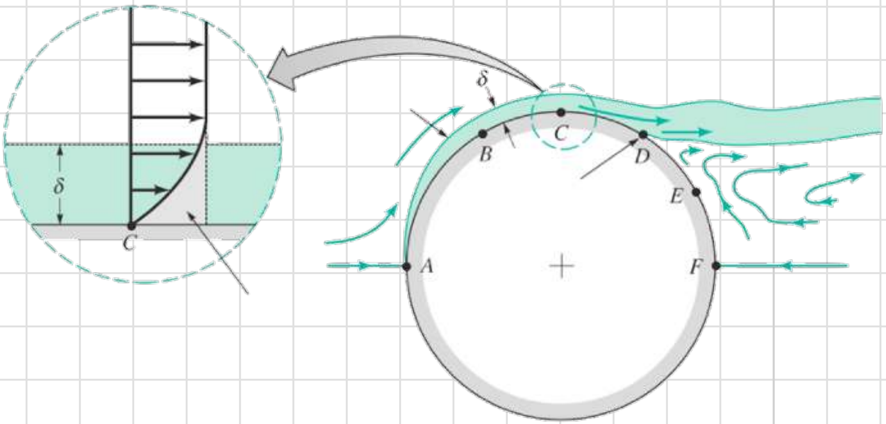


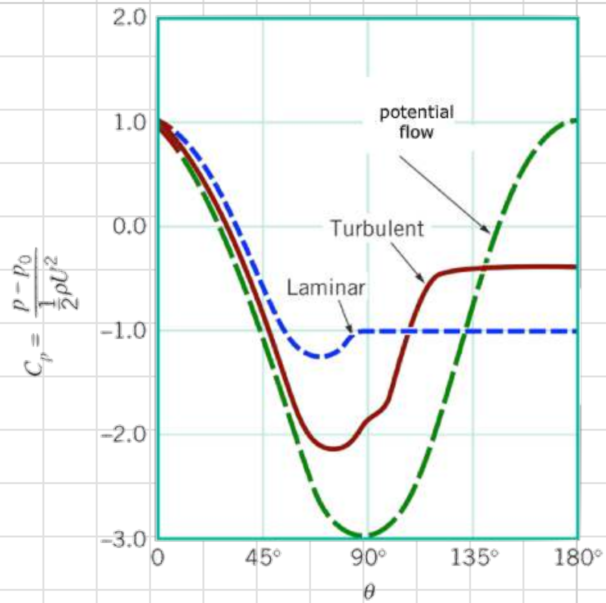
Which curve belongs to the laminar boundary layer?

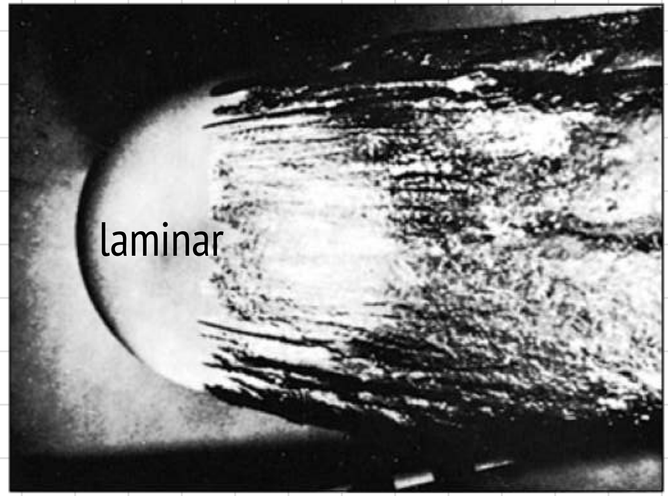
(A) blue

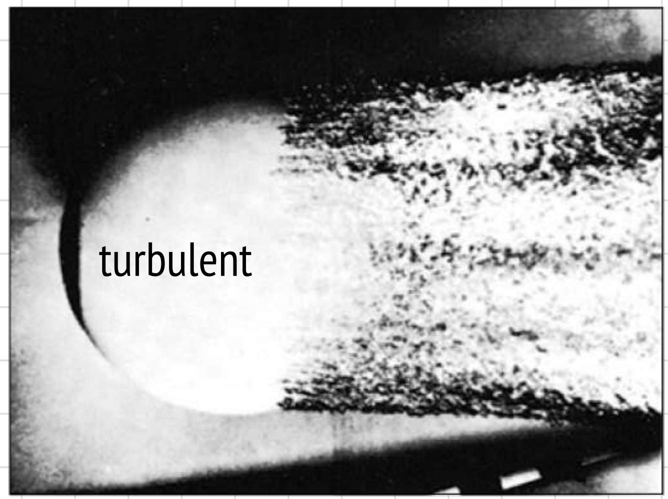
(B) red

Boundary layer separation

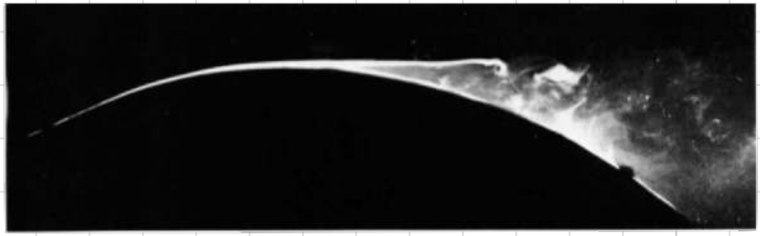


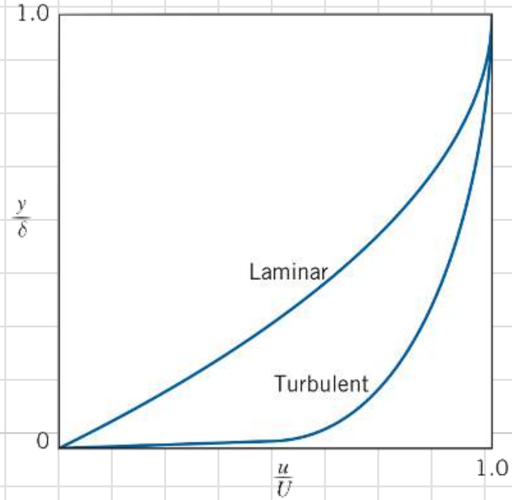


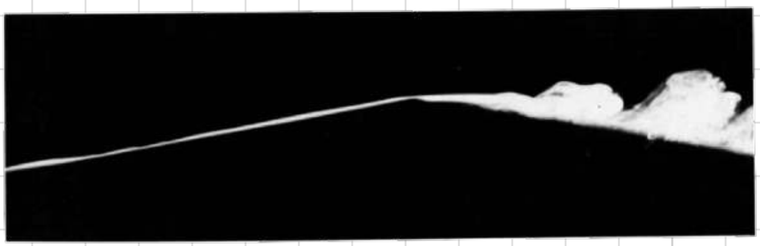


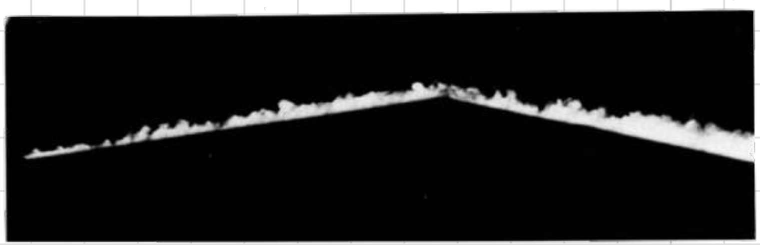


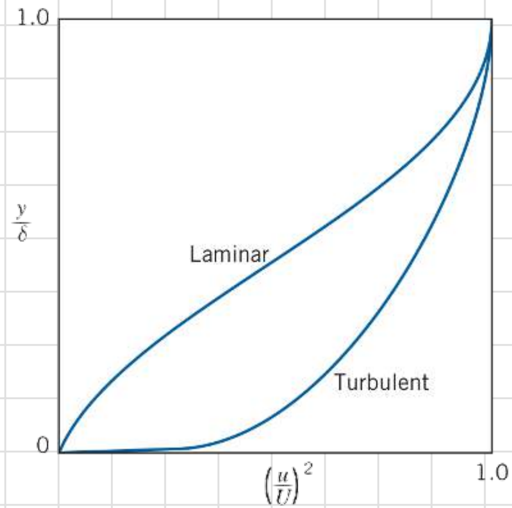
Boundary layer separation







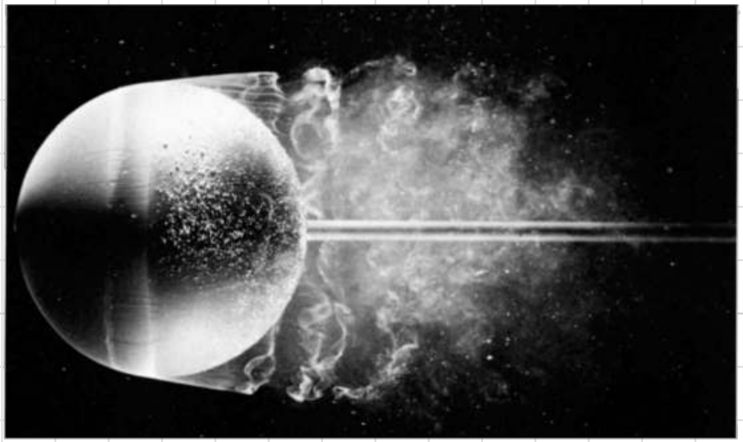


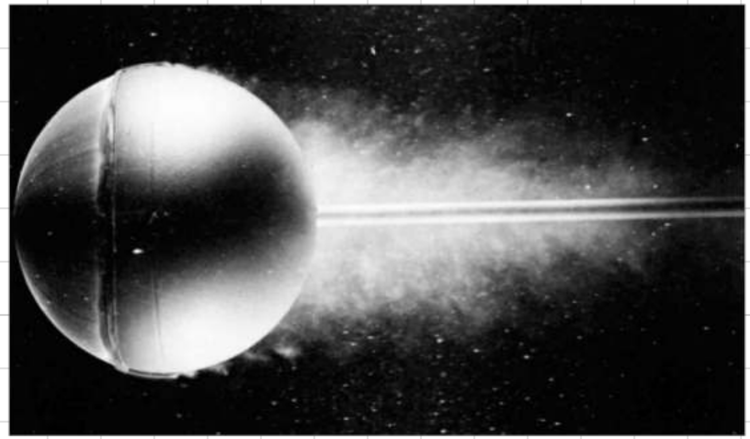


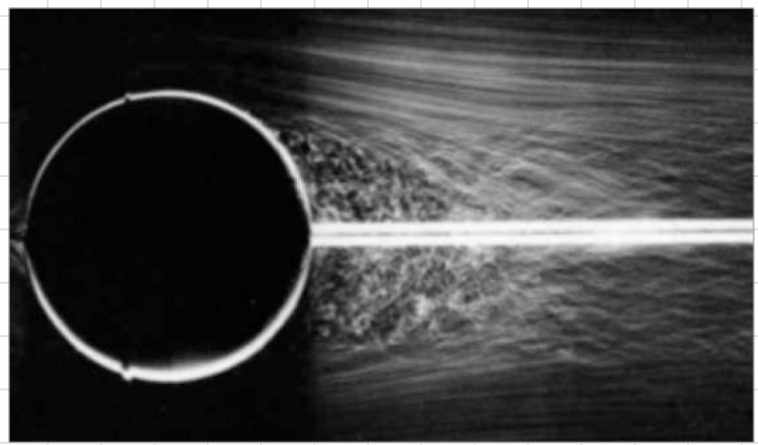
Boundary layer separation

laminar

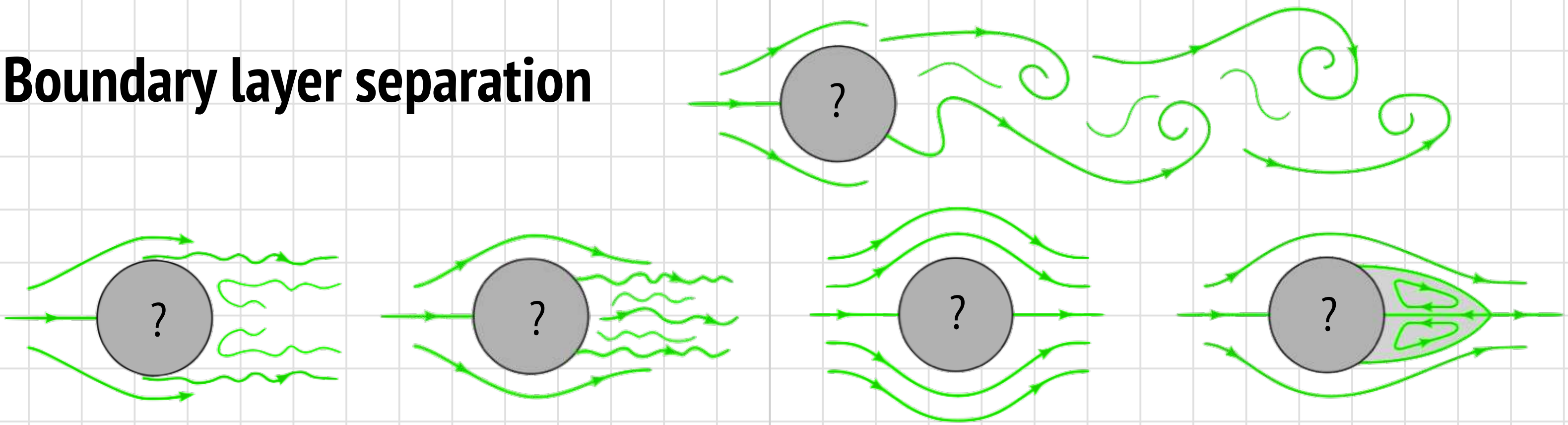
turbulent

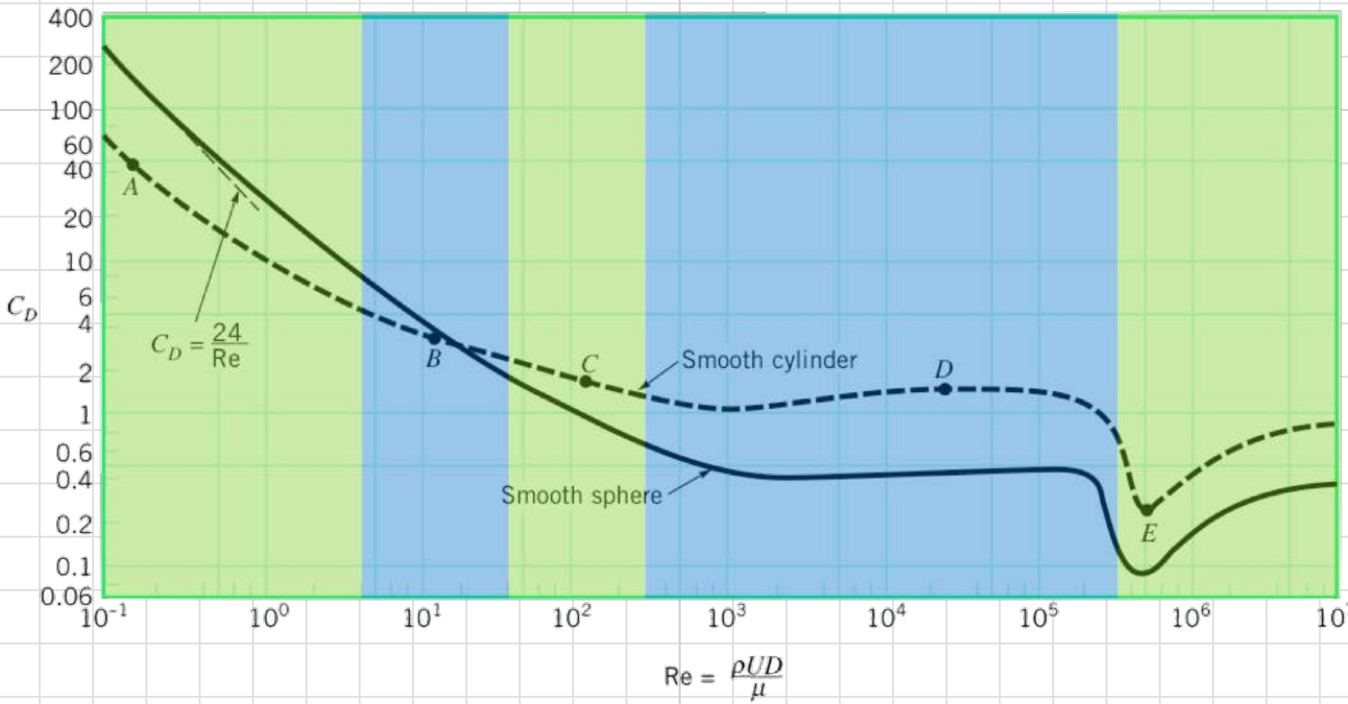




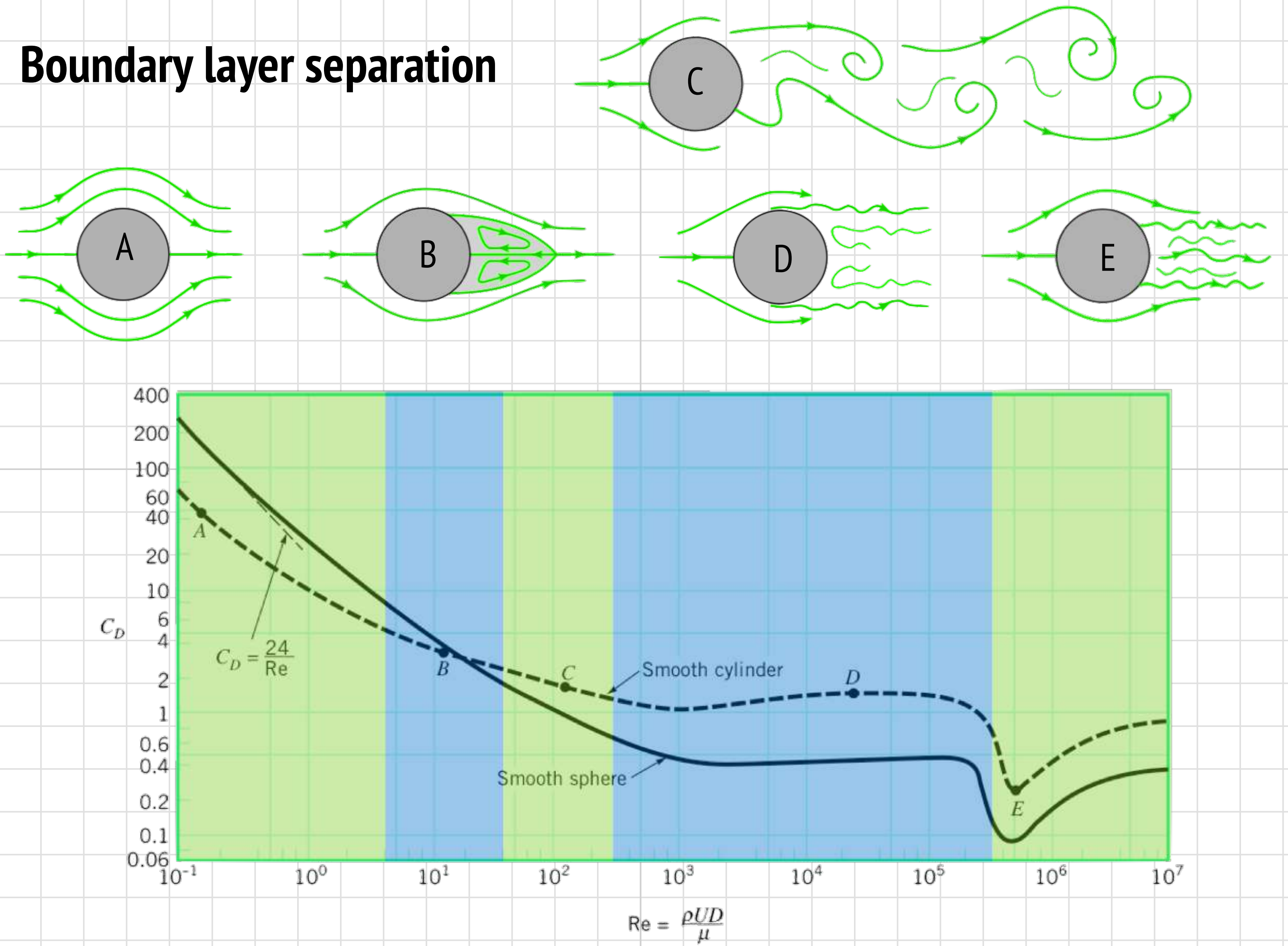


Boundary layer separation

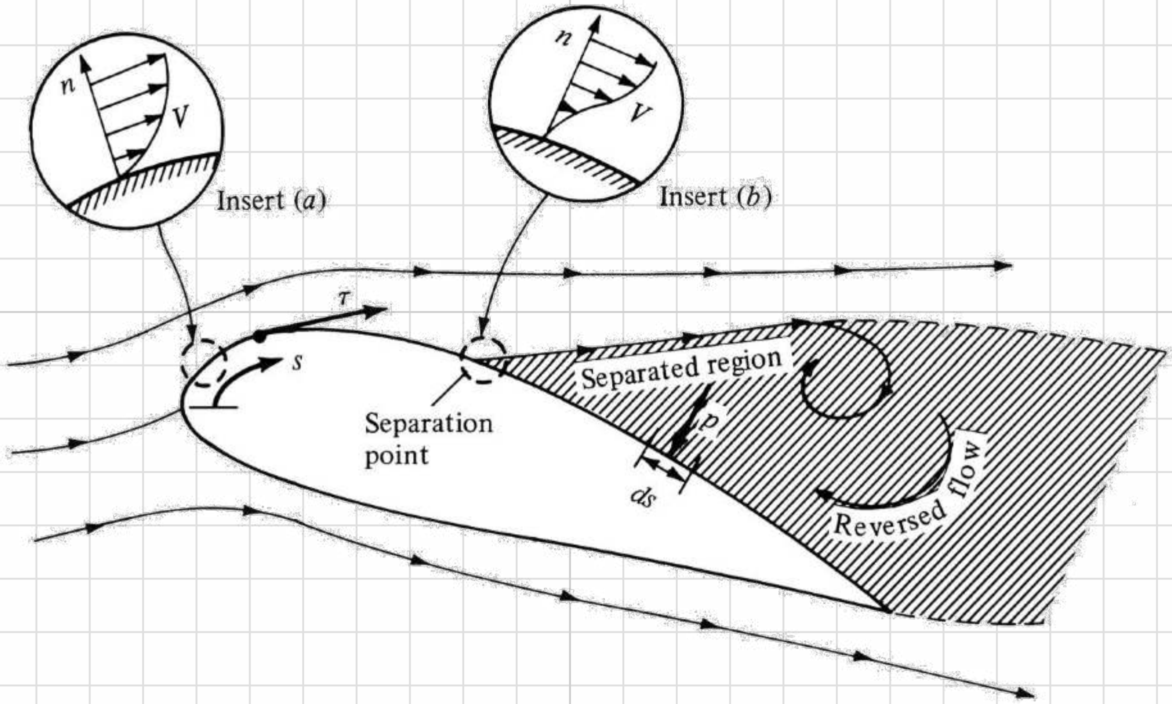




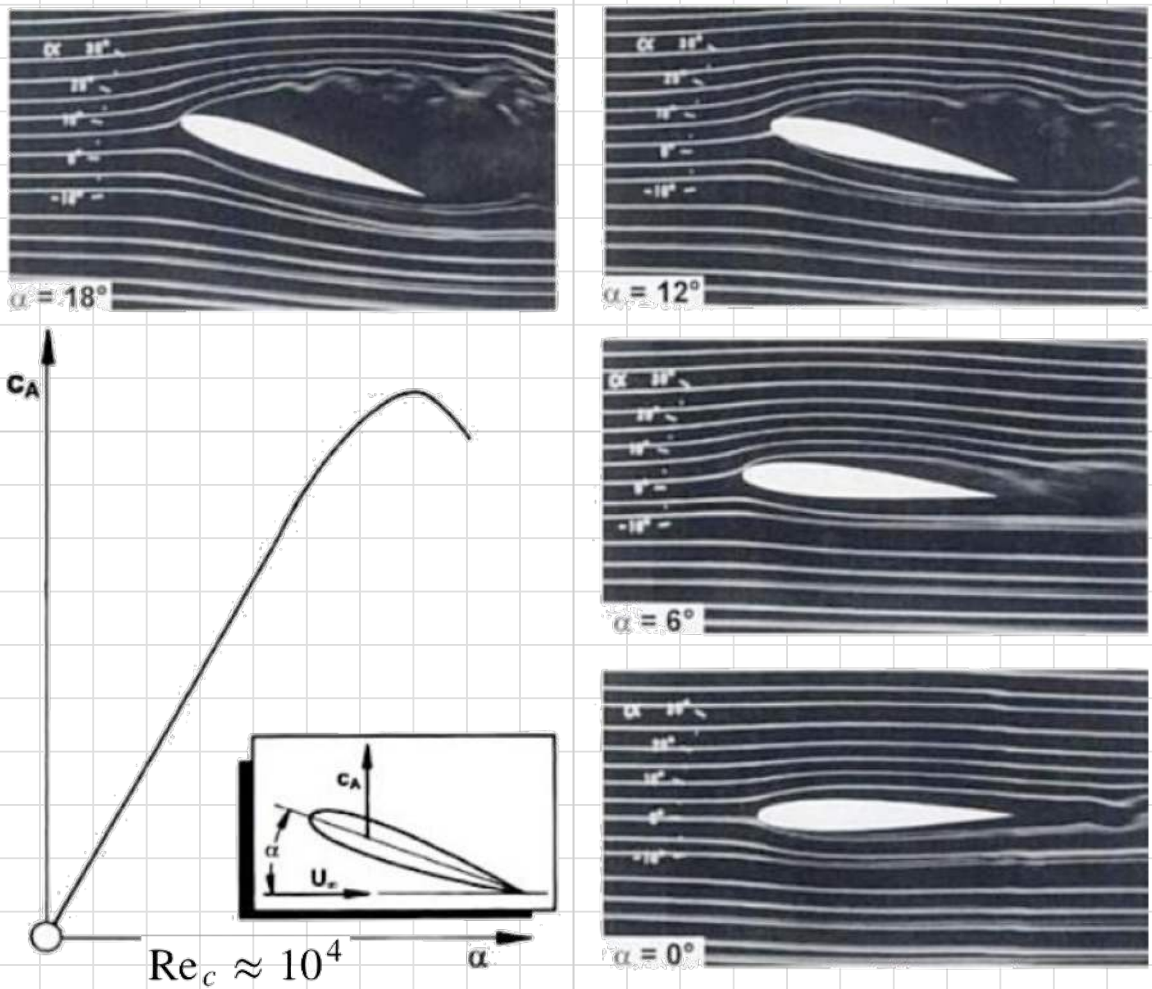
Boundary layer separation



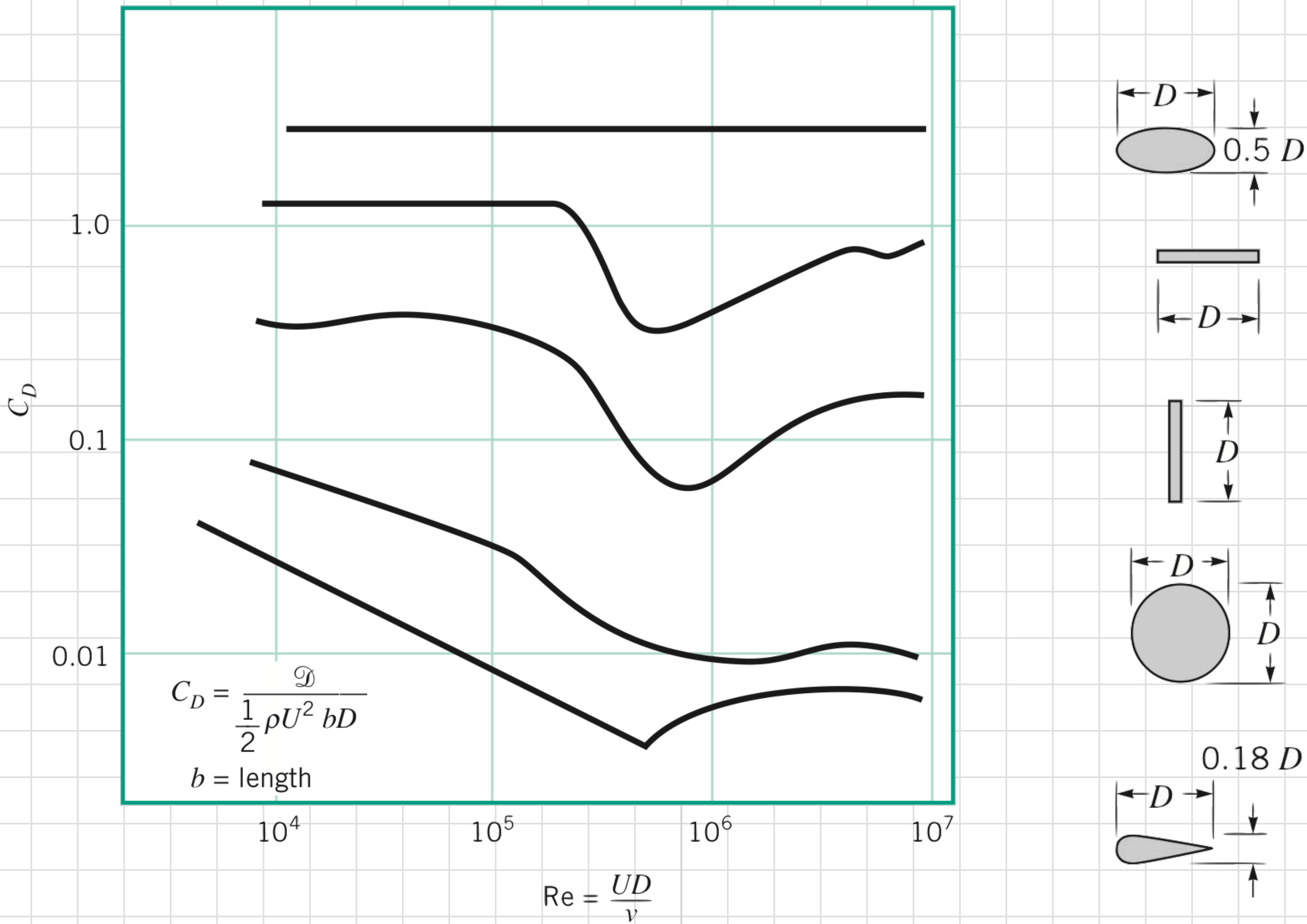
Boundary layer separation on airfoils



Boundary layer separation on airfoils



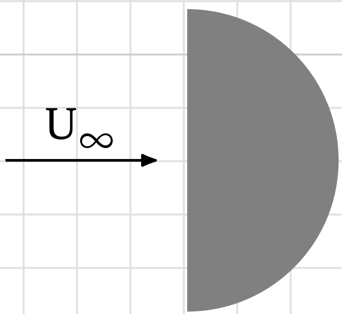
Back to drag

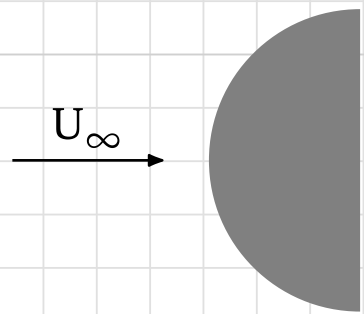


1.0
0.1
0.01
C_D
$C_D = \frac{\mathscr{D}}{\frac{1}{2}\rho U^2 bD}$
b = length
10^4
10^5
10^6
10^7
Re = $\frac{UD}{\nu}$
D
0.5 D
D
D
D
D
0.18 D
D

More drag

$$C_D \qquad <,\ >,\ \text{or}\ = \qquad C_D$$

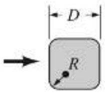
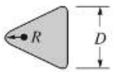
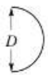
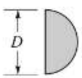
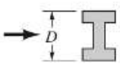
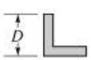
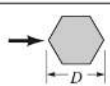
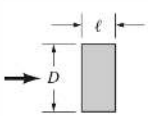


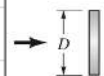
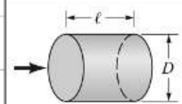
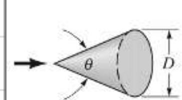
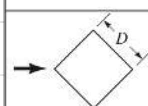
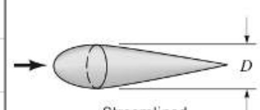


More drag

Solid vs open hemisphere

More drag

Shape	Reference area A (b = length)	Drag coefficient $C_D = \dfrac{\mathcal{D}}{\frac{1}{2}\rho U^2 A}$	Reynolds number $\text{Re} = \rho UD/\mu$
 Square rod with rounded corners	$A = bD$	R/D : C_D 0 : 2.2 0.02 : 2.0 0.17 : 1.2 0.33 : 1.0	$\text{Re} = 10^5$
 Rounded equilateral triangle	$A = bD$	R/D : C_D (→) : C_D (←) 0 : 1.4 : 2.1 0.02 : 1.2 : 2.0 0.08 : 1.3 : 1.9 0.25 : 1.1 : 1.3	$\text{Re} = 10^5$
 Semicircular shell	$A = bD$	→ 2.3 ← 1.1	$\text{Re} = 2 \times 10^4$
 Semicircular cylinder	$A = bD$	→ 2.15 ← 1.15	$\text{Re} > 10^4$
 T-beam	$A = bD$	→ 1.80 ← 1.65	$\text{Re} > 10^4$
 I-beam	$A = bD$	2.05	$\text{Re} > 10^4$
 Angle	$A = bD$	→ 1.98 ← 1.82	$\text{Re} > 10^4$
 Hexagon	$A = bD$	1.0	$\text{Re} > 10^4$
 Rectangle	$A = bD$	ℓ/D : C_D ≤ 0.1 : 1.9 0.5 : 2.5 0.65 : 2.9 1.0 : 2.2 2.0 : 1.6 3.0 : 1.3	$\text{Re} = 10^5$

Shape	Reference area A	Drag coefficient C_D	Reynolds number $\text{Re} = \rho UD/\mu$
 Solid hemisphere	$A = \frac{\pi}{4}D^2$	→ 1.17 ← 0.42	$\text{Re} > 10^4$
 Hollow hemisphere	$A = \frac{\pi}{4}D^2$	→ 1.42 ← 0.38	$\text{Re} > 10^4$
 Thin disk	$A = \frac{\pi}{4}D^2$	1.1	$\text{Re} > 10^3$
 Circular rod parallel to flow	$A = \frac{\pi}{4}D^2$	ℓ/D : C_D 0.5 : 1.1 1.0 : 0.93 2.0 : 0.83 4.0 : 0.85	$\text{Re} > 10^5$
 Cone	$A = \frac{\pi}{4}D^2$	θ, degrees : C_D 10 : 0.30 30 : 0.55 60 : 0.80 90 : 1.15	$\text{Re} > 10^4$
 Cube	$A = D^2$	1.05	$\text{Re} > 10^4$
 Cube	$A = D^2$	0.80	$\text{Re} > 10^4$
 Streamlined body	$A = \frac{\pi}{4}D^2$	0.04	$\text{Re} > 10^5$

Take away message

$$\boxed{D = D_f + D_p}$$