Fall 2024

ME-445 AERODYNAMICS

04 - Thin airfoil theory

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Lord Kelvin and the starting vortex

Vortex dynamics intermezzo

Introduction to thin airfoil theory

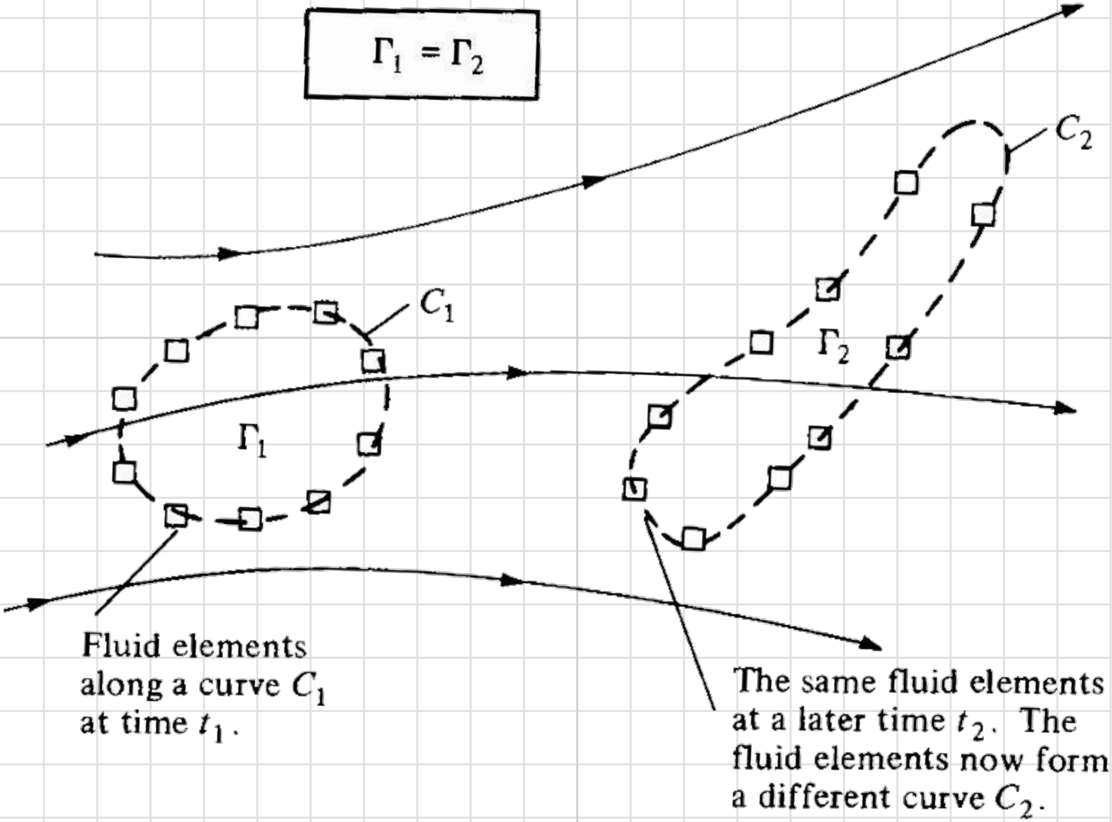
Classical thin airfoil theory

General thin airfoil section

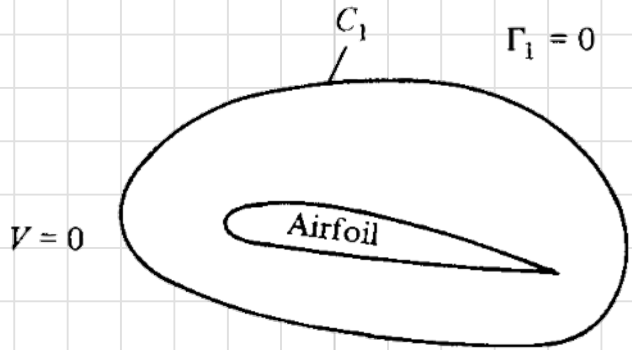
Trailing edge flap

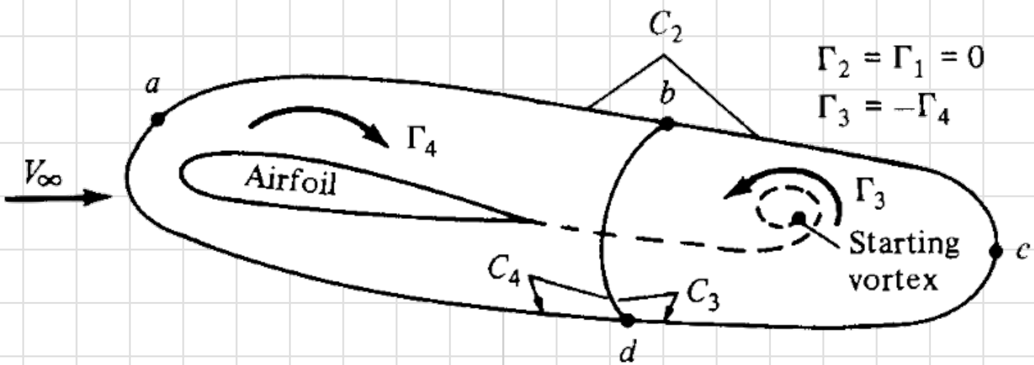
Kelvin's circulation theorem

$$\frac{D\Gamma}{Dt} = 0$$



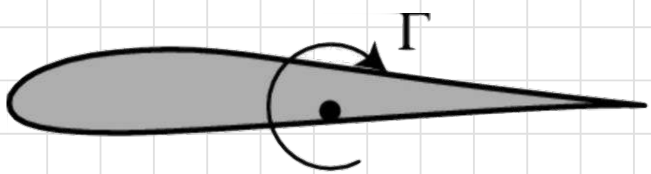
Kelvin’s circulation theorem and the starting vortex

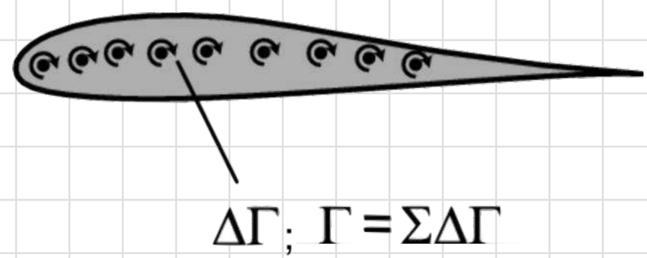




Circulation and lift

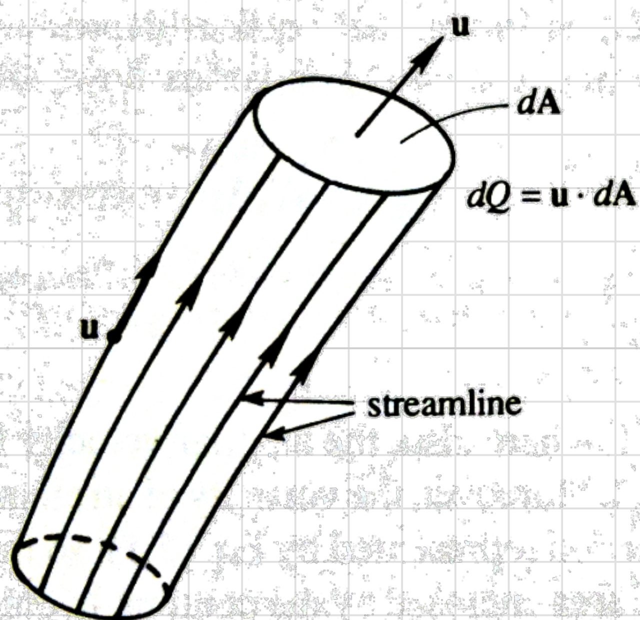
It is not necessary to suppose that the vorticity that gives rise to the circulation is due to a single vortex



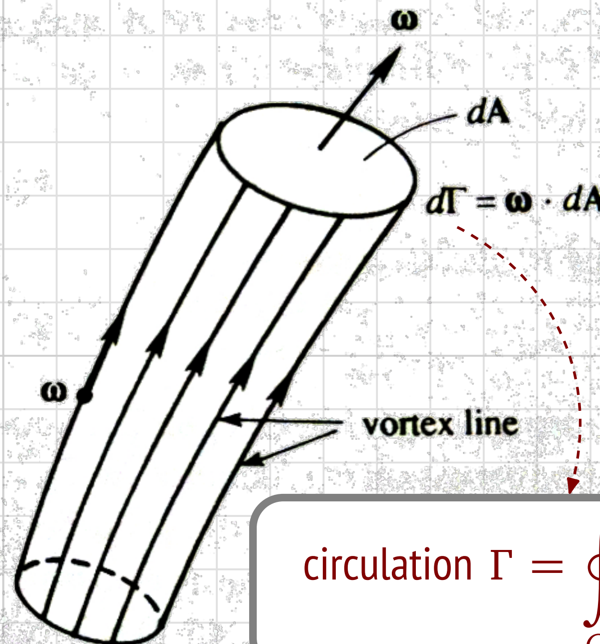


Vortex lines and tubes

vorticity $\vec{\omega} = \nabla \times \vec{u}$



stream tube



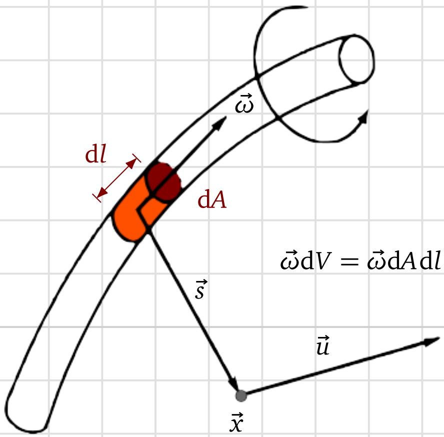
vortex tube

circulation $\Gamma = \oint_C \vec{u} \cdot \mathrm{d}\vec{s}$

$$\Gamma = \oint_C \vec{u} \cdot \mathrm{d}\vec{s} = \iint_A \vec{n} \cdot (\nabla \times \vec{u})\,\mathrm{d}A = \iint_A \vec{n} \cdot \vec{\omega}\,\mathrm{d}A$$

Biot-Savart

determines the velocity field associated with given vorticity field



$$\vec{\omega}\mathrm{d}V = \vec{\omega}\mathrm{d}A\,\mathrm{d}l = \vec{\Gamma}\mathrm{d}l$$

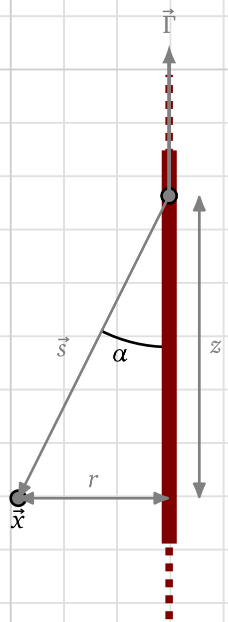
$$\vec{u}(\vec{x}) = \frac{1}{4\pi}\iiint_V \frac{\vec{\omega}\times\vec{s}}{s^3}\mathrm{d}V = \frac{1}{4\pi}\oint_L \frac{\vec{\Gamma}\times\vec{s}}{s^3}\mathrm{d}l$$

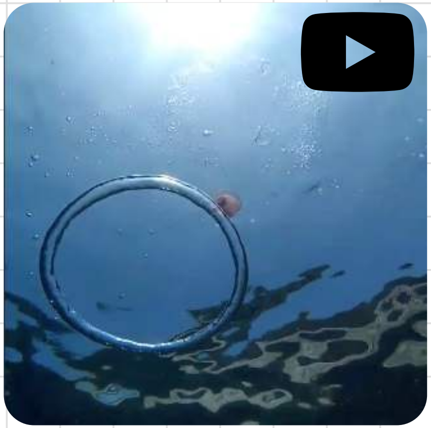
Example: Induced velocity by point vortices

$$u(\vec{x}) = \frac{\Gamma r}{4\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}z}{(r^2+z^2)^{3/2}} = \frac{\Gamma}{2\pi r}$$

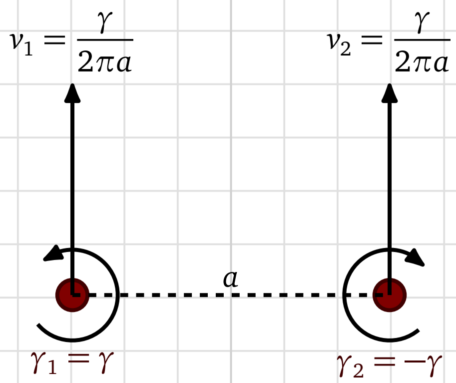
semi-infinite vortex line:

$$u(\vec{x}) = \frac{\Gamma r}{4\pi}\int_{0}^{\infty}\frac{\mathrm{d}z}{(r^2+z^2)^{3/2}} = \frac{\Gamma}{4\pi r}$$



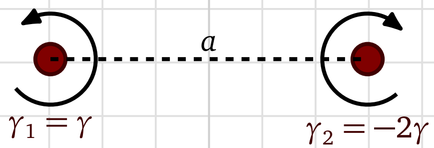


Interaction of point vortices



Interaction of point vortices

What would be the trajectory of this vortex pair?



Interaction of point vortices

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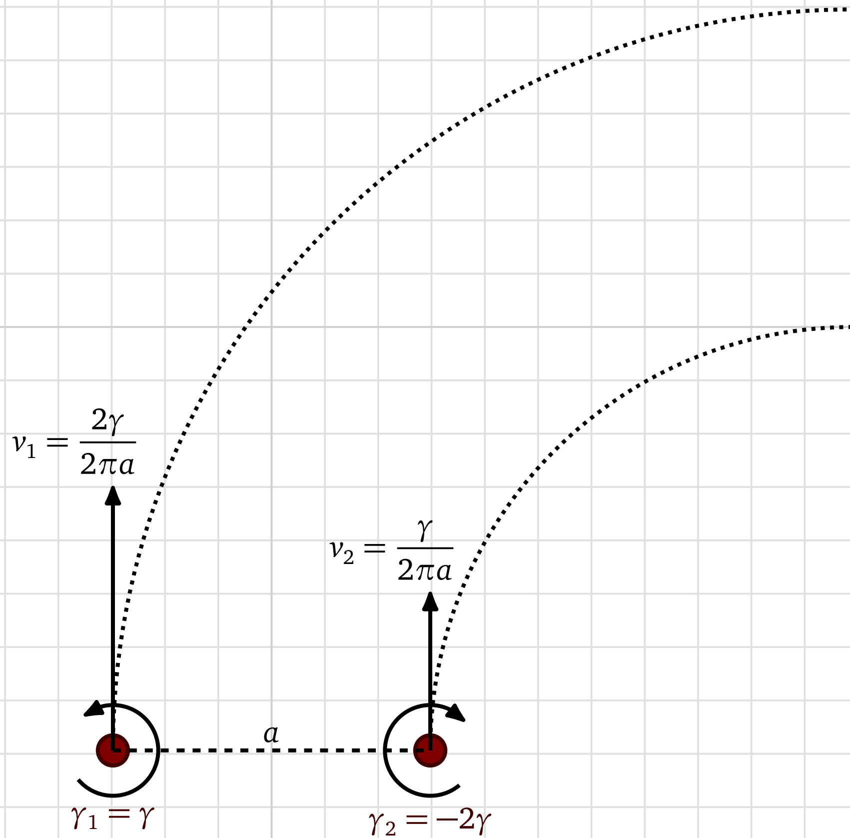


Interaction of point vortices with walls

What would happen to this vortex?

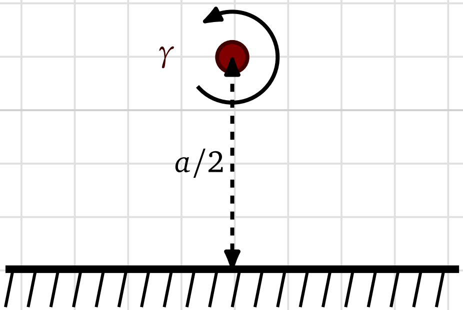
(A) it splits in 2 parts

(B) it disintegrates

(C) it moves to the left

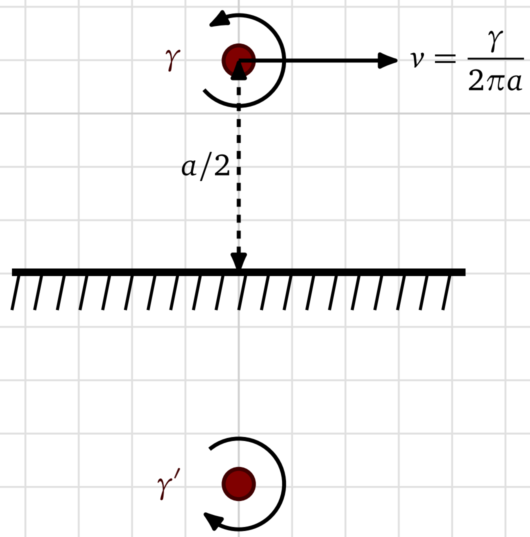
(D) it moves to the right

(E) it bounces back up

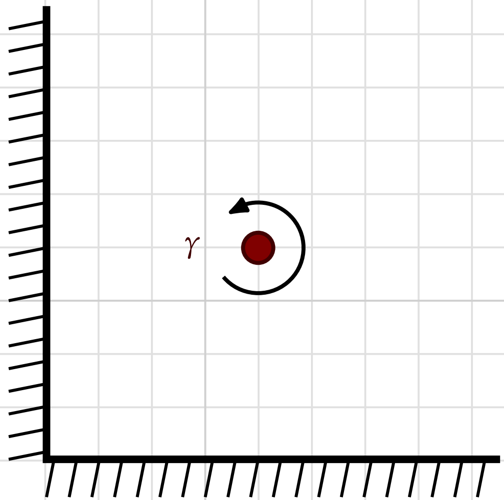


Interaction of point vortices with walls

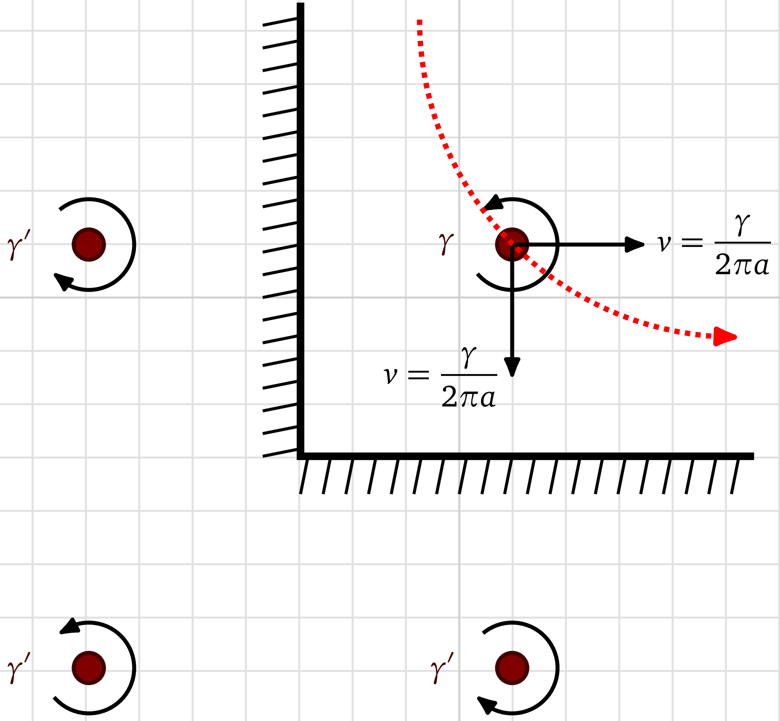
What would happen to this vortex?



Interaction of point vortices with walls



Interaction of point vortices with walls



Calculating lift

Vortex sheet method

$\frac{u^2{}_{l,ps}}{U_\infty^2}, \frac{u^2{}_{l,ss}}{U_\infty^2}$ □

Calculating lift

Vortex sheet method

Calculating lift

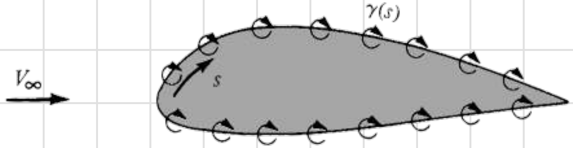
Vortex sheet method

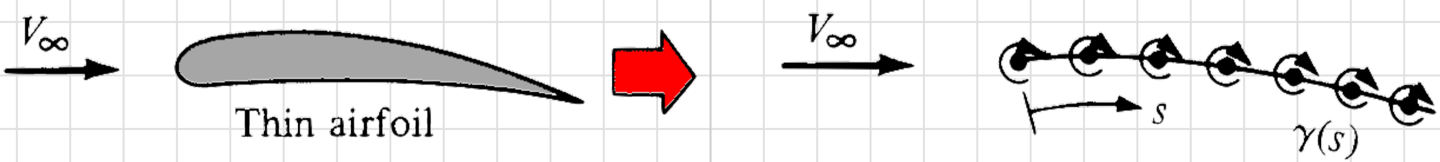
$$c_{p,ps} = 1 - \frac{u^2{}_{l,ps}}{U_\infty^2} \quad \square$$

$$c_l = \int_0^1 \left(c_{p,ps} - c_{p,ss}\right) \mathrm{d}x' \quad \square$$

Calculating lift

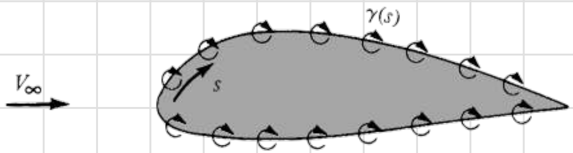
Classical thin airfoil theory

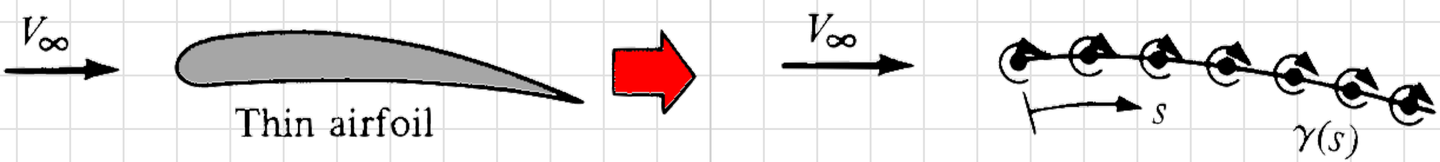




Calculating lift

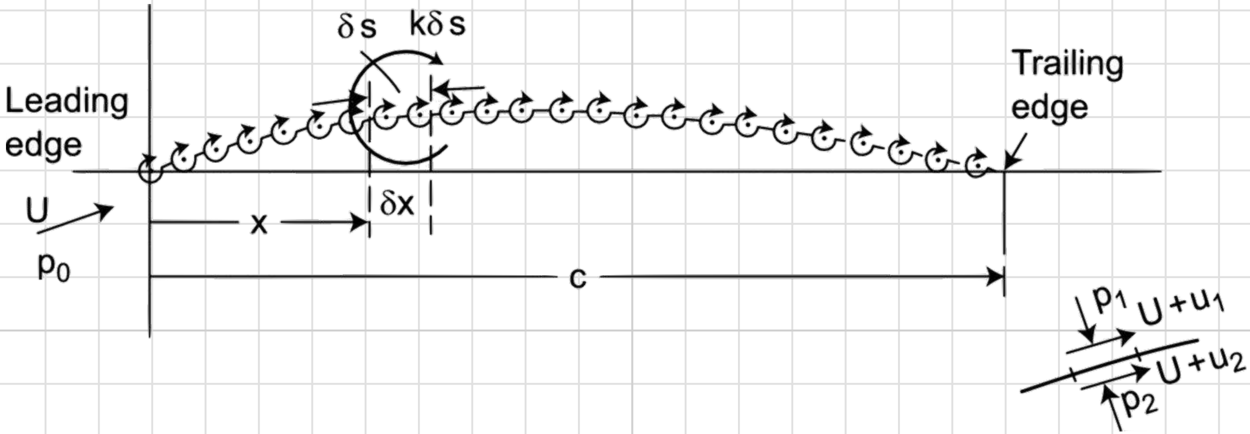
Classical thin airfoil theory





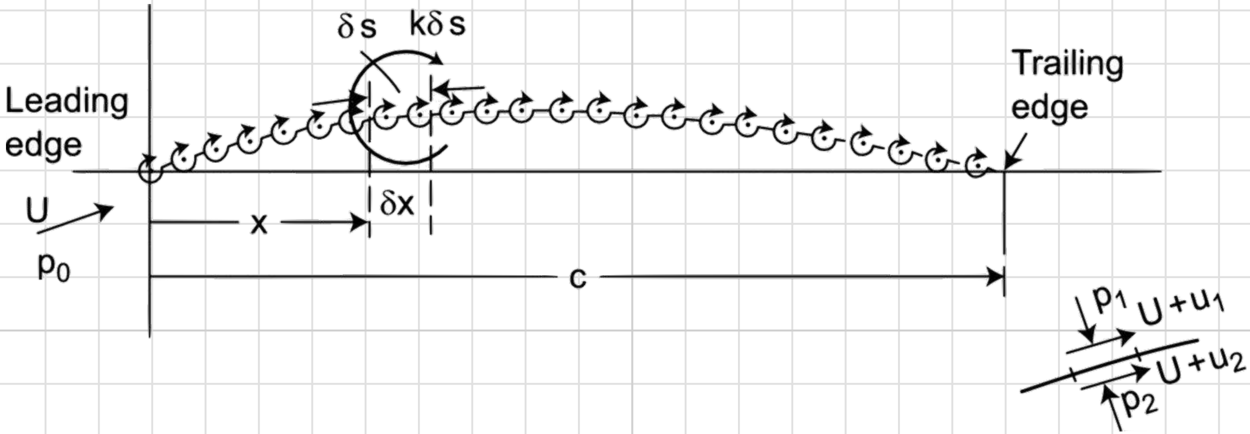
Classical thin airfoil theory

General idea



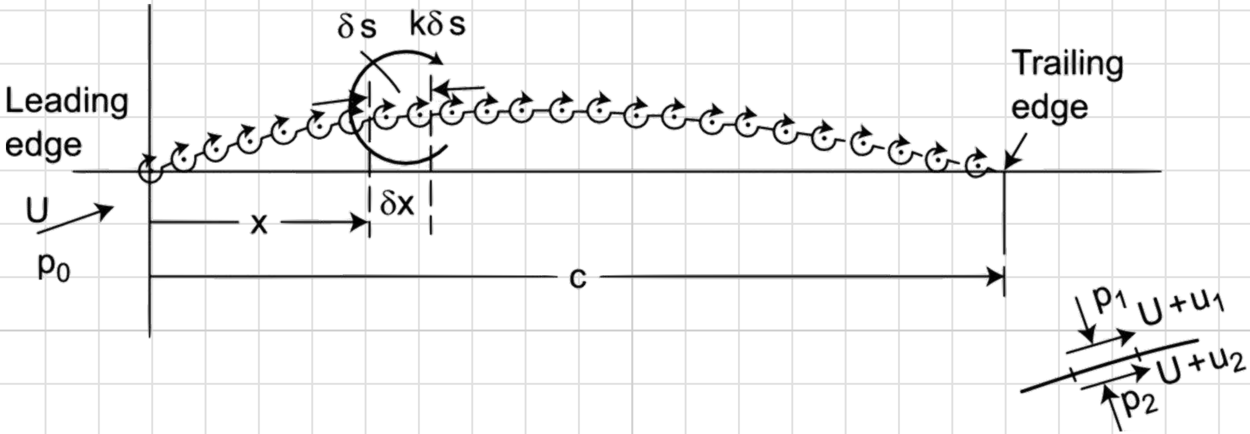
Classical thin airfoil theory

General idea



Classical thin airfoil theory

General idea



Classical thin airfoil theory

General idea

Classical thin airfoil theory

$$\boxed{U_\infty\left[\frac{\mathrm{d}y_c}{\mathrm{d}x}-\alpha\right]=\frac{1}{2\pi}\int_0^c \frac{k\,\mathrm{d}x}{x-x_1}}$$

Solving the integral ...

$$\frac{x}{c}=\frac{(1-\cos\theta)}{2}$$

Kutta condition: $k(\theta=\pi)=0$

$$\int_0^c \frac{k\,\mathrm{d}x}{x-x_1}=-\int_0^\pi \frac{k\sin\theta\,\mathrm{d}\theta}{\cos\theta-\cos\theta_1}$$

$$\int_0^\pi \frac{\cos n\theta}{\cos\theta-\cos\theta_1}\mathrm{d}\theta=\pi\frac{\sin n\theta_1}{\sin\theta_1} \qquad n=0,1,2,\dots$$

$$\int_0^\pi \frac{\sin n\theta\sin\theta}{\cos\theta-\cos\theta_1}\mathrm{d}\theta=-\pi\cos n\theta_1 \qquad n=1,2,3,\dots$$

Classical thin airfoil theory

Symmetrical thin airfoil

$$\boxed{U_\infty\left[\frac{\mathrm{d}y_c}{\mathrm{d}x}-\alpha\right]=-\frac{1}{2\pi}\int_0^\pi \frac{k\sin\theta\,\mathrm{d}\theta}{\cos\theta-\cos\theta_1}}$$

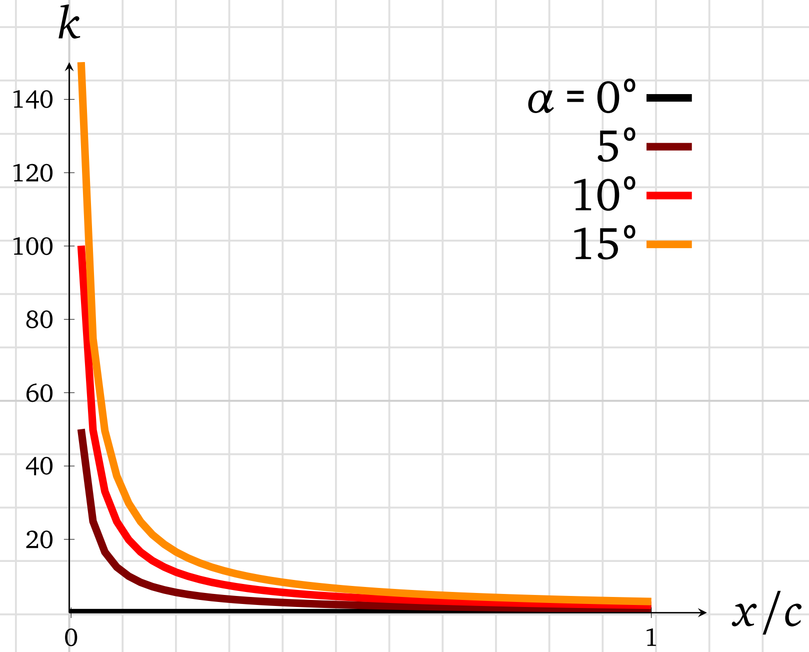
$$\int_0^\pi \frac{\cos n\theta}{\cos\theta-\cos\theta_1}\mathrm{d}\theta=\pi\,\frac{\sin n\theta_1}{\sin\theta_1} \qquad n=0,1,2,\ldots$$

$$\int_0^\pi \frac{\sin n\theta\sin\theta}{\cos\theta-\cos\theta_1}\mathrm{d}\theta=-\pi\cos n\theta_1 \qquad n=1,2,3,\ldots$$

Classical thin airfoil theory

Symmetrical thin airfoil

$$\boxed{k = 2U_\infty \alpha \left(\frac{\cos\theta + 1}{\sin\theta} \right)}$$



Classical thin airfoil theory

Symmetrical thin airfoil

Classical thin airfoil theory

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Symmetrical thin airfoil

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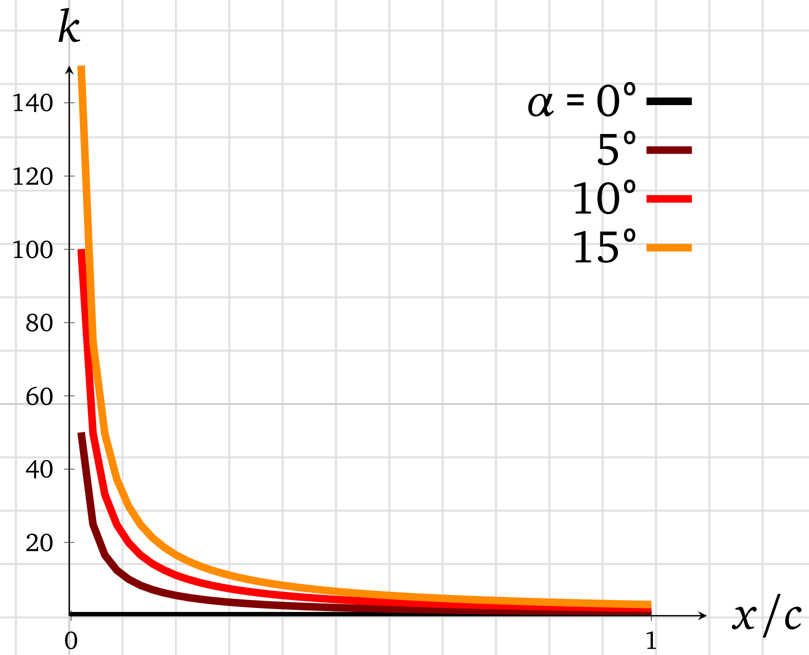


Classical thin airfoil theory

Symmetrical thin airfoil

Classical thin airfoil theory

Symmetrical thin airfoil

Classical thin airfoil theory

Symmetrical thin airfoil

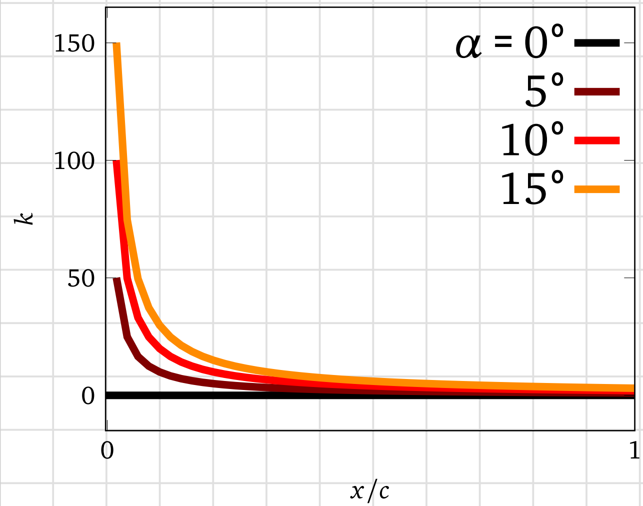
$$\frac{dy_c}{dx} = 0$$

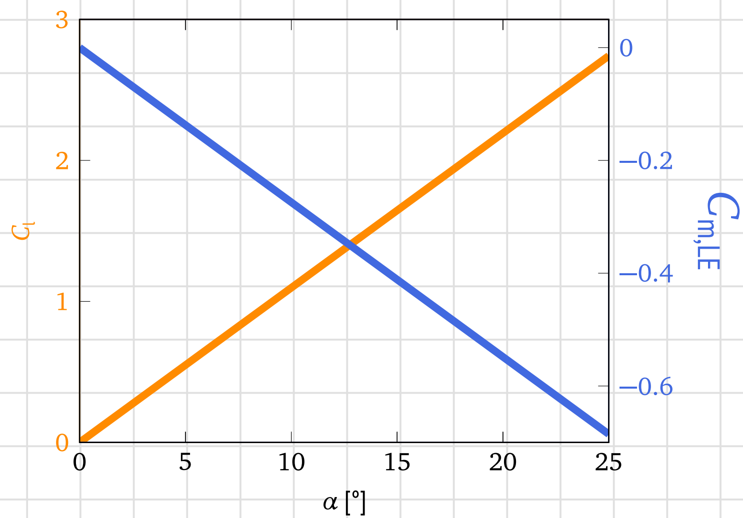
$$k = 2U_\infty \alpha \left(\frac{\cos\theta + 1}{\sin\theta} \right)$$

$$C_l = 2\pi\alpha$$

$$C_{m,LE} = -\frac{\pi}{2}\alpha = -\frac{C_l}{4}$$

$$\frac{x_{CP}}{c} = \frac{-C_{m,LE}}{C_l} = \frac{1}{4}$$





Classical thin airfoil theory

General thin airfoil section

$$\boxed{U_\infty\left[\frac{\mathrm{d}y_c}{\mathrm{d}x}-\alpha\right]=-\frac{1}{2\pi}\int_0^\pi \frac{k\sin\theta\,\mathrm{d}\theta}{\cos\theta-\cos\theta_1}}$$

$$\int_0^\pi \frac{\cos n\theta}{\cos\theta-\cos\theta_1}\mathrm{d}\theta=\pi\frac{\sin n\theta_1}{\sin\theta_1} \qquad n=0,1,2,\ldots$$

$$\int_0^\pi \frac{\sin n\theta\sin\theta}{\cos\theta-\cos\theta_1}\mathrm{d}\theta=-\pi\cos n\theta_1 \qquad n=1,2,3,\ldots$$

Classical thin airfoil theory

General thin airfoil section

$$U_\infty(\alpha - A_0) - U_\infty \sum_{n=1}^{\infty} A_n \cos n\theta = \frac{1}{2\pi}\int_0^\pi k \frac{\sin\theta}{\cos\theta - \cos\theta_1}\, d\theta$$

find solution $k = k_1 + k_2 + k_3$ with

$$\frac{1}{2\pi}\int_0^\pi k_1 \frac{\sin\theta}{\cos\theta - \cos\theta_1}\, d\theta = U_\infty(\alpha - A_0)$$

$$\frac{1}{2\pi}\int_0^\pi k_2 \frac{\sin\theta}{\cos\theta - \cos\theta_1}\, d\theta = 0$$

$$\frac{1}{2\pi}\int_0^\pi k_3 \frac{\sin\theta}{\cos\theta - \cos\theta_1}\, d\theta = -U_\infty \sum_{n=1}^{\infty} A_n \cos n\theta$$

Classical thin airfoil theory

General thin airfoil section

$$\boxed{k = k_1 + k_2 + k_3 = 2U_\infty \left[(\alpha - A_0)\frac{\cos\theta + 1}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta \right]}$$

Classical thin airfoil theory

$$\frac{dy_c}{dx} = A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta$$

How to find A_n?

Classical thin airfoil theory

Calculating lift

Classical thin airfoil theory

Calculating pitching moment

Classical thin airfoil theory

General thin airfoil section

$$\frac{dy_{\mathrm{c}}}{dx} = A_0 + \sum_{n=1}^{\infty} A_{\mathrm{n}} \cos n\theta$$

$$k = 2\mathrm{U}_{\infty}\left[(\alpha - A_0)\frac{\cos\theta + 1}{\sin\theta} + \sum_{n=1}^{\infty} A_{\mathrm{n}} \sin n\theta\right]$$

$$A_0 = \frac{1}{\pi}\int_0^{\pi} \frac{\mathrm{d}y_{\mathrm{c}}}{\mathrm{d}x}\mathrm{d}\theta \qquad A_{\mathrm{n}} = \frac{2}{\pi}\int_0^{\pi} \frac{\mathrm{d}y_{\mathrm{c}}}{\mathrm{d}x}\cos n\theta\,\mathrm{d}\theta$$

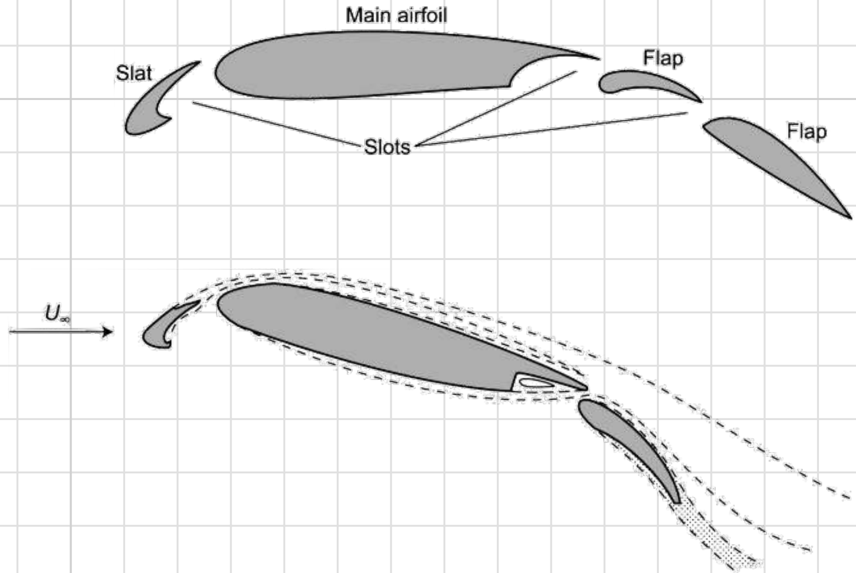
$$C_{\mathrm{l}} = 2\pi\alpha + \pi(A_1 - 2A_0)$$

$$C_{\mathrm{m,LE}} = -\frac{\pi}{2}\left[(\alpha - A_0) + A_1 - \frac{A_2}{2}\right] = -\frac{C_{\mathrm{l}}}{4}\left[1 + \frac{A_1 - A_2}{C_{\mathrm{l}}/\pi}\right]$$

$$\frac{x_{\mathrm{CP}}}{c} = \frac{-C_{\mathrm{m,LE}}}{C_{\mathrm{l}}} = \frac{1}{4} + \frac{\pi}{4C_{\mathrm{l}}}(A_1 - A_2)$$

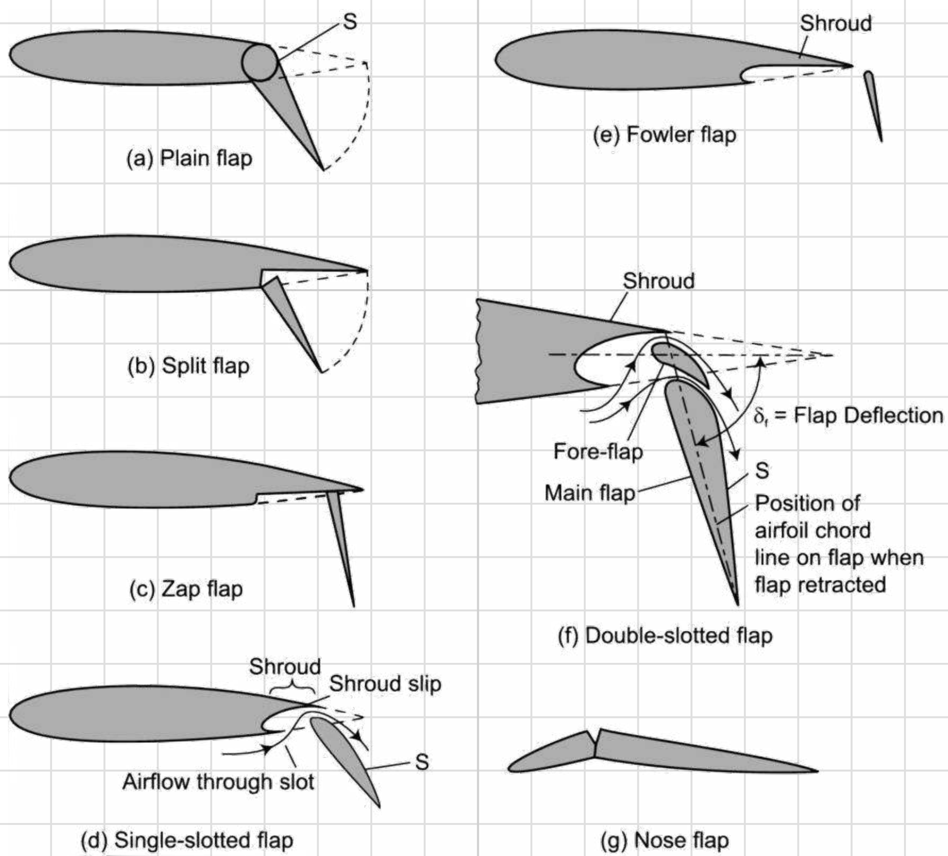
Thin airfoil theory - Multi-element airfoils

In aircraft design, high-lift devices are used to increase the amount of lift produced by the wing. Obtaining higher lift can be required to complete certain flight missions, such as take-off.

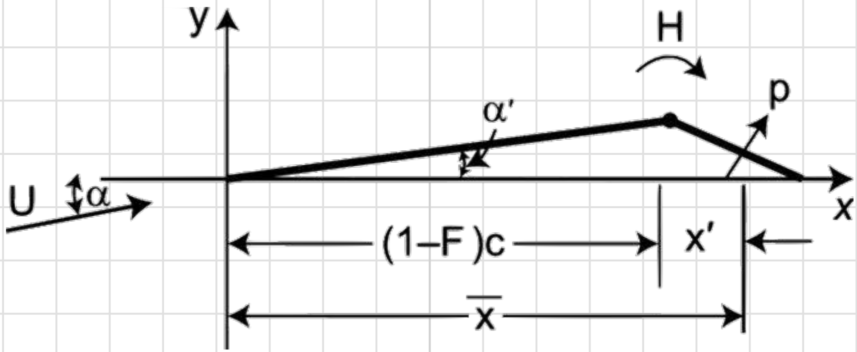


Thin airfoil theory - trailing edge flap

Flaps are common actuated high-lift devices. Part of the airfoil deflects from the camber line's original slope to alter circulation around the wing.

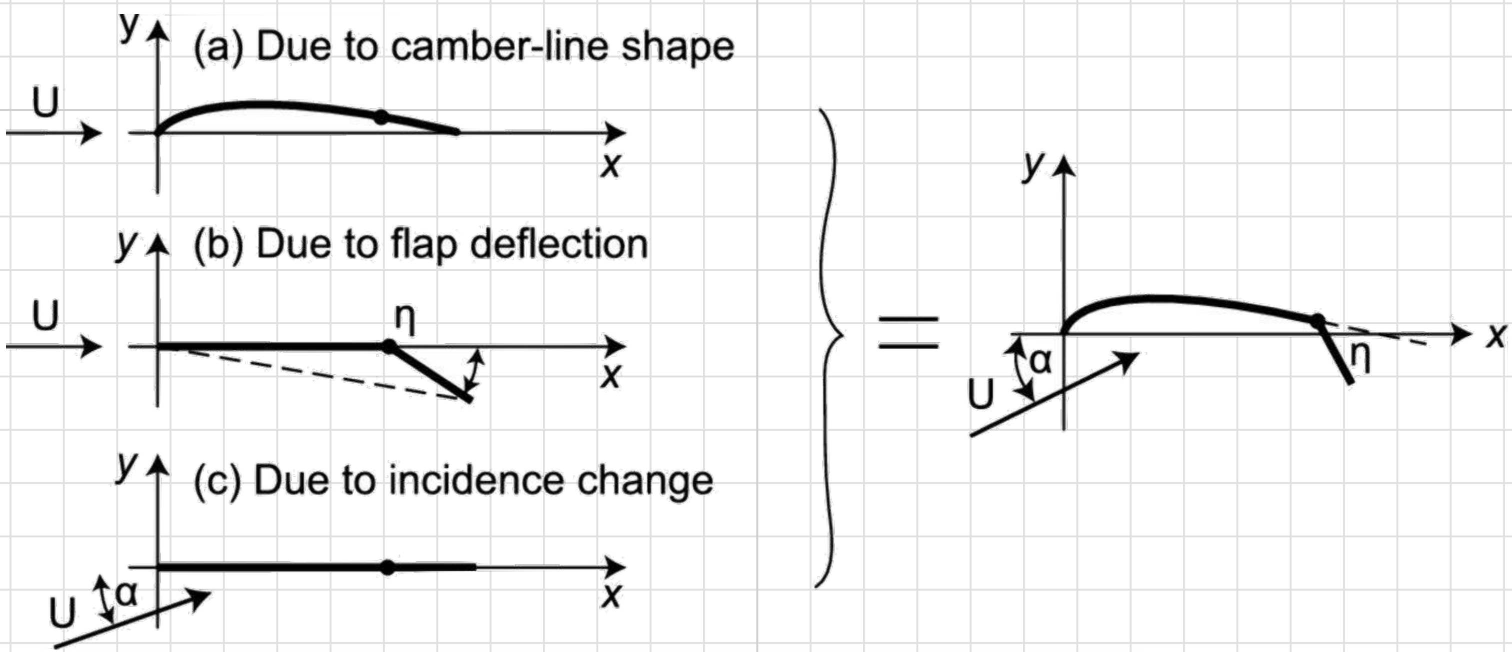


Thin airfoil theory - trailing edge flap



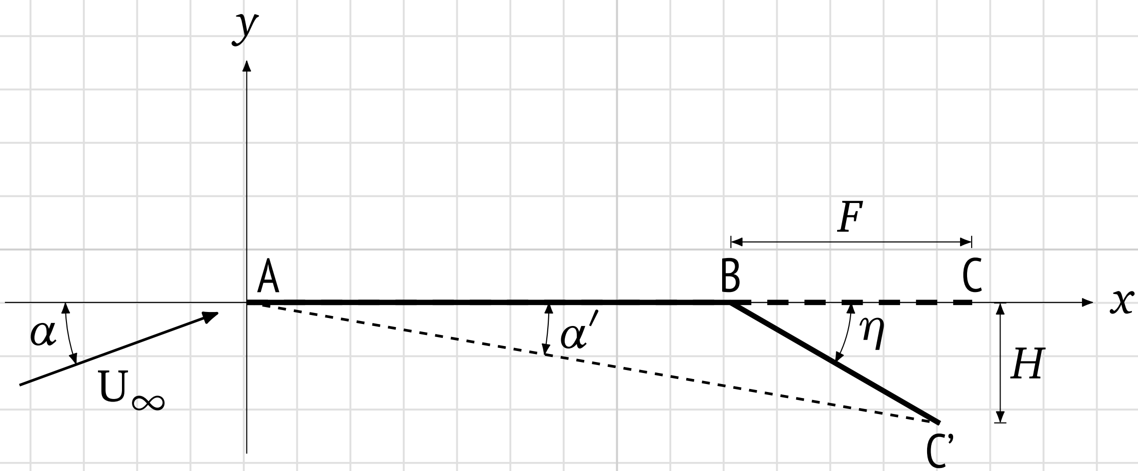
Thin airfoil theory - trailing edge flap

For the general cambered airfoil, the distribution of circulation along the camber line consists of the sum of a component due to a symmetric airfoil at incidence and a component due to the camber line shape. Potential flow allows you to simply sum a circulation component due to the flap to the previous two components to obtain the circulation distribution along a cambered airfoil with a flap.



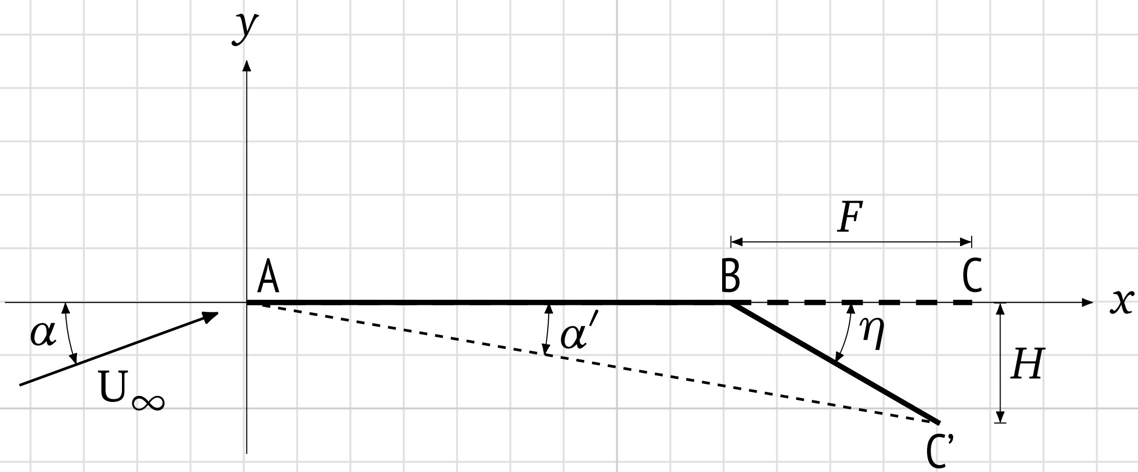
Thin airfoil theory - trailing edge flap

We consider a flap deflected downwards by an angle η relative to the chord line. The flap rotates about a hinge that is positioned at a distance $\dfrac{x}{c} = F$ from the trailing edge.



Thin airfoil theory - trailing edge flap

Thin airfoil theory - trailing edge flap

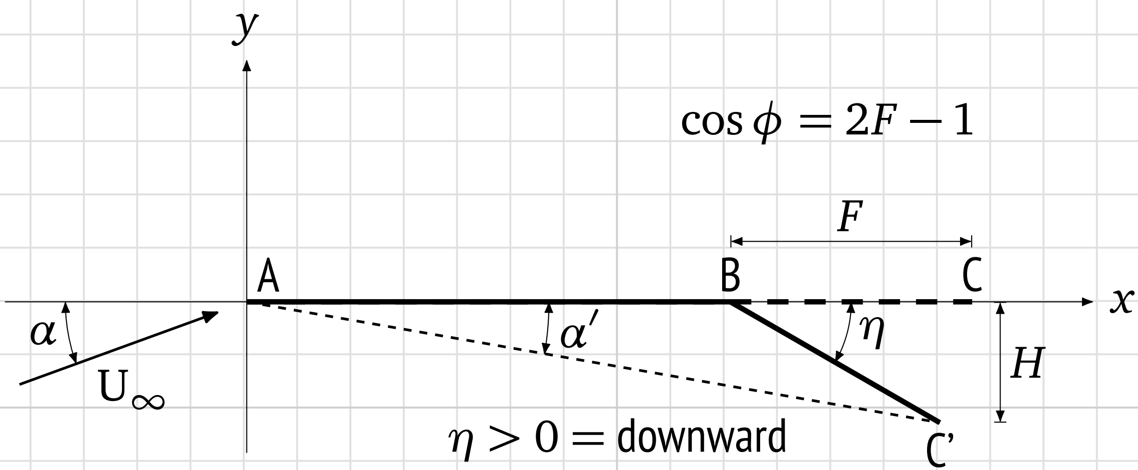


$$\begin{cases} A_0 = -(1-\dfrac{\phi}{\pi})\eta \\ A_1 = \dfrac{2\sin\phi}{\pi}\eta \\ A_2 = \dfrac{\sin 2\phi}{\pi}\eta \end{cases}$$

where ϕ is the value of θ at the hinge.

Thin airfoil theory - trailing edge flap

Aerodynamics for engineering students 4.5(6ed.)

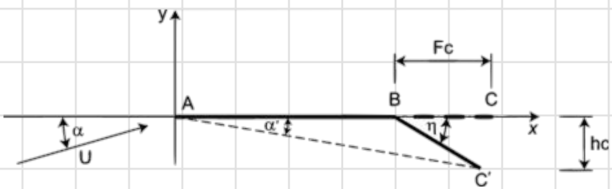


$$k = 2U_\infty \alpha \frac{\cos\theta + 1}{\sin\theta} + 2U_\infty \left[\left(1 - \frac{\phi}{\pi}\right) \frac{\cos\theta + 1}{\sin\theta} + \sum_{1}^{\infty} \frac{2\sin n\phi}{n\pi} \sin n\theta \right] \eta$$

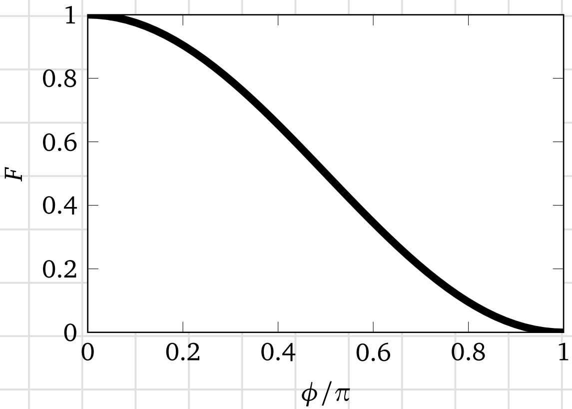
$$C_l = 2\pi\alpha + 2(\pi - \phi + \sin\phi)\eta$$

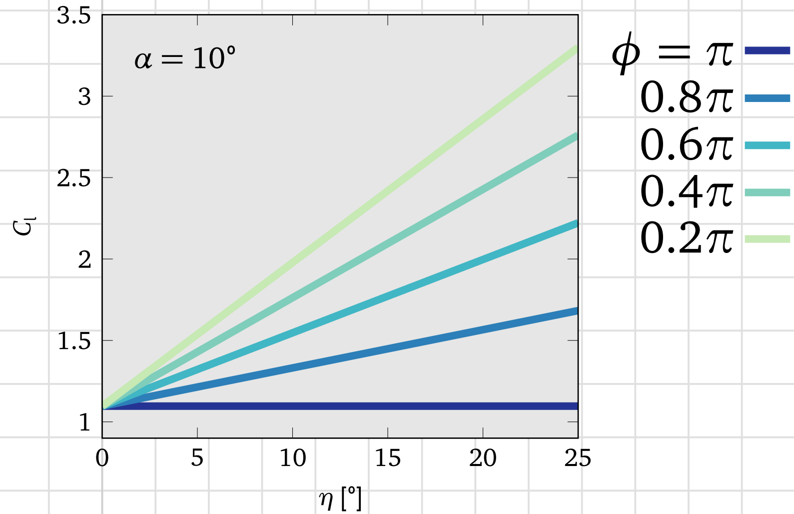
$$C_{m,LE} = -\frac{\pi}{2}\alpha - \frac{1}{2}[\pi - \phi + \sin\phi\,(2 - \cos\phi)]\eta$$

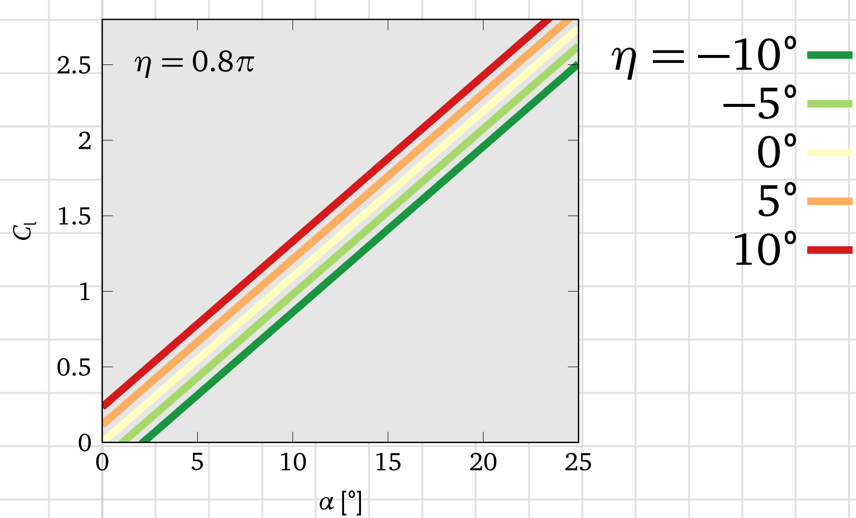
Trailing edge flap



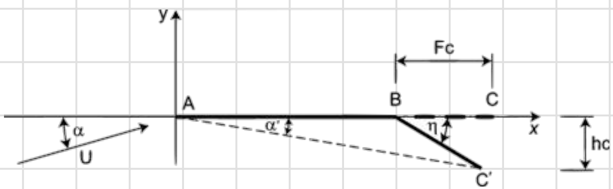
$$\boxed{C_l = 2\pi\alpha + 2\,(\pi - \phi + \sin\phi)\,\eta}$$







Trailing edge flap



$$C_{m,LE} = -\frac{\pi}{2}\alpha - \frac{1}{2}\left[\pi - \phi + \sin\phi\,(2 - \cos\phi)\right]\eta$$