Fall 2024

ME-445 AERODYNAMICS

05 - Wing theory

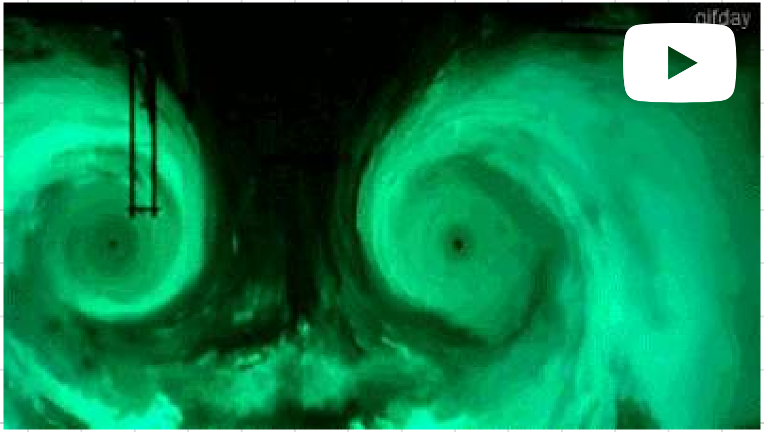
K. Mulleners

Finite wings

- Examples
- Wing nomenclature
- Finite wing aerodynamics
- Helmholtz’s theorems
- Prandtl’s lifting line theory
- Prandtl’s lifting line theory

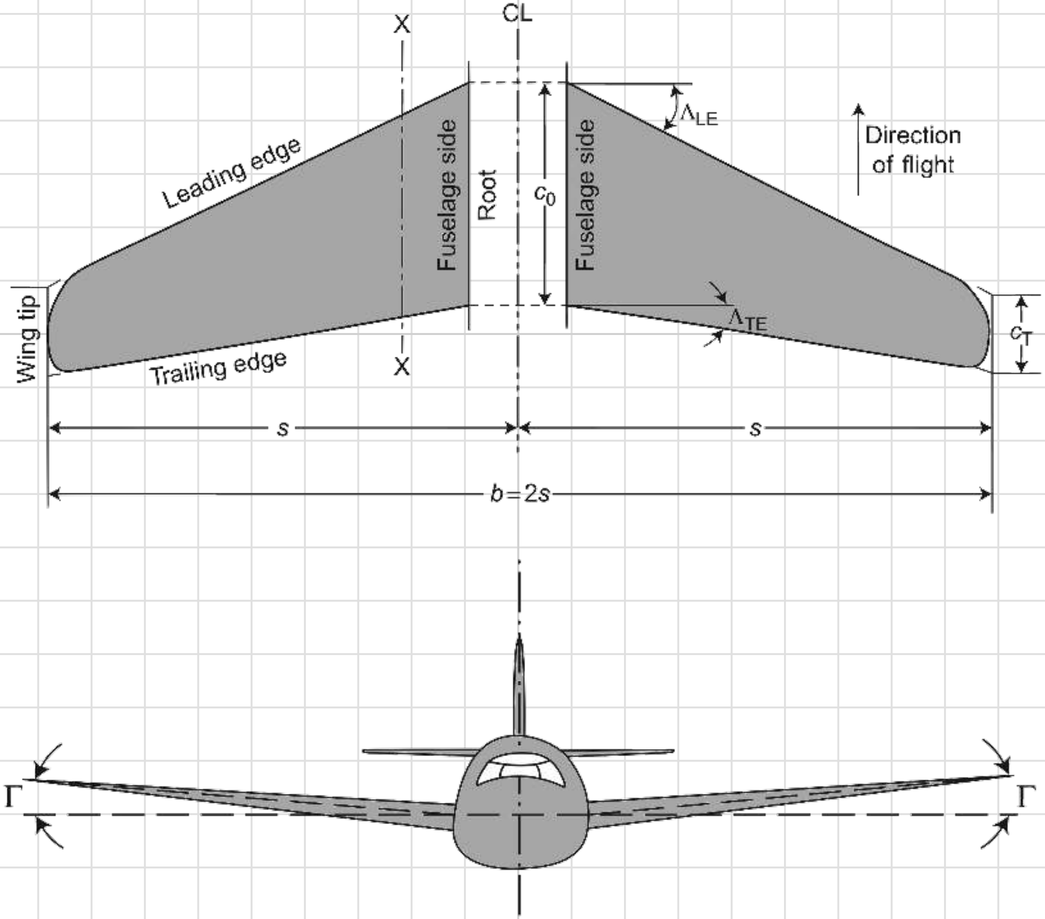
Influence of AR

AR



Nomenclature

Wing geometry

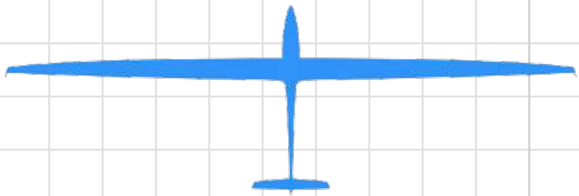


- wing tip ☐
- wing span ☐
- aspect ratio ☐
- taper ☐
- thickness ratio ☐
- sweep angle ☐
- dihedral angle ☐
- twist ☐

Wing geometry

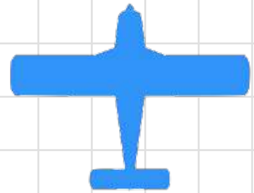
Aspect ratio

AR = 33.5



Schleicher ASH 31 Glider

AR = 5.6



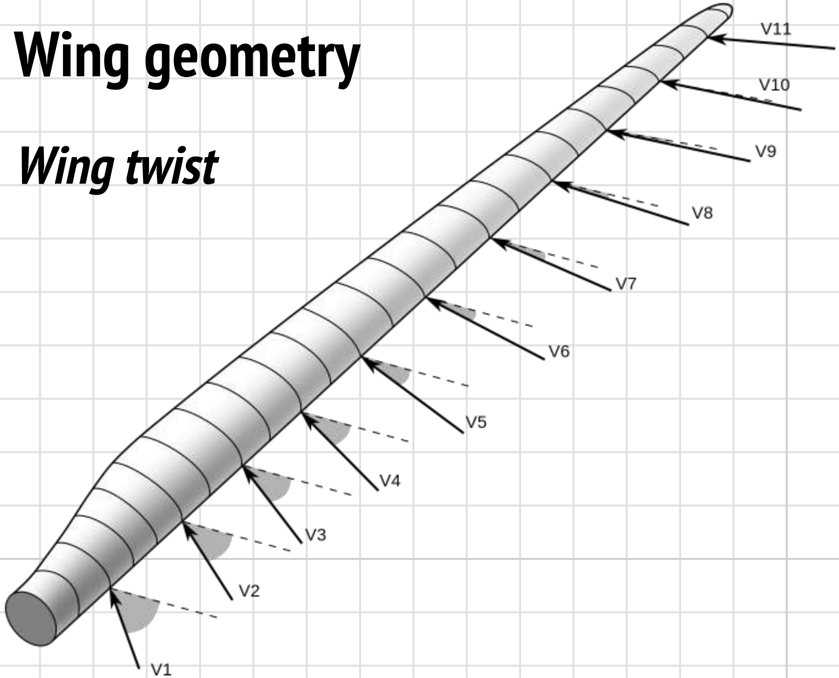
Piper Cherokee

Wing geometry

Wing taper

Wing geometry

Wing twist



Wing geometry

Thickness ratio

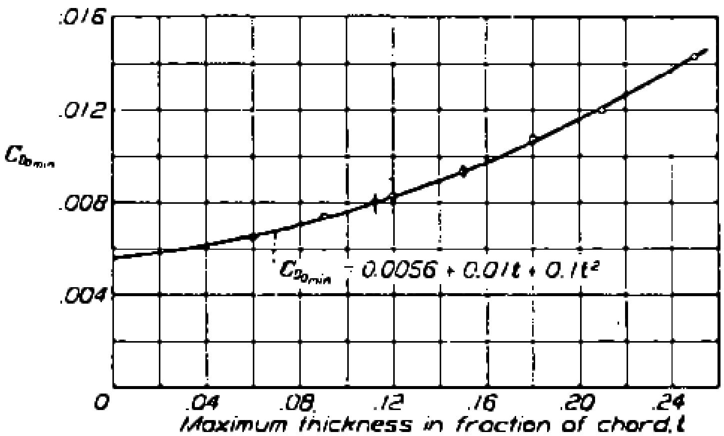
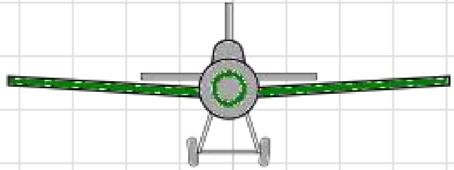


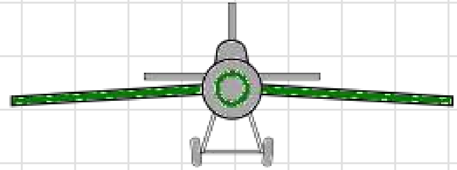
FIGURE 91.—Variation of minimum profile drag with thickness for the symmetrical airfoils.

Wing geometry

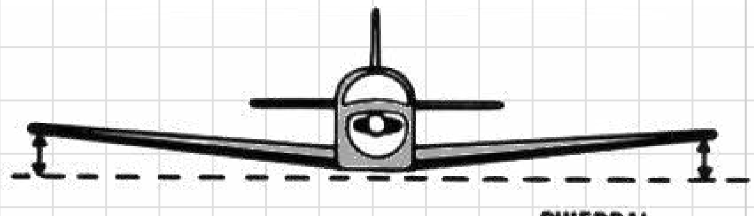
Wing sweep

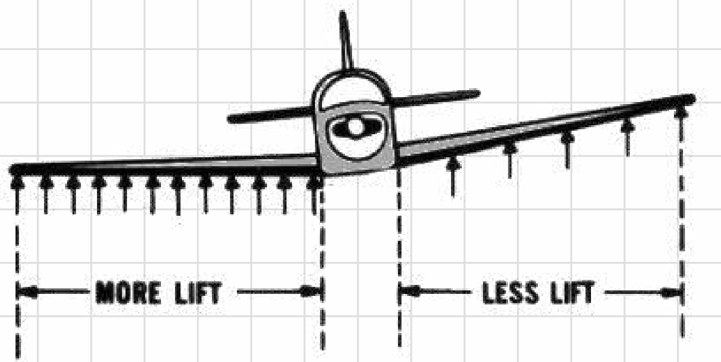
Wing geometry





Wing dihedral/anhedral

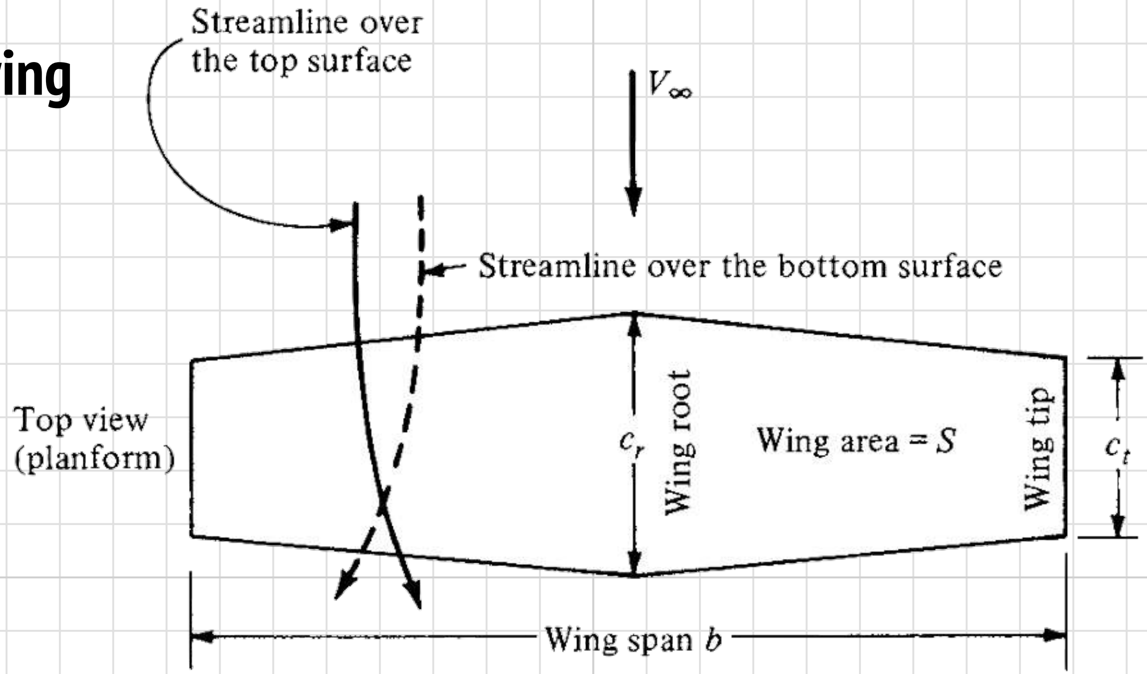


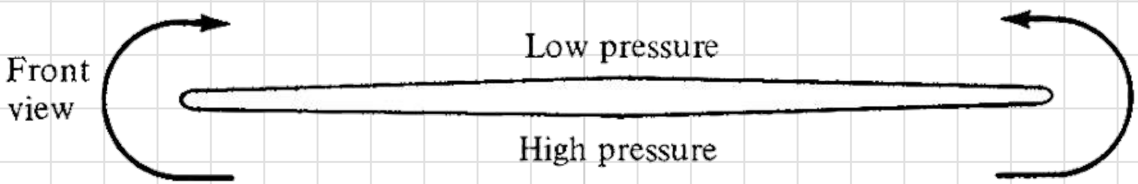


Finite wing

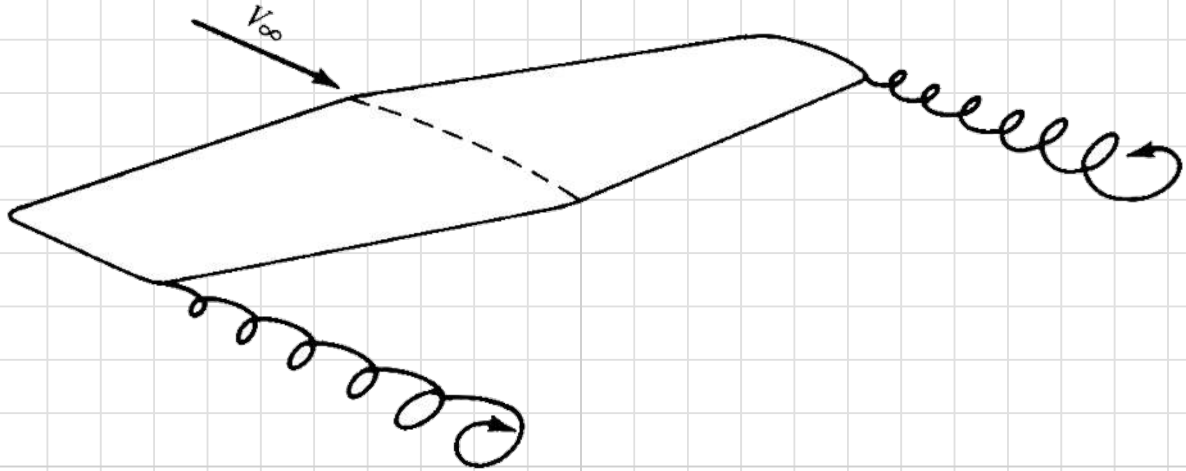
spanwise flow ☐

tip vortices ☐





Finite wing



Tip vortices

BEWARE!

Which plane creates the stronger down-wash?

(A) Airbus A380

(B) Challenger 604

(C) F16 Falcon fighter jet

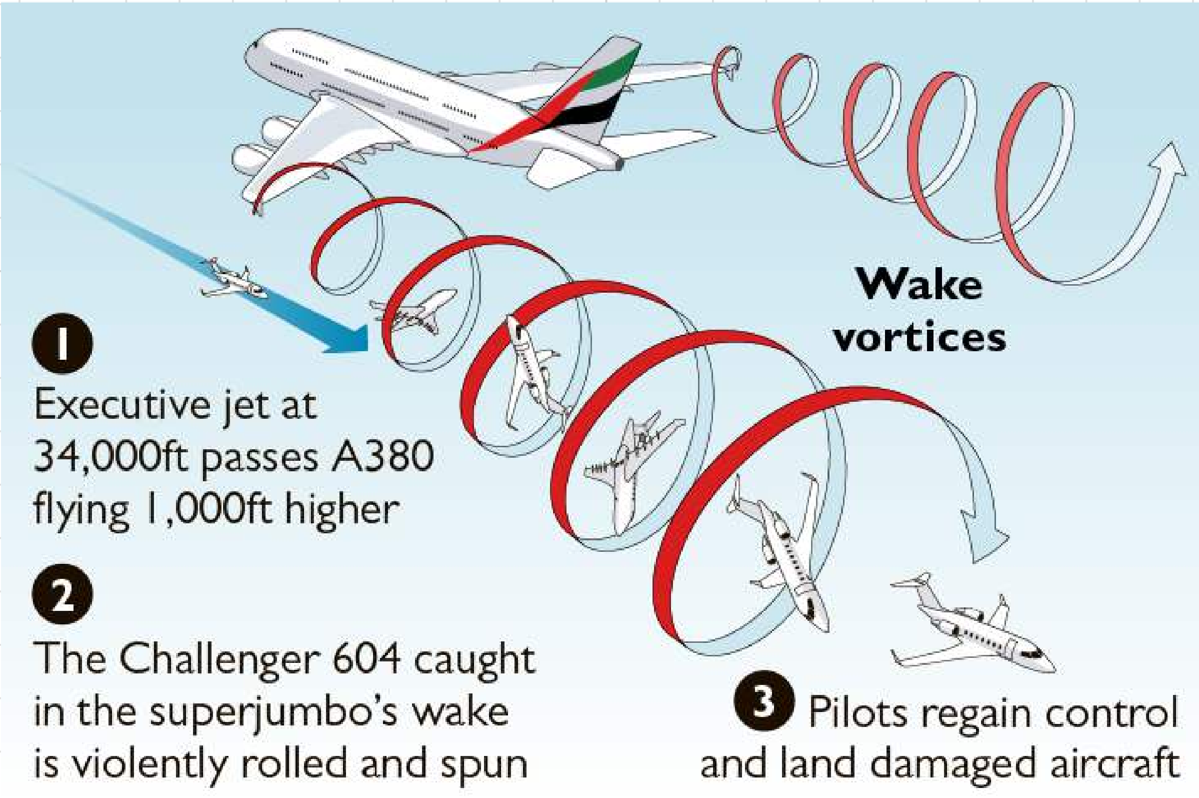
(D) that depends on the weather conditions

Tip vortices

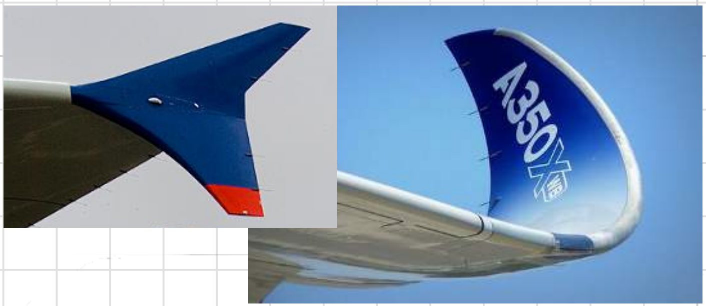
https://www.thetimes.co.uk/article/private-jet-flipped-in-superjumbo-s-wake-bgr2s5blc

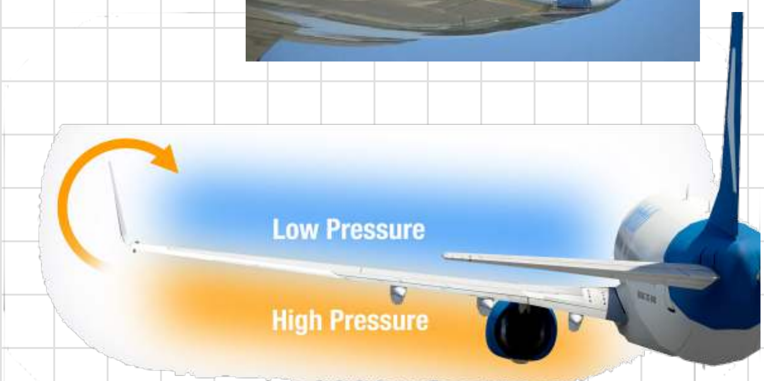
http://loungtastic.com/2017/03/25/private-jet-crash-saw-from-emirates-a380-in-sky/

BEWARE!

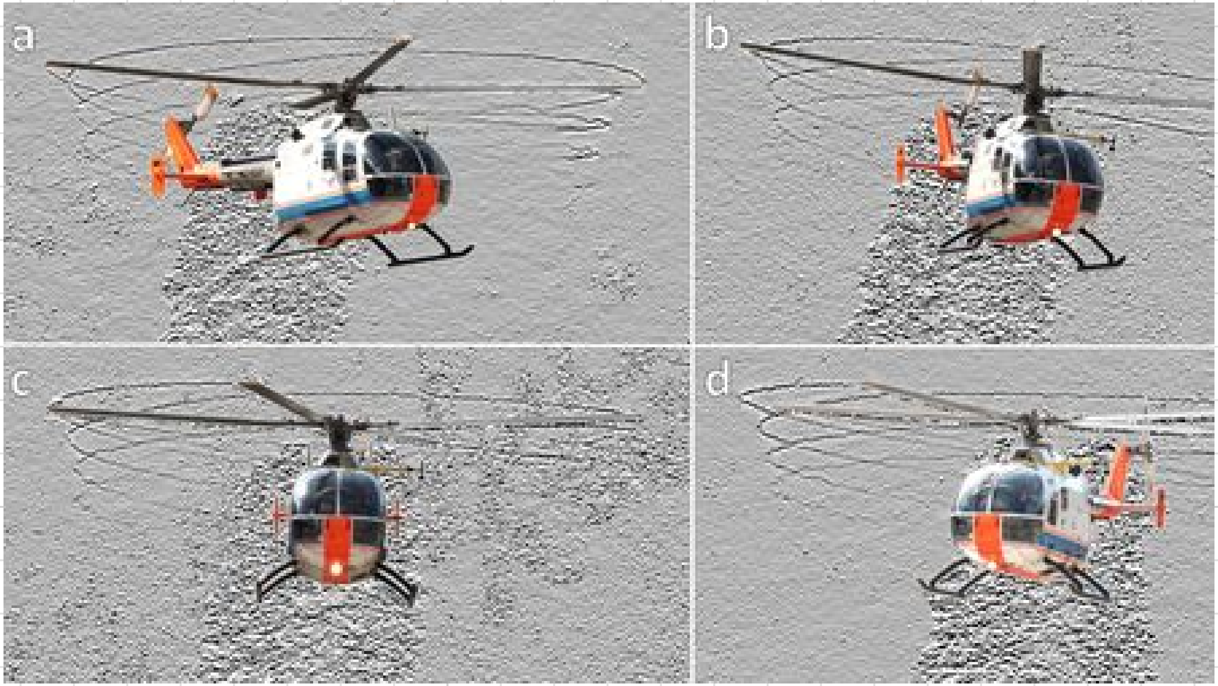


Tip vortices

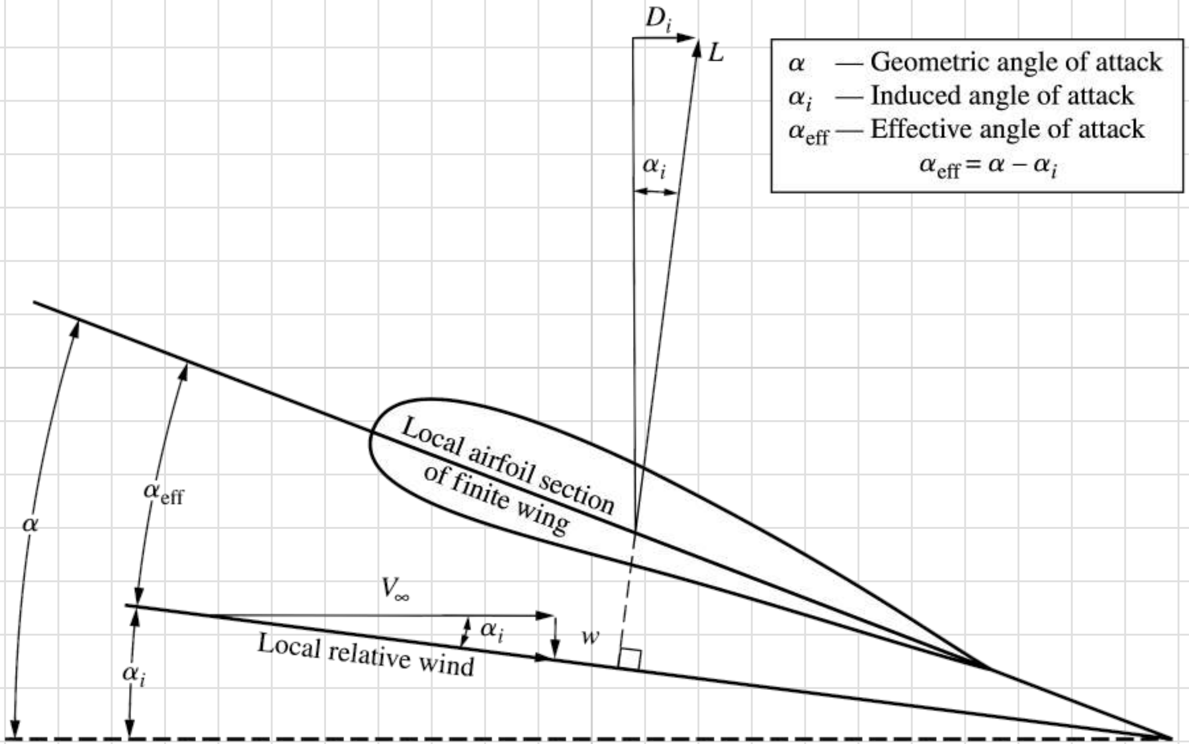




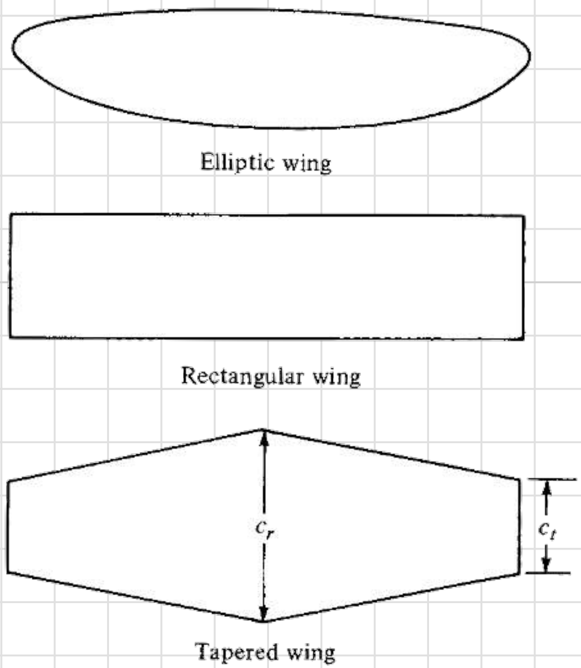
Tip vortices

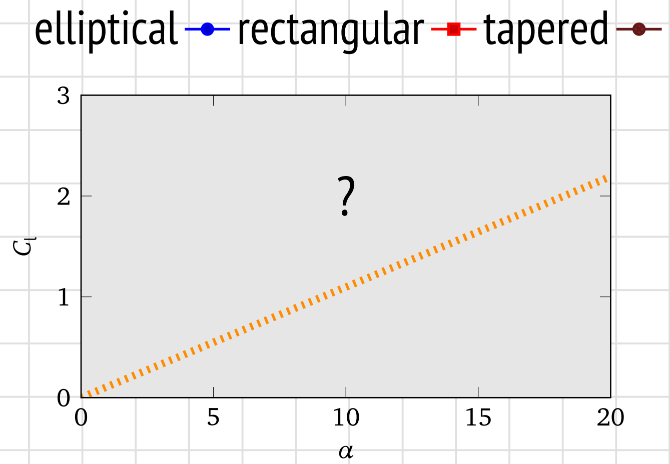


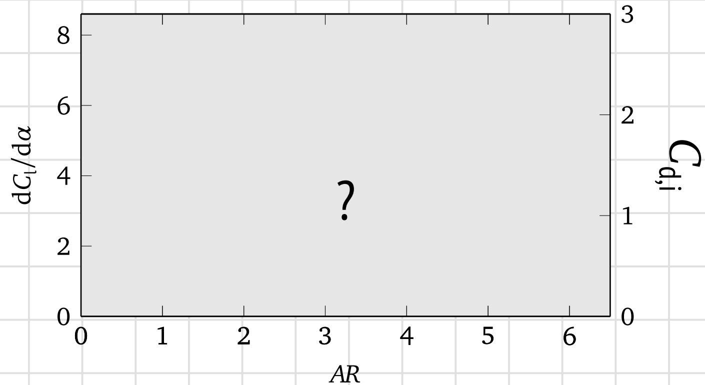
Downwash, effective angle of attack, and induced drag



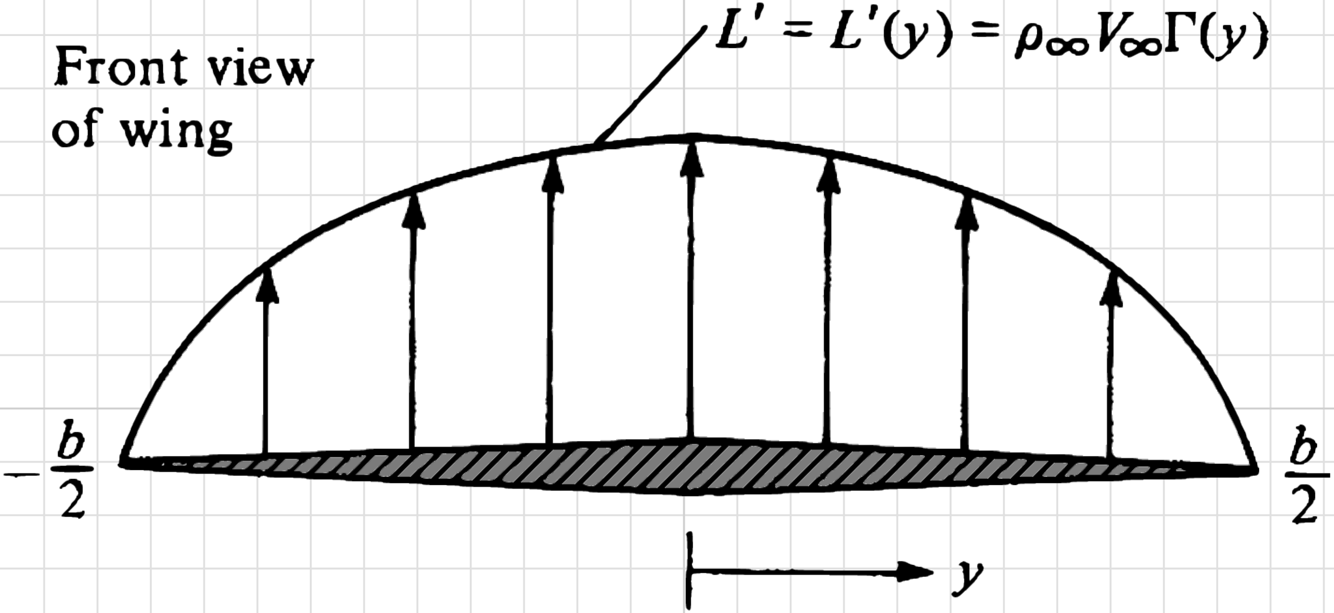
Finite wing aerodynamic loads







Span-wise lift distribution

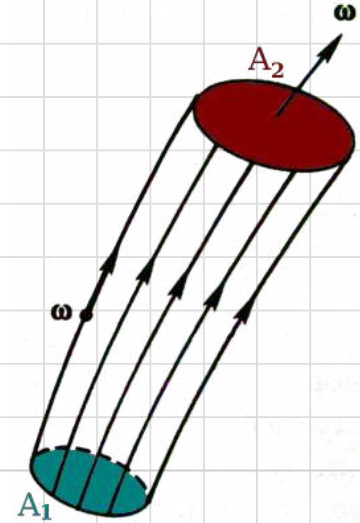


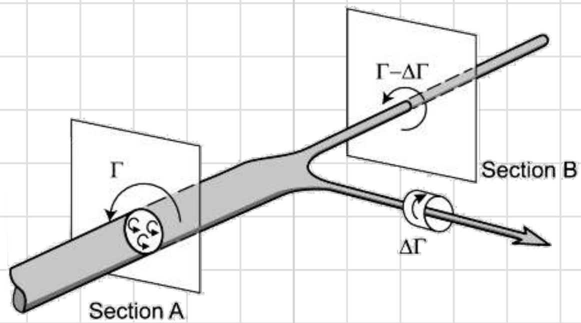
Helmholtz's theorems

1. The strength of a vortex tube is constant along its length

The integrated vorticity flux over a cross surface of a vorticity tube or the circulation of the tube is constant, independent of the shape and location of the cross surface or its boundary over which the integrals are estimated:

$$\iint_{A_1} \vec{\omega} \cdot \vec{n} \, \mathrm{d}A = \iint_{A_2} \vec{\omega} \cdot \vec{n} \, \mathrm{d}A$$



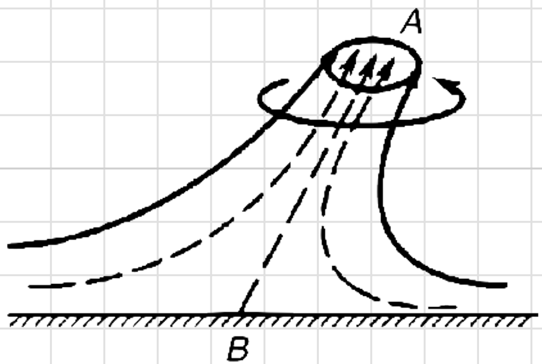


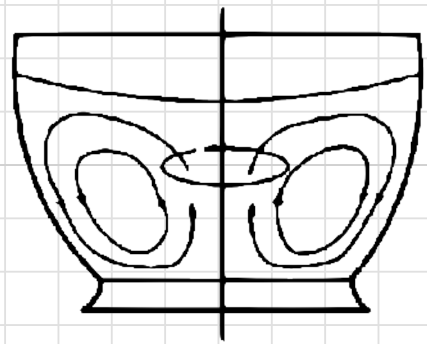
⇒ circulation is a proper measure of the strength of a vorticity tube

Helmholtz's theorems

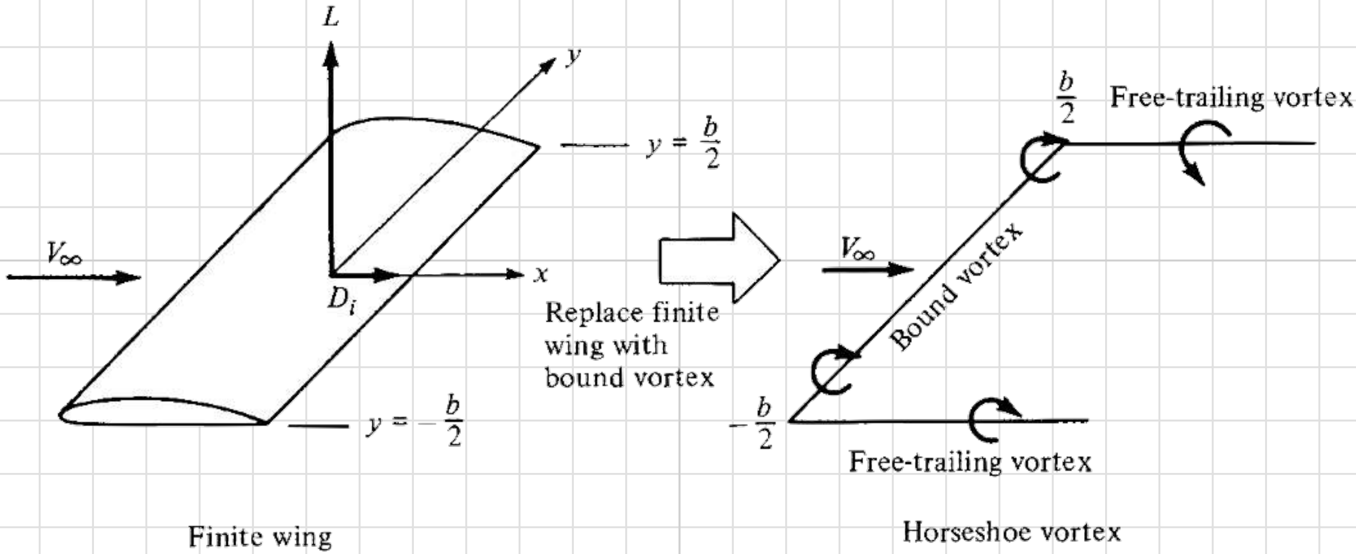
2. A vortex line cannot end in the fluid

⇒ it must form a closed loop or originate/terminate in a discontinuity in the fluid such as a solid body or a surface of separation.

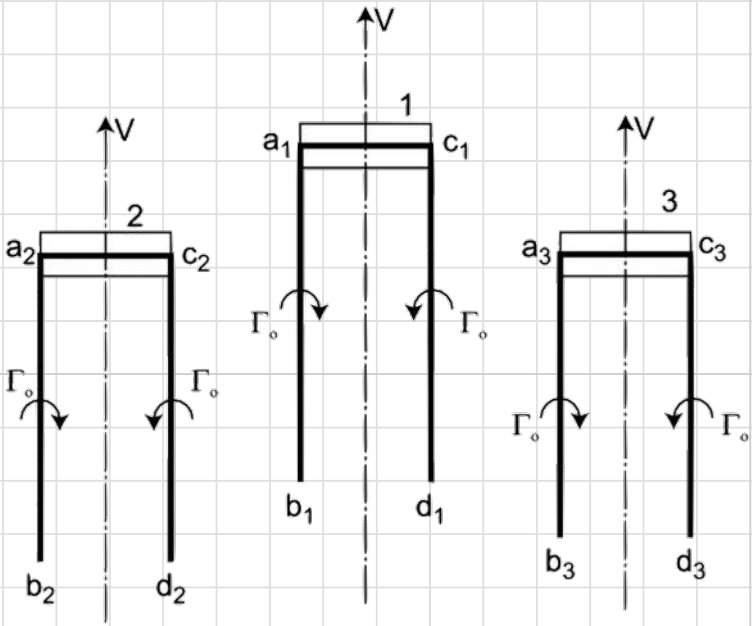




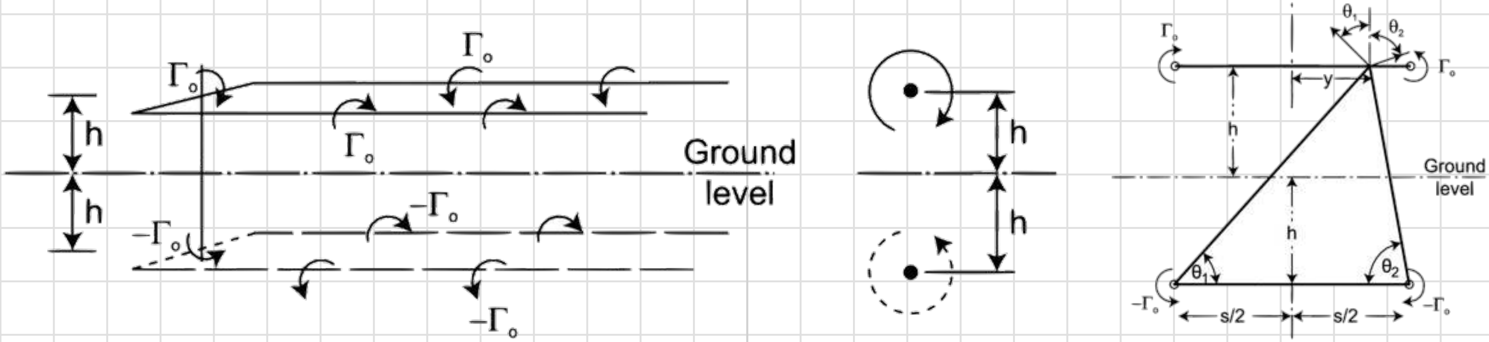
Prandtl's lifting line theory



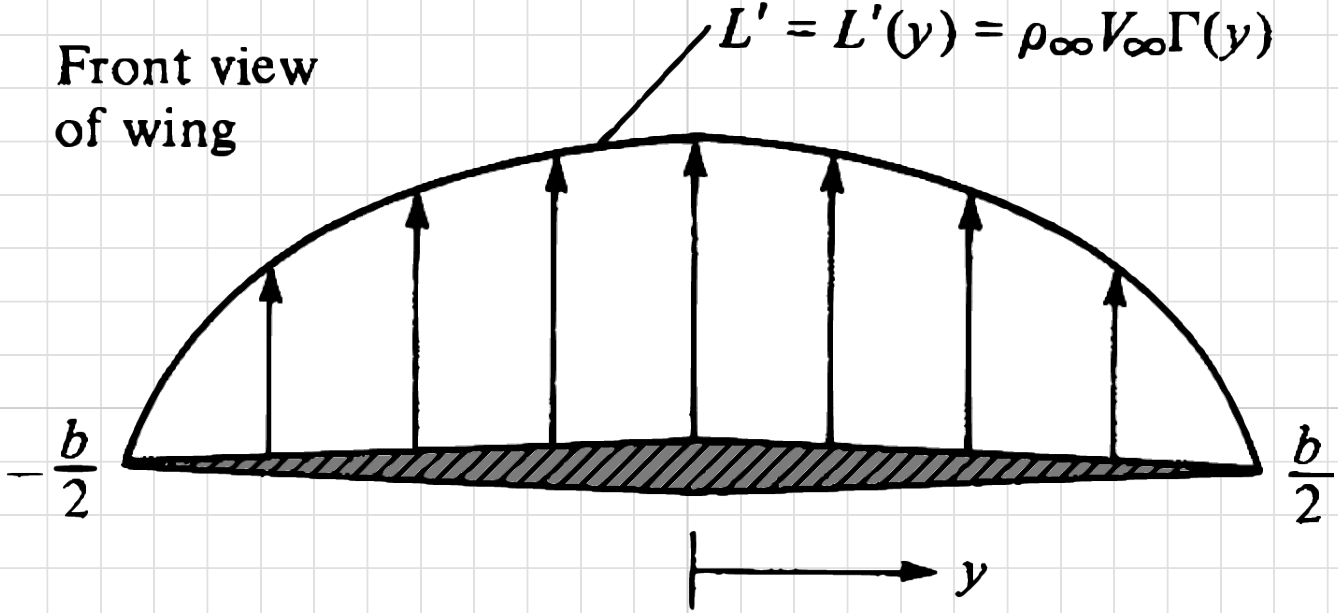
Formation flight



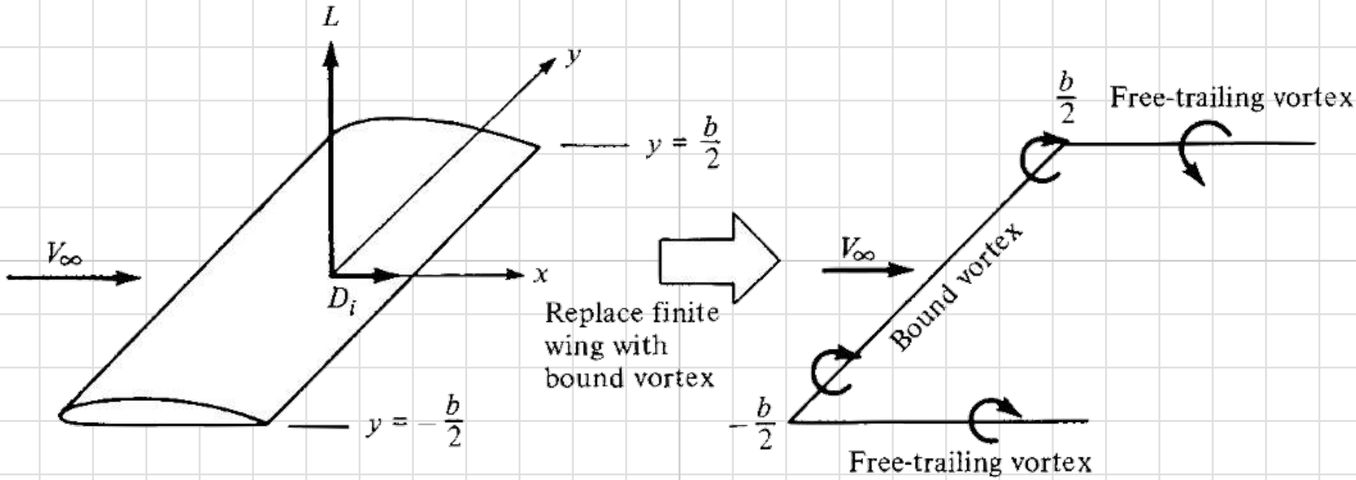
Ground effect



Span-wise lift distribution



Prandtl's lifting line theory

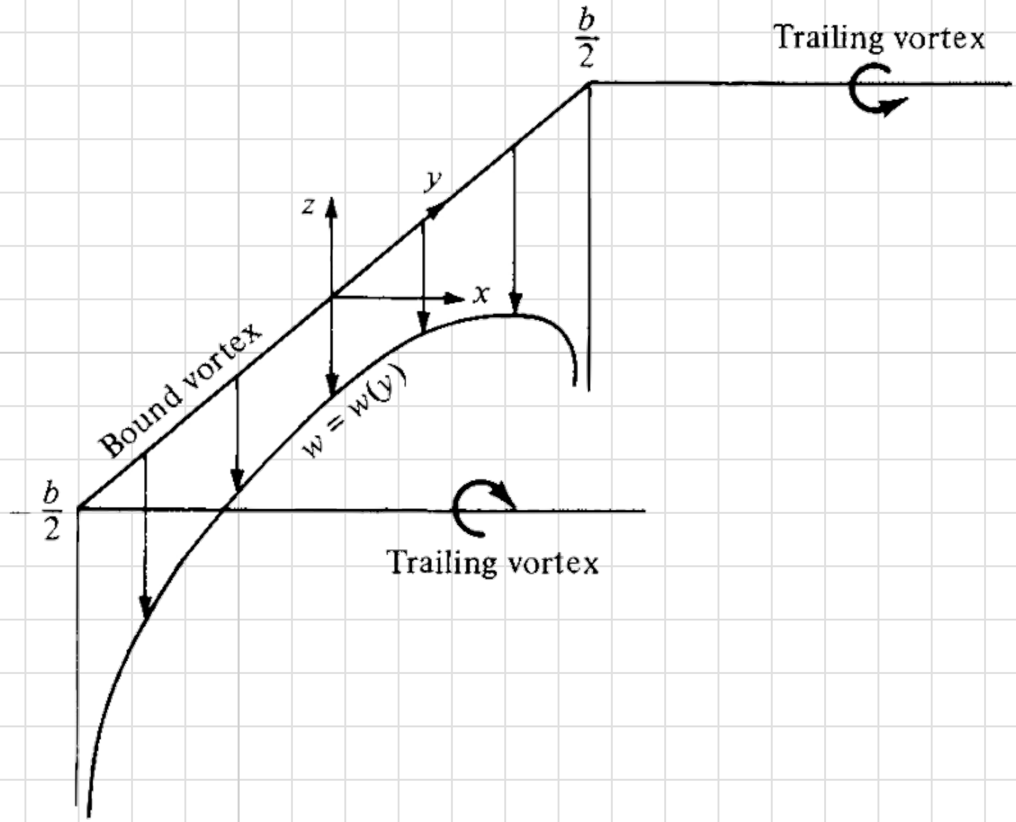


Finite wing

Horseshoe vortex

Prandtl's lifting line theory

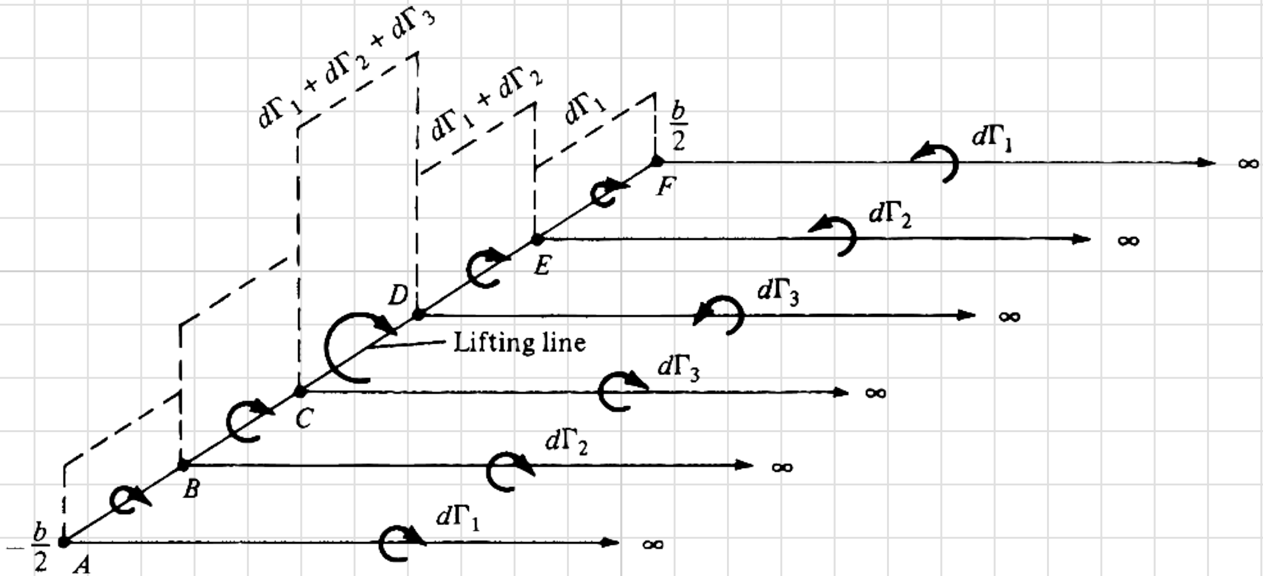
Downwash $w(y)$



$$w(y) = -\frac{\Gamma}{4\pi}\frac{b}{(b/2)^2 - y^2}$$

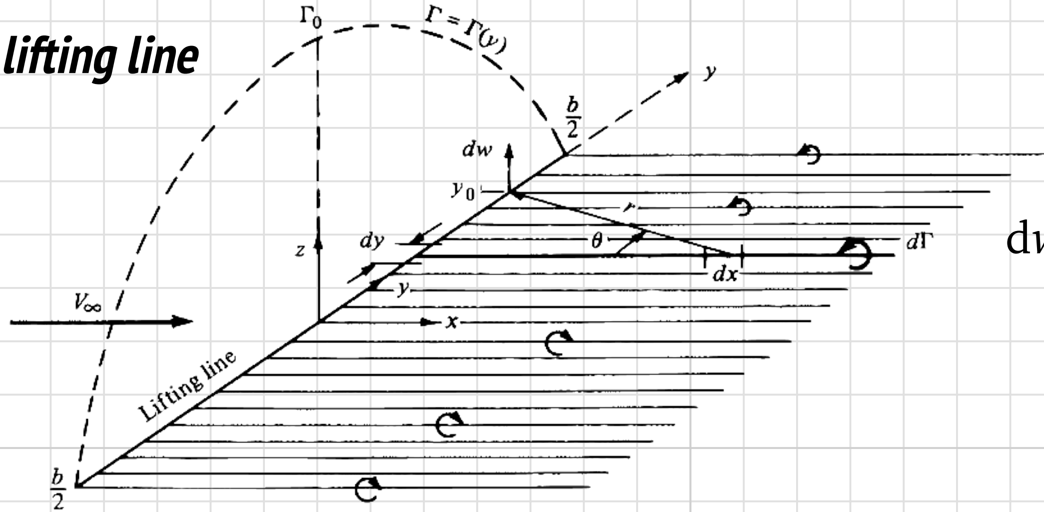
Prandtl's lifting line theory

The lifting line



Prandtl's lifting line theory

The lifting line



$$\mathrm{d}w(y_0) = -\frac{\mathrm{d}\Gamma}{4\pi(y_0 - y)}$$

Sign convention Anderson's book: $w(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2} \frac{(\mathrm{d}\Gamma/\mathrm{d}y)}{(y_0 - y)}\mathrm{d}y$

Sign convention Houghton's book: $\boxed{w(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2} \frac{(\mathrm{d}\Gamma/\mathrm{d}y)}{(y - y_0)}\mathrm{d}y}$

We will follow Houghton's sign convention and consider the downwash positive when the induced velocity points downward.

Prandtl's lifting line theory

Prandtl's lifting line theory

Prandtl's lifting line theory

$\Gamma(y)$ is the key to obtaining the aerodynamic characteristics of a finite wing

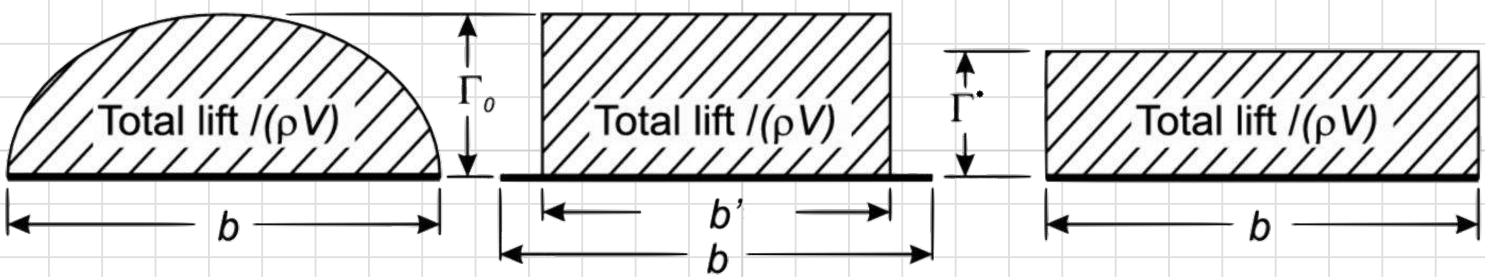
1. $$C_\mathrm{L} = \frac{2}{\mathrm{U}_\infty S} \int_{-b/2}^{b/2} \Gamma(y)\,\mathrm{d}y$$

2. $$\alpha(y_0) = \alpha_\mathrm{eff}(y_0) + \alpha_\mathrm{i}(y_0) = \frac{\Gamma(y_0)}{\pi \mathrm{U}_\infty c(y_0)} + \alpha_{\mathrm{L}=0}(y_0) - \frac{1}{4\pi \mathrm{U}_\infty} \int_{-b/2}^{b/2} \frac{(\mathrm{d}\Gamma/\mathrm{d}y)}{y - y_0}\mathrm{d}y$$

3. $$C_{\mathrm{D,i}} = \frac{2}{\mathrm{U}_\infty S} \int_{-b/2}^{b/2} \Gamma(y)\alpha_\mathrm{i}(y)\,\mathrm{d}y$$

Prandtl's lifting line theory

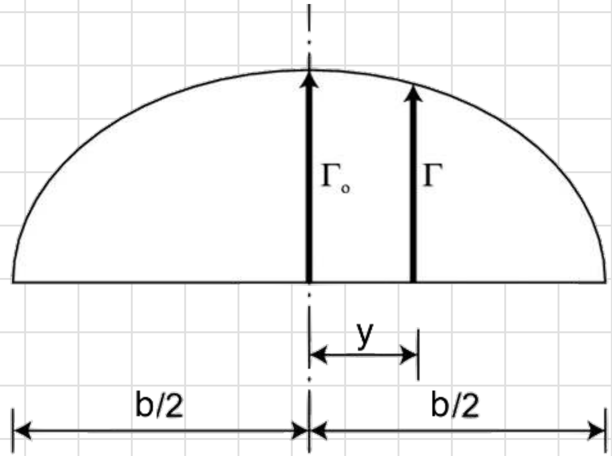
Lift distribution



Prandtl's lifting line theory

Elliptical distribution

$$\Gamma(y) = \Gamma_0\sqrt{1-\left(\frac{2y}{b}\right)^2}$$



Prandtl's lifting line theory

Elliptical distribution

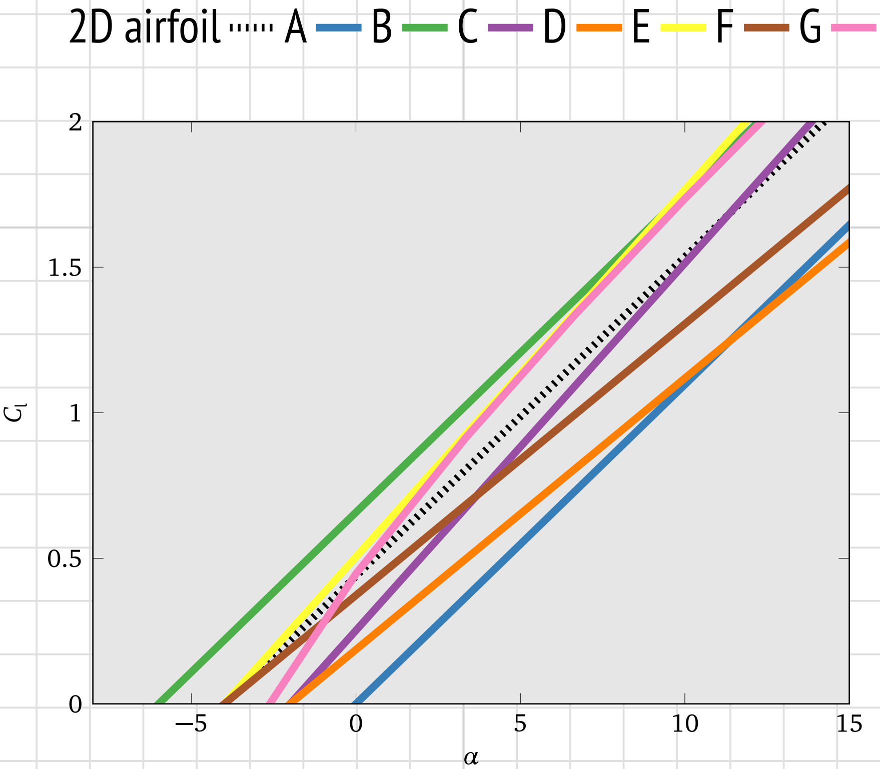
Prandtl's lifting line theory

Elliptical distribution

Prandtl's lifting line theory

Elliptical distribution

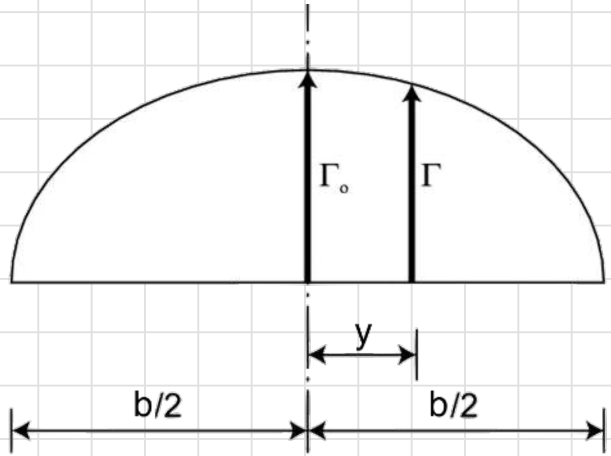
The most probable lift curve for a finite wing with elliptical lift distribution is



Prandtl's lifting line theory

Elliptical distribution

$$\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$



$$\boxed{w(\theta_0) = \frac{\Gamma_0}{2b}}$$

$$\boxed{\alpha_\text{i} = \frac{C_\text{L}}{\pi AR}}$$

$$\boxed{C_{\text{D,i}} = \frac{C_\text{L}^2}{\pi AR}}$$

Sign convention:

if induced velocity points downward:

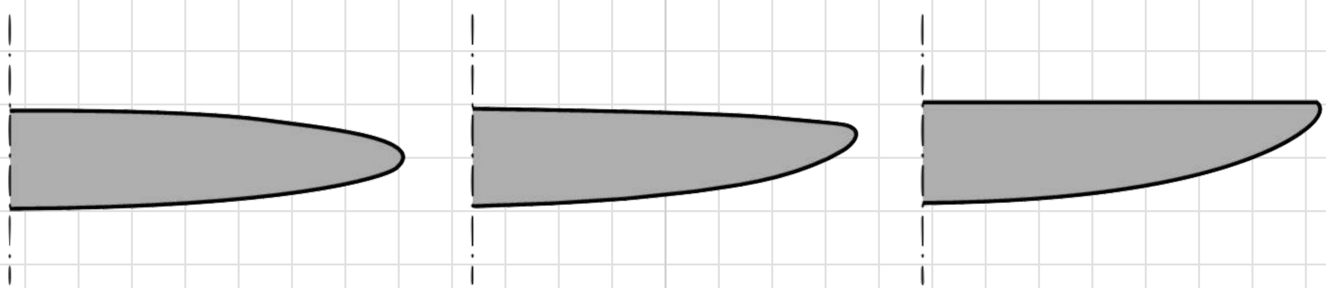
$w > 0 \quad \alpha_\text{i} > 0 \quad \alpha_\text{eff} < \alpha$

if induced velocity points upward:

$w < 0 \quad \alpha_\text{i} < 0 \quad \alpha_\text{eff} > \alpha$

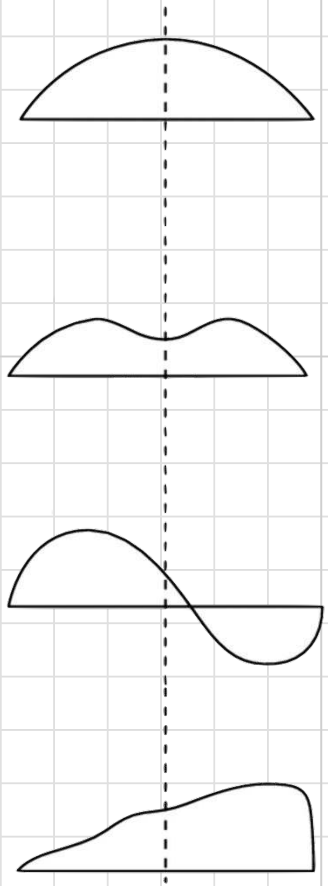
Prandtl's lifting line theory

Elliptical distribution



Prandtl's lifting line theory

General load distributions



Isolated wing in
steady symmetric
flight

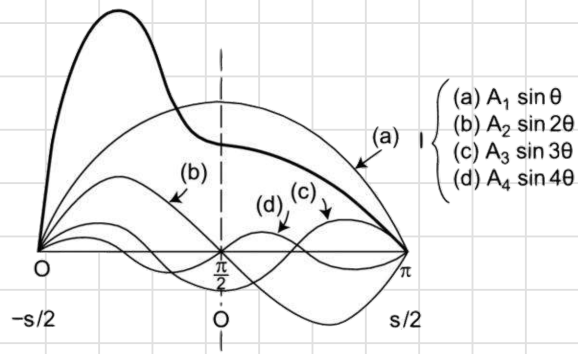
Lift distribution
modified by
fuselage effects

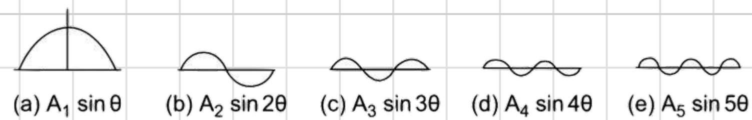
Lift distribution
in antisymmetric
flight

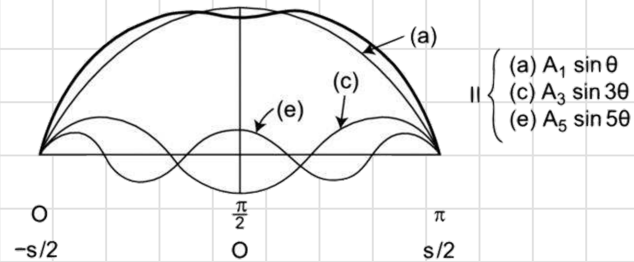
Antisymmetric flight
with ailerons
in operation

General load distributions

$$\Gamma(\theta) = 2b\mathrm{U}_\infty \sum_{n=1}^{N} A_n \sin n\theta$$







General load distributions

Lift on the wing

$$L = \int_{-s/2}^{s/2} \rho U_\infty \Gamma \mathrm{d}y$$

$$\Gamma(\theta) = 2bU_\infty \sum_{n=1}^{N} A_n \sin n\theta$$

$$y = b/2 \cos\theta$$

$$\mathrm{d}y = -b/2 \sin\theta \, \mathrm{d}\theta$$

$$\int_0^\pi \sin n\theta \sin\theta \mathrm{d}\theta = \pi/2 \text{ if } n = 1 \text{ and } 0 \text{ otherwise}$$

General load distributions

$$\Gamma(\theta) = 2bU_\infty \sum_{n=1}^{N} A_n \sin n\theta$$

$$y = b/2 \cos\theta$$

$$dy = -b/2 \sin\theta \, d\theta$$

$$\int_0^\pi \frac{\cos n\theta}{\cos\theta - \cos\theta_1} d\theta = \pi \frac{\sin n\theta_1}{\sin\theta_1} \text{ with n=0,1,2,...}$$

Downwash

$$w(y_0) = -\frac{1}{4\pi} \int_{-b/2}^{b/2} \frac{(d\Gamma/dy)}{(y - y_0)} dy$$

$$w(\theta_0) = \frac{1}{2\pi b} \int_0^\pi \frac{(d\Gamma/d\theta)}{(\cos\theta - \cos\theta_0)} d\theta$$

General load distributions

$$\Gamma(\theta) = 2bU_\infty \sum_{n=1}^{N} A_n \sin n\theta$$

$$y = b/2 \cos\theta$$

$$\mathrm{d}y = -b/2 \sin\theta \,\mathrm{d}\theta$$

$$D_i = L\alpha_i = Lw/U_\infty = \rho\Gamma w$$

$$C_L = \pi A_1 \,\mathrm{AR}$$

Induced drag

$$D_i = \int_{-s/2}^{s/2} \rho\,\Gamma\,w\,\mathrm{d}y$$

$$= \ldots$$

$$= \ldots$$

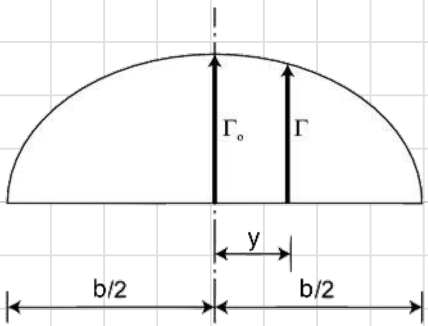
$$= \ldots$$

$$= \frac{\pi\rho U_\infty^2 b^2}{2} \sum_{n=1}^{\infty} nA_n^2$$

Minimum induced drag

elliptical distribution

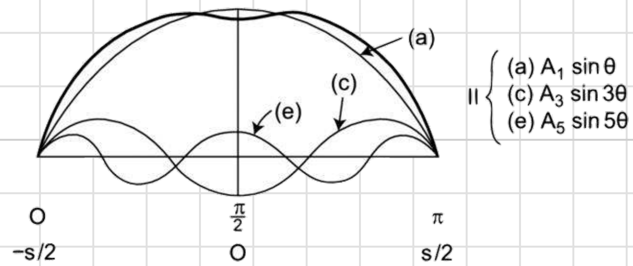
$$\Gamma = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2} = \Gamma_0 \sin\theta$$



$$\boxed{C_{\mathrm{D,i}} = \frac{C_{\mathrm{L}}^2}{\pi\,\mathrm{AR}}}$$

general load distribution

$$\Gamma(\theta) = 2b\mathrm{U}_\infty \sum_{n=1}^{N} A_{\mathrm{n}} \sin n\theta$$



$$\boxed{C_{\mathrm{D,i}} = \frac{C_{\mathrm{L}}^2}{\pi\,\mathrm{AR}}(1+\delta)}$$

with $\delta = \sum_{n=2}^{N} n(A_{\mathrm{n}}/A_1)^2$

General load distributions

Summary

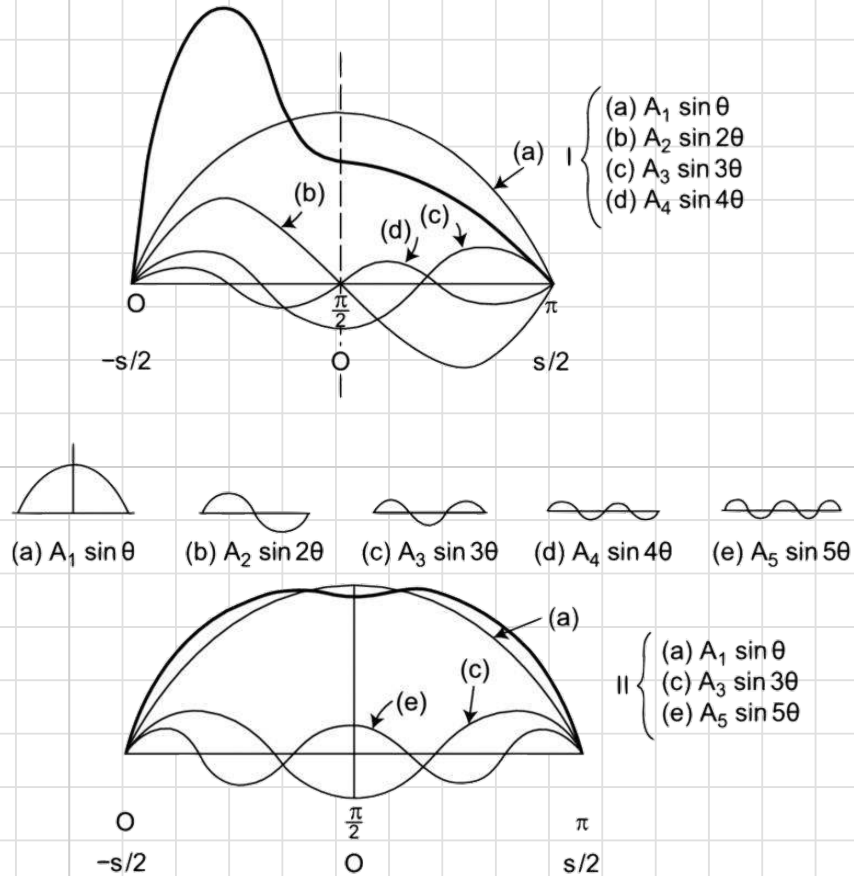
$$\boxed{\Gamma(\theta) = 2b\mathrm{U}_\infty \sum_{n=1}^{N} A_\mathrm{n} \sin n\theta}$$

$$\boxed{C_\mathrm{L} = \pi\,\mathrm{AR}\,A_1}$$

$$\boxed{w(\theta) = \mathrm{U}_\infty \sum_{n=1}^{\infty} nA_\mathrm{n} \frac{\sin n\theta}{\sin\theta}}$$

$$\boxed{C_\mathrm{D,i} = \frac{C_\mathrm{L}^2}{\pi\,\mathrm{AR}}(1+\delta)}$$

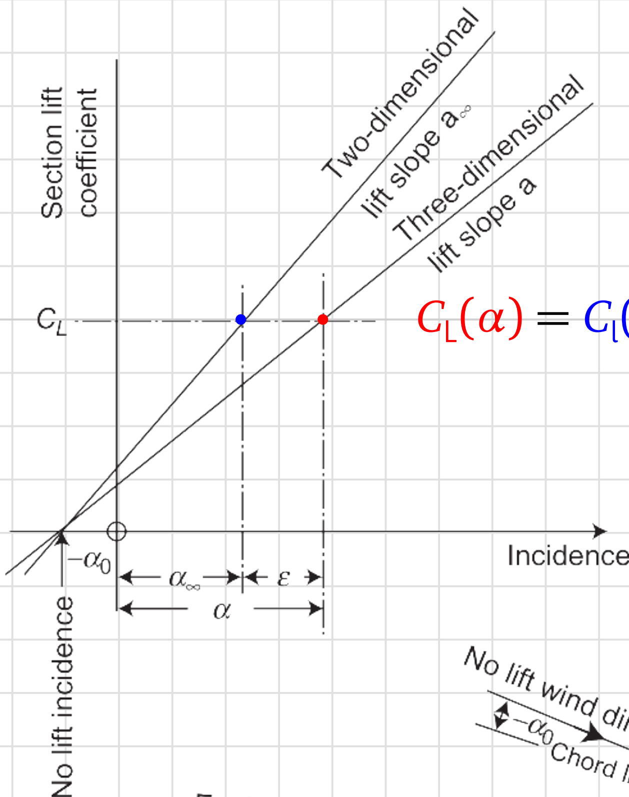
$$\text{with } \delta = \sum_{n=2}^{N} n(A_\mathrm{n}/A_1)^2$$



Load distribution on a given wing

2D: $C_l(\alpha) = a_\infty\,(\alpha_\infty - \alpha_0)$

3D: $C_L(\alpha) = a\,(\alpha - \alpha_0)$



$$C_L(\alpha) = C_l(\alpha_\infty) = a_\infty(\alpha_\infty - \alpha_0) = a_\infty(\alpha - \alpha_i - \alpha_0)$$