Fall 2024

ME-445 AERODYNAMICS

02 - Basic concepts

K. Mulleners

Basic concepts

Flight and aerodynamic forces

How do planes fly?

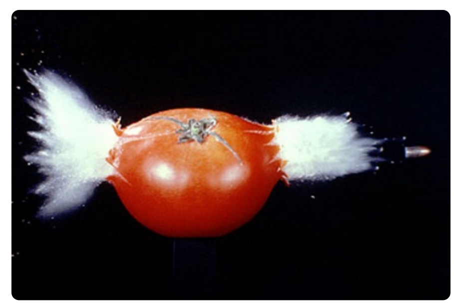
Airfoil nomenclature

Calculating aerodynamic forces

Calculating aerodynamic moments

What is flight?

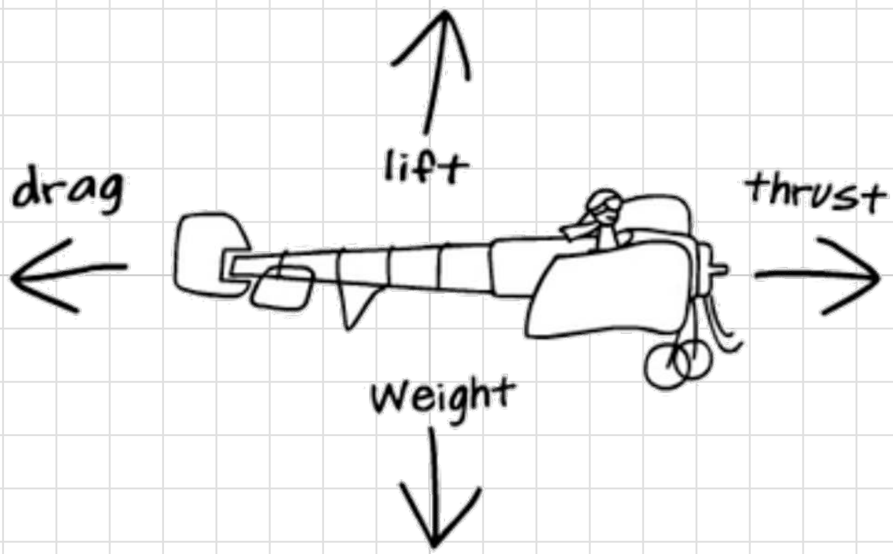
> Flight is the process by which an object moves, through an atmosphere or beyond it. This can be achieved by generating **aerodynamic lift**, **propulsive thrust**, hydrostatically using **buoyancy**, or by **ballistic movement**.
>
> *- wikipedia*



Aerodynamic lift and other forces

lift $\perp U_\infty$

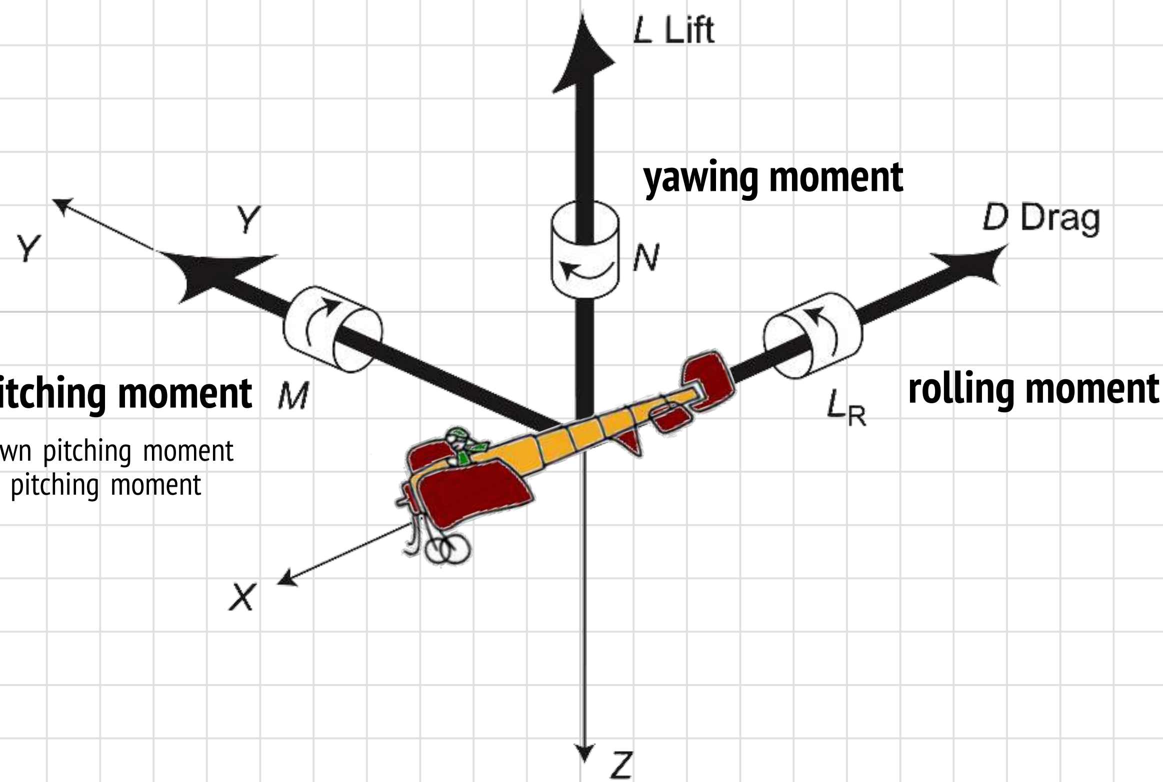
drag $\parallel U_\infty$



in cruise: lift = weight

Aerodynamic forces and moments

pitching moment ☐
rolling moment ☐
yawing moment ☐



$M < 0$: nose-down pitching moment
$M > 0$: nose-up pitching moment

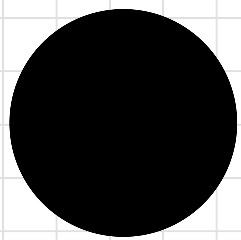
Sources of aerodynamic forces

Which objects can generate net aerodynamic lift in flight?

(A)

(B)

(C)

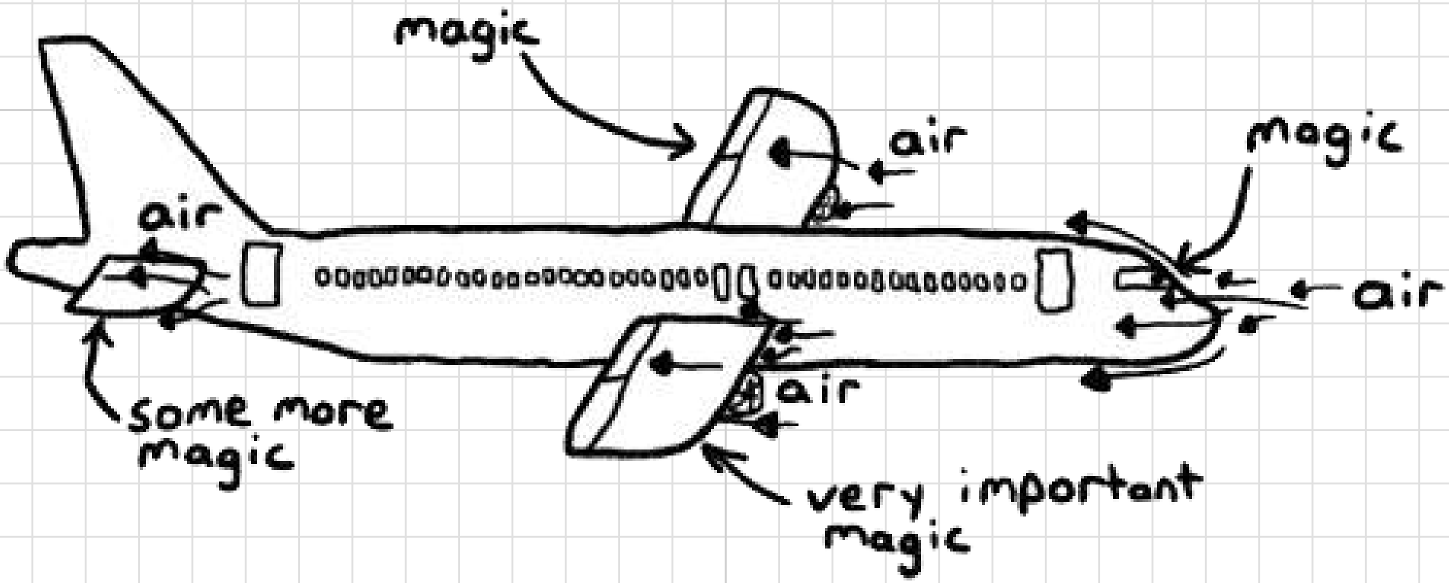


(D)

Explaining lift

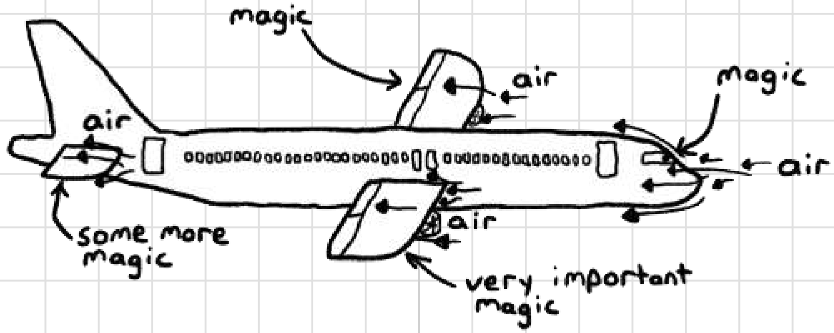
It is easy to explain how a rocket works, but explaining how a wing works takes a rocket scientist

- *Philippe SPALART*



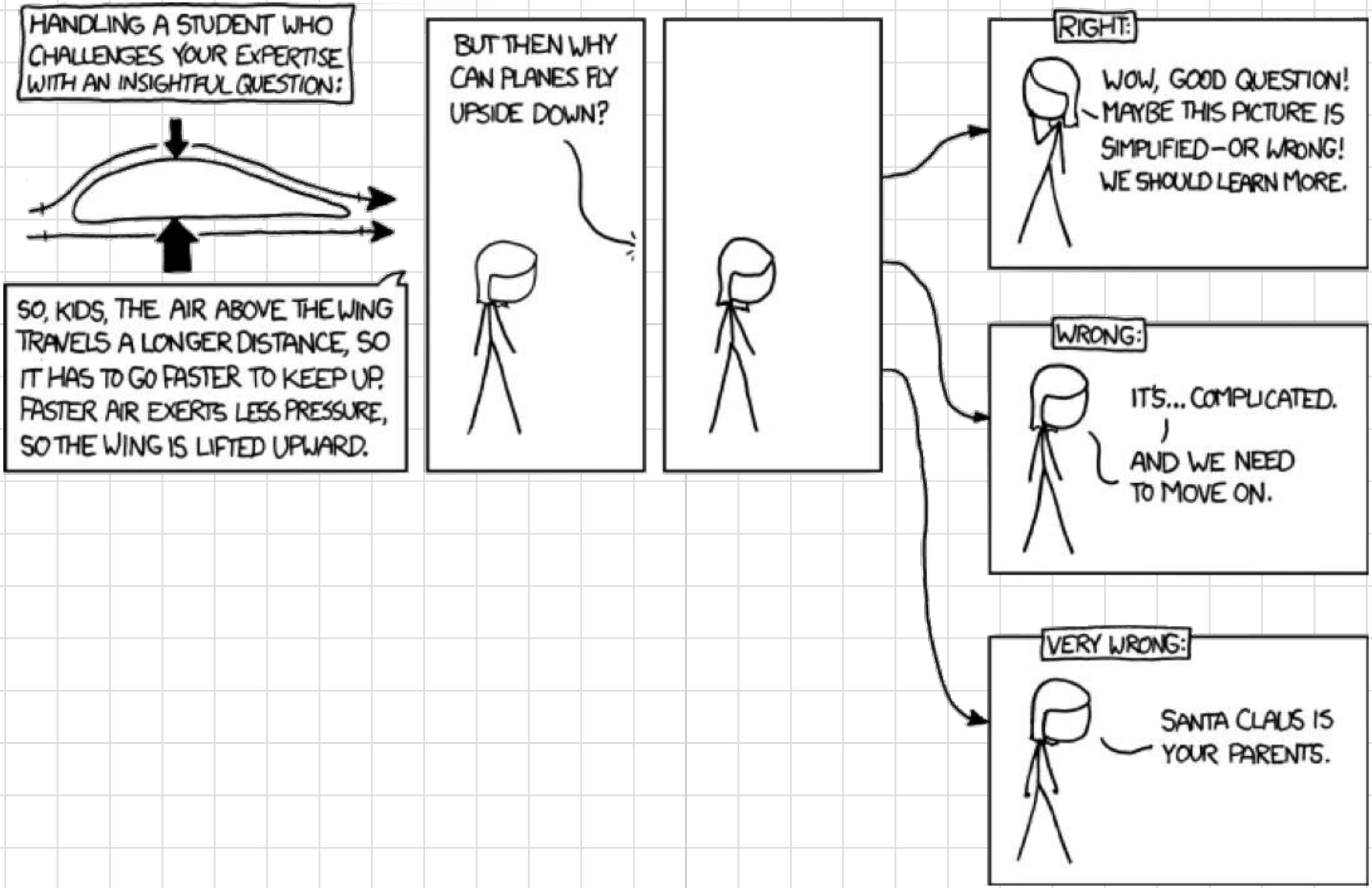
Explaining lift

Can you explain the magic?



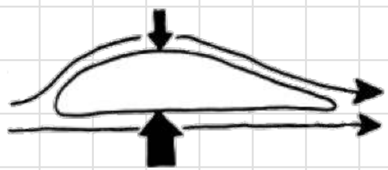
Explaining lift

Misconceptions

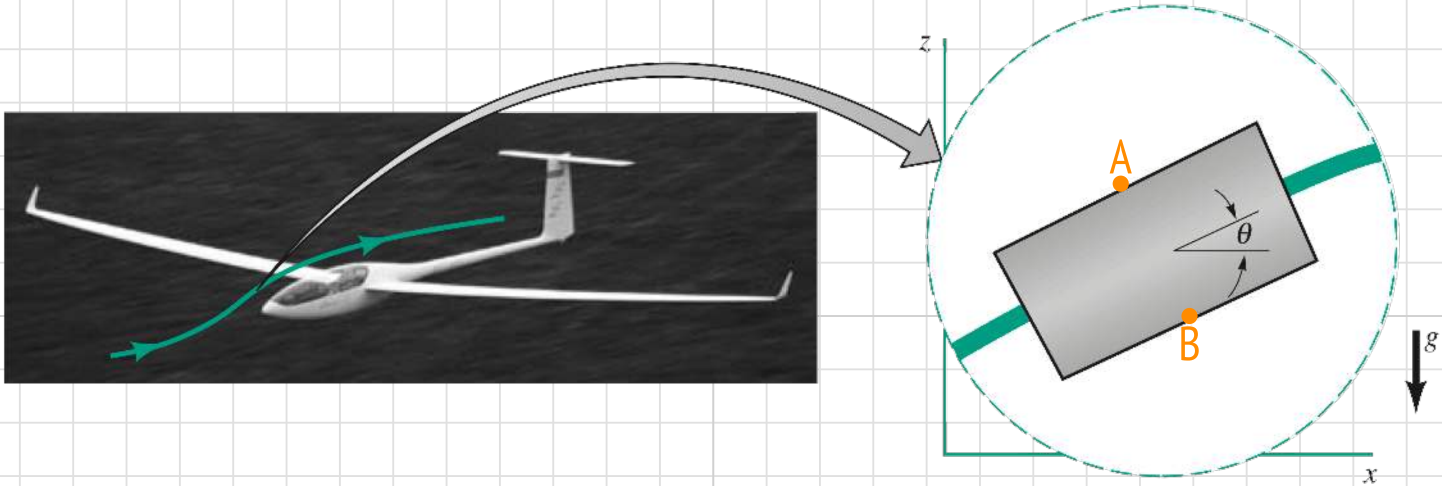


Explaining lift

Misconceptions



Explaining lift correctly



Which statement about the pressure across a curved streamline is correct?

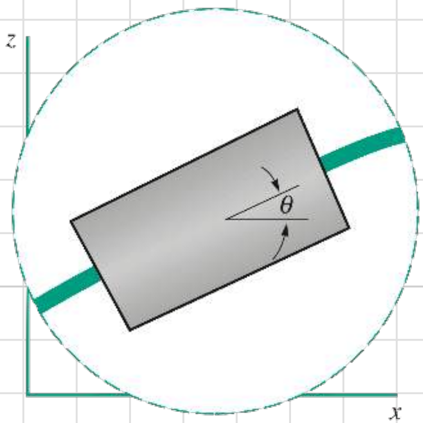
(A) $p_A < p_B$

(B) $p_A = p_B$

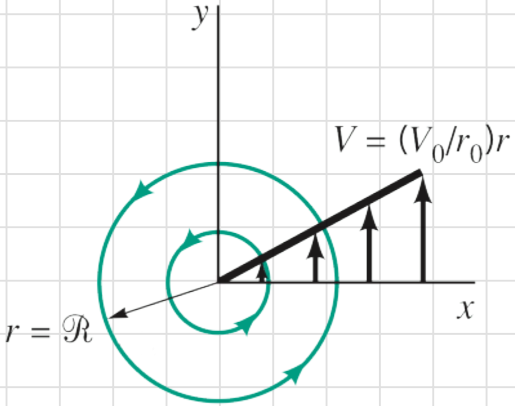
(C) $p_A > p_B$

Explaining lift correctly

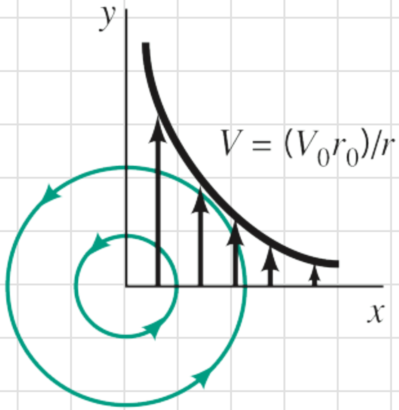
Force balance across a curved streamline



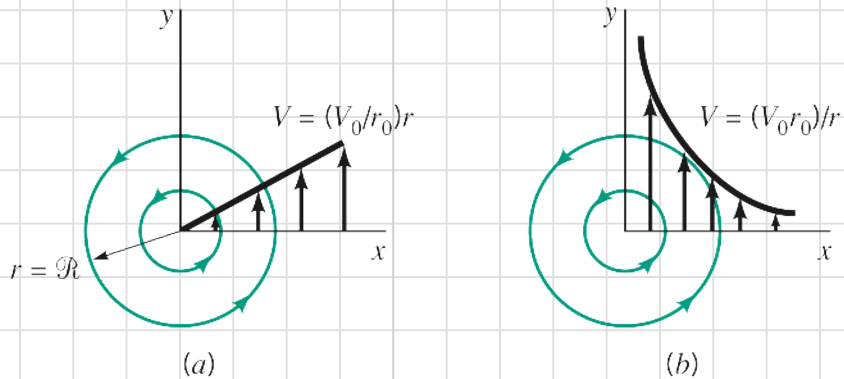
Curved streamlines

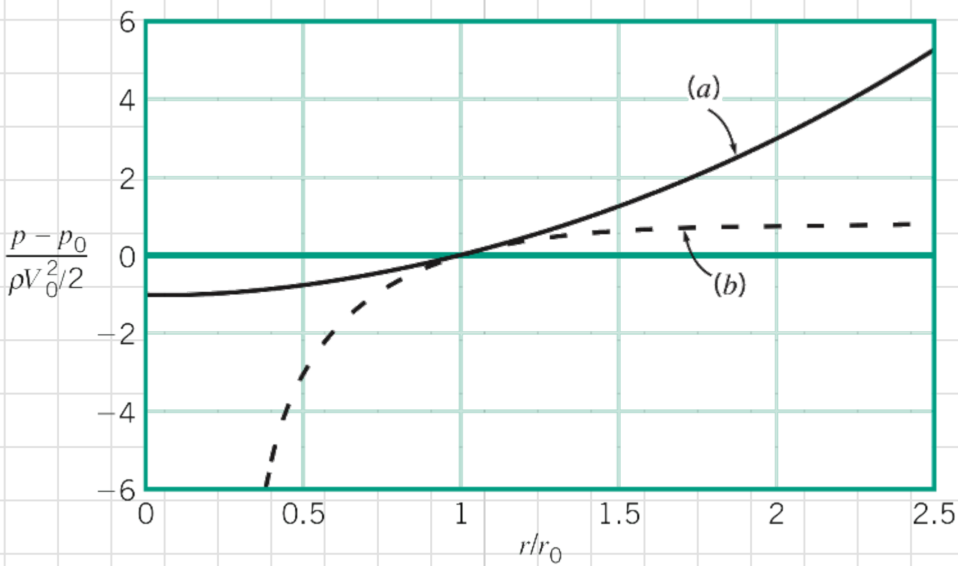


Curved streamlines

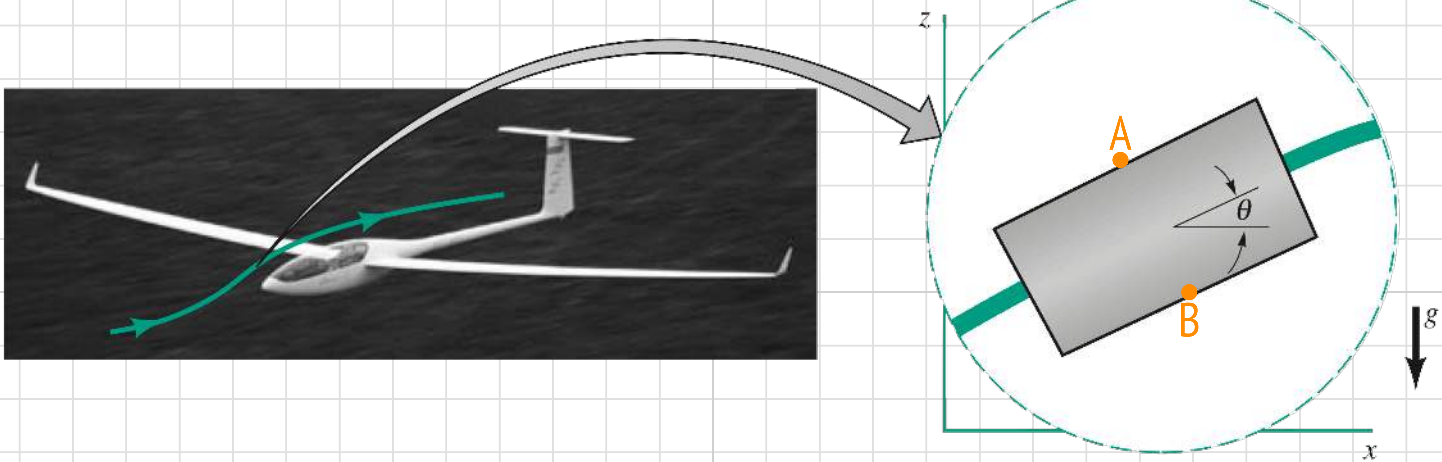


Curved streamlines





Curved streamlines



Which statement about the pressure across a curved streamline is correct?

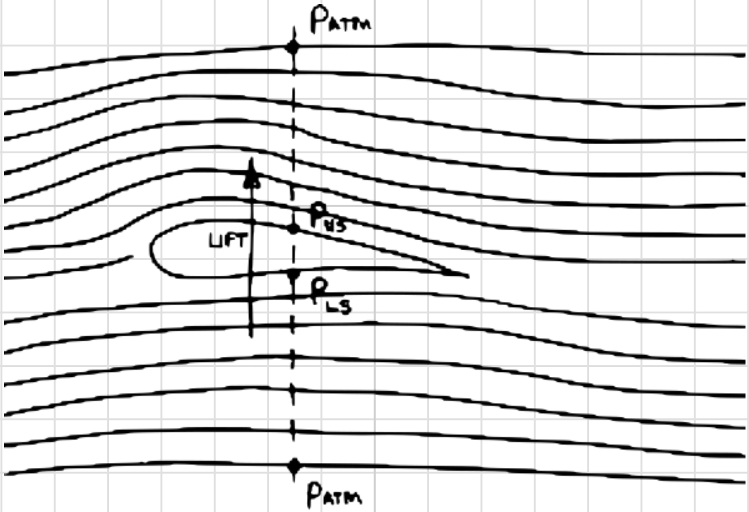
(A) $p_A < p_B$

(B) $p_A = p_B$

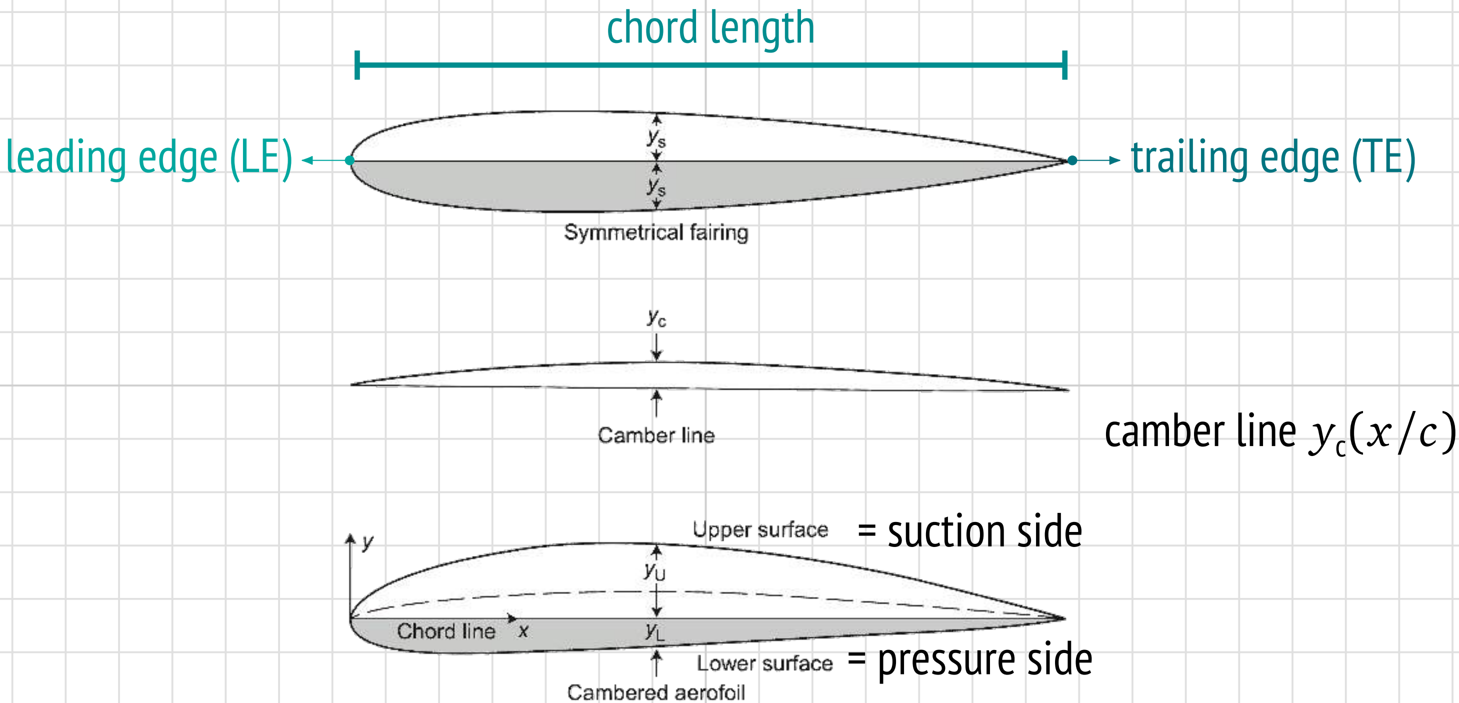
(C) $p_A > p_B$

Explaining lift correctly

Lift is due to flow curvature



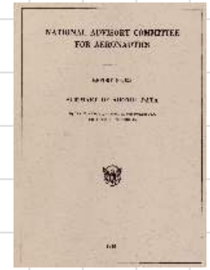
Airfoil nomenclature



$$y_c(x/c) = y_u(x/c) - t(x/c)/2 = y_l(x/c) + t(x/c)/2$$

with $t(x/c)$ the thickness of the airfoil along the chord (c)

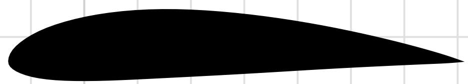
NACA 4-digit airfoil series



- A NACA XYTT has a maximum camber of $X\%$ located $(Y * 10)\%$ $(0.Y$ chords) from the leading edge with a maximum thickness of $TT\%$ of the chord. Four-digit series airfoils by default have maximum thickness at $30\,\%$ of the chord $(0.3$ chords) from the leading edge.

- A NACA0015 is a symmetric airfoil with no camber and a maximum thickness of $15\,\%$ of the chord.

- A NACA4415 has a maximum camber of $4\,\%$ located $40\,\%$ (0.4 chords) from the leading edge with a maximum thickness of $15\,\%$ of the chord.



Airfoil databases:
http://airfoiltools.com/
http://m-selig.ae.illinois.edu/ads/coord_database.html
https://wind.nrel.gov/airfoils/

Aerodynamic forces and moments on airfoils

R = resultant force ☐

N = normal & A = axial force ☐

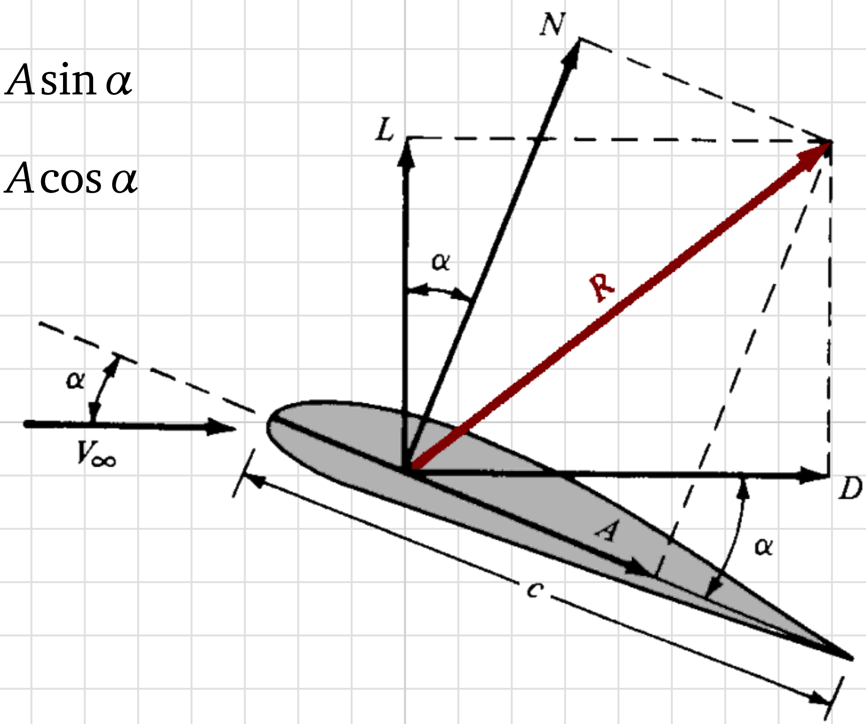
L = lift & D = drag ☐

α = angle of attack ☐

U_∞ = freestream velocity ☐

$$L = N\cos\alpha - A\sin\alpha$$

$$D = N\sin\alpha + A\cos\alpha$$



Convention:

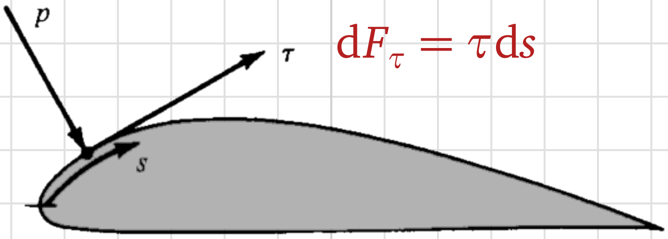
2D: $C_l = \dfrac{L}{\frac{1}{2}\rho U_\infty^2 c}$ with L lift per unit span $\qquad C_m = \dfrac{M}{\frac{1}{2}\rho U_\infty^2 c^2}$

3D: $C_L = \dfrac{L}{\frac{1}{2}\rho U_\infty^2 S}$ with S the wing's surface $\qquad C_M = \dfrac{M}{\frac{1}{2}\rho U_\infty^2 S c}$

Sources of aerodynamic forces

- pressure distribution $p(x)$
- wall shear stress distribution $\tau(x)$

$\mathrm{d}F_{\mathrm{p}} = p\mathrm{d}s$



$\mathrm{d}F_{\tau} = \tau\mathrm{d}s$

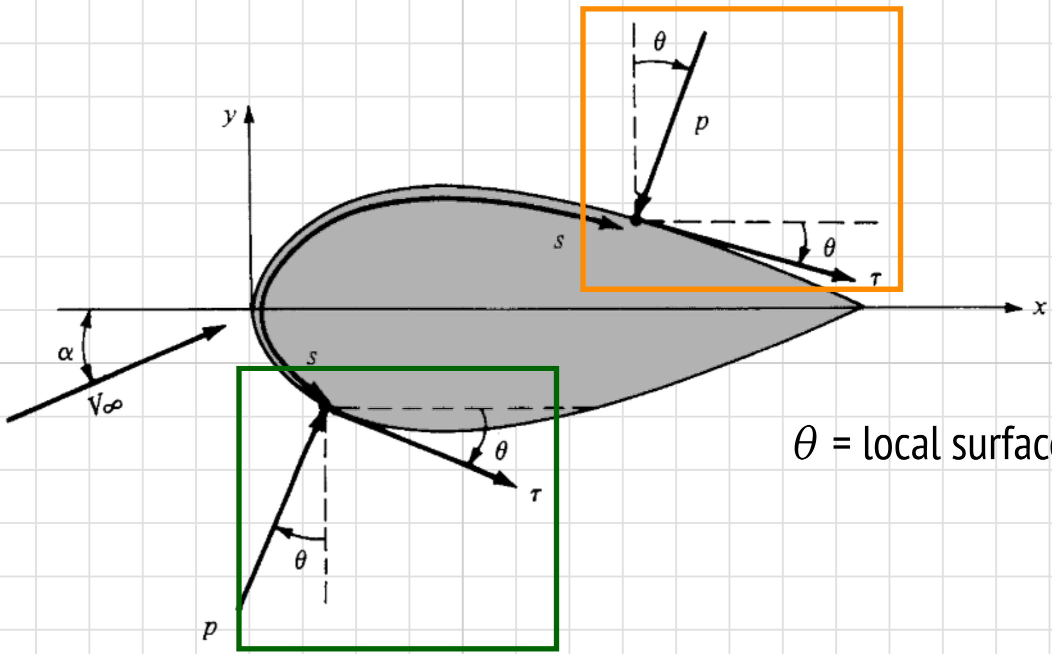
We will often use a low α approximation:

- $\cos\alpha \approx 1$
- $\sin\alpha \approx 0$ or $\sin\alpha \approx \alpha$

For $\alpha = 20° = 0.34906$:

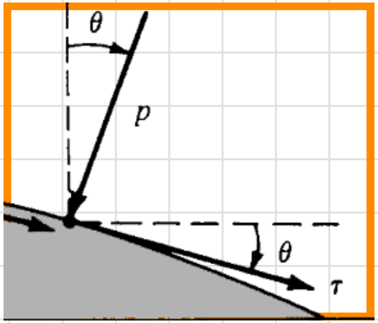
- $\cos\alpha = 0.93969$
- $\sin\alpha = 0.34202$

Calculating aerodynamic forces



θ = local surface curvature

Calculating aerodynamic forces on the suction side (ss)



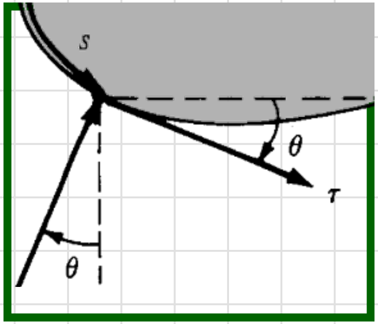
Chord-normal projections of the pressure and shear stress force components:

$$\mathrm{d}N_{\mathrm{ss}} = -|\mathrm{d}F_{\mathrm{p,N}}| - |\mathrm{d}F_{\tau,\mathrm{N}}| = -p_{\mathrm{ss}}\mathrm{d}s\cos\theta - \tau_{\mathrm{ss}}\mathrm{d}s\sin\theta$$

Axial projections of the pressure and shear stress force components:

$$\mathrm{d}A_{\mathrm{ss}} = -|\mathrm{d}F_{\mathrm{p,A}}| + |\mathrm{d}F_{\tau,\mathrm{A}}| = -p_{\mathrm{ss}}\mathrm{d}s\sin\theta + \tau_{\mathrm{ss}}\mathrm{d}s\cos\theta$$

Calculating aerodynamic forces on the pressure side (ps)



Chord-normal projections of the pressure and shear stress force components:

$$\mathrm{d}N_{\mathrm{ps}} = |\mathrm{d}F_{\mathrm{p,N}}| - |\mathrm{d}F_{\tau,\mathrm{N}}| = p_{\mathrm{ps}}\mathrm{d}s\cos\theta - \tau_{\mathrm{ps}}\mathrm{d}s\sin\theta$$

Axial projections of the pressure and shear stress force components:

$$\mathrm{d}A_{\mathrm{ps}} = |\mathrm{d}F_{\mathrm{p,A}}| + |\mathrm{d}F_{\tau,\mathrm{A}}| = p_{\mathrm{ps}}\mathrm{d}s\sin\theta + \tau_{\mathrm{ps}}\mathrm{d}s\cos\theta$$

Calculating aerodynamic forces

$$N = \int_{LE}^{TE} \mathrm{d}N_{\mathrm{ss}} + \int_{LE}^{TE} \mathrm{d}N_{\mathrm{ps}}$$

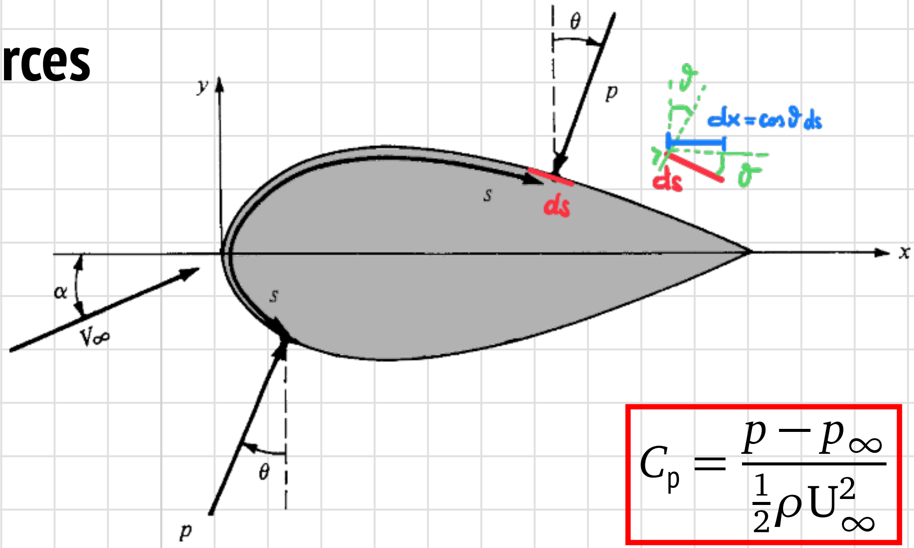
$$\int_{LE}^{TE} (-p_{\mathrm{ss}} \cos\theta - \tau_{\mathrm{ss}} \sin\theta)\, \mathrm{d}s_{\mathrm{ss}} + \int_{LE}^{TE} \left(p_{\mathrm{ps}} \cos\theta - \tau_{\mathrm{ps}} \sin\theta\right) \mathrm{d}s_{\mathrm{ps}}$$

$$A = \int_{LE}^{TE} \mathrm{d}A_{\mathrm{ss}} + \int_{LE}^{TE} \mathrm{d}A_{\mathrm{ps}}$$

$$\int_{LE}^{TE} (-p_{\mathrm{ss}} \sin\theta + \tau_{\mathrm{ss}} \cos\theta)\, \mathrm{d}s_{\mathrm{ss}} + \int_{LE}^{TE} \left(p_{\mathrm{ps}} \sin\theta - \tau_{\mathrm{ps}} \cos\theta\right) \mathrm{d}s_{\mathrm{ps}}$$

Calculating aerodynamic forces

from pressure distribution only



$$\boxed{C_{\mathrm{p}} = \frac{p - p_\infty}{\frac{1}{2}\rho U_\infty^2}}$$

$$N = \int_{LE}^{TE} -p_{\mathrm{ss}} \cos\theta \, \mathrm{d}s_{\mathrm{ss}} + \int_{LE}^{TE} p_{\mathrm{ps}} \cos\theta \, \mathrm{d}s_{\mathrm{ps}}$$

$$\frac{N}{\frac{1}{2}\rho U_\infty^2 c} = \int_{LE}^{TE} \frac{(p_\infty - p_{\mathrm{ss}})}{\frac{1}{2}\rho U_\infty^2} \mathrm{d}x/c + \int_{LE}^{TE} \frac{(p_{\mathrm{ps}} - p_\infty)}{\frac{1}{2}\rho U_\infty^2} \mathrm{d}x/c$$

$$C_{\mathrm{n}} = \int_0^1 -C_{\mathrm{p,ss}} \mathrm{d}\left(\frac{x}{c}\right) + \int_0^1 C_{\mathrm{p,ps}} \mathrm{d}\left(\frac{x}{c}\right) = \int_0^1 \left(C_{\mathrm{p,ps}} - C_{\mathrm{p,ss}}\right) \mathrm{d}\left(\frac{x}{c}\right)$$

Calculating aerodynamic moments about the leading edge M_{LE}

sign convention:

$M < 0$: nose-down pitching moment
$M > 0$: nose-up pitching moment

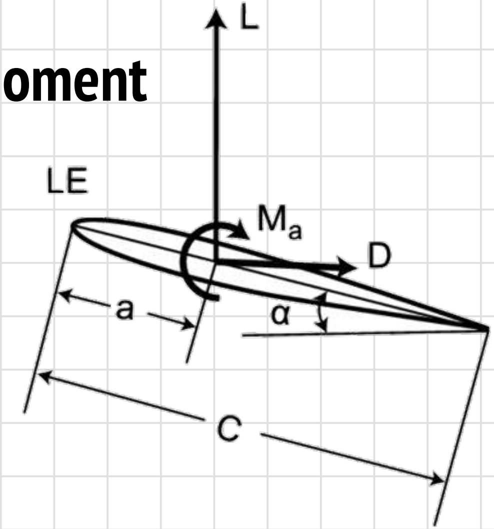
$$
\begin{aligned}
M_{\mathrm{LE}} &= \text{moment around the origin} \\
&= \int_{LE}^{TE} x\,\mathrm{d}N_{\mathrm{ss}} - \int_{LE}^{TE} x\,\mathrm{d}N_{\mathrm{ps}} - \int_{LE}^{TE} y\,\mathrm{d}A_{\mathrm{ss}} - \int_{LE}^{TE} y\,\mathrm{d}A_{\mathrm{ps}} \\
&= \int_{LE}^{TE} \left(x p_{\mathrm{ss}} \cos\theta\,\mathrm{d}s_{\mathrm{ss}} - x p_{\mathrm{ps}} \cos\theta\,\mathrm{d}s_{\mathrm{ps}} - y p_{\mathrm{ss}} \sin\theta\,\mathrm{d}s_{\mathrm{ss}} - y p_{\mathrm{ps}} \sin\theta\,\mathrm{d}s_{\mathrm{ps}} \right)
\end{aligned}
$$

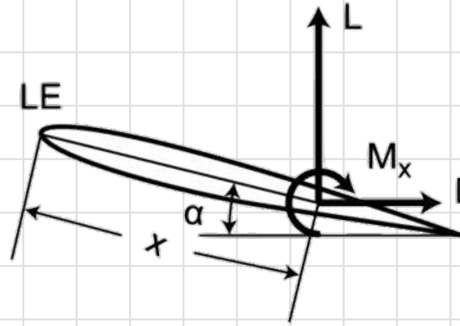
for most airfoil shapes or streamlined objects: $\sin\theta \ll \cos\theta$

$$
\Rightarrow \frac{M_{\mathrm{LE}}}{\frac{1}{2}\rho \mathrm{U}_\infty^2 c^2} = \int_{LE}^{TE} \frac{(p_{\mathrm{ss}} - p_\infty)x\,\mathrm{d}x}{\frac{1}{2}\rho \mathrm{U}_\infty^2 c^2} - \frac{(p_{\mathrm{ps}} - p_\infty)x\,\mathrm{d}x}{\frac{1}{2}\rho \mathrm{U}_\infty^2 c^2}
$$

$$
C_{\mathrm{m,LE}} = \int_0^1 \left(C_{\mathrm{p,ss}} - C_{\mathrm{p,ps}}\right) \frac{x}{c}\,\mathrm{d}\left(\frac{x}{c}\right)
$$

Pitching moment





The pitching motion is measured about a reference point.

To convert from one reference point a to another reference point x:

$$M_{\mathrm{LE}} = M_{\mathrm{a}} - aL\cos\alpha - aD\sin\alpha = M_{\mathrm{x}} - xL\cos\alpha - xD\sin\alpha$$

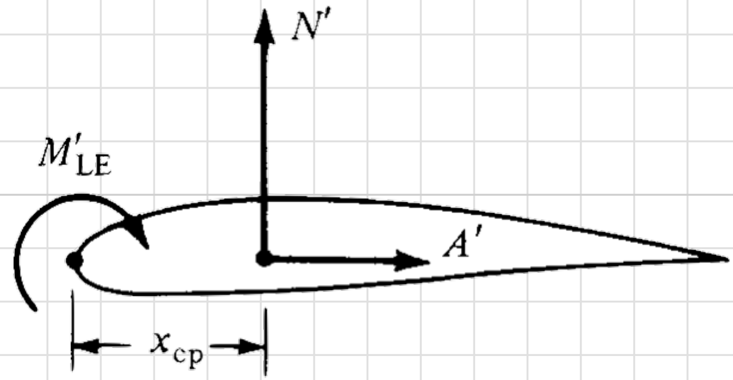
$$\Rightarrow M_{\mathrm{x}} = M_{\mathrm{a}} - (a-x)(L\cos\alpha + D\sin\alpha) = M_{\mathrm{a}} - (a-x)N$$

$$\Rightarrow c_{\mathrm{m,x}} = c_{\mathrm{m,a}} - \left(\frac{a}{c} - \frac{x}{c}\right)c_{\mathrm{n}}$$

Centre of pressure

Centre of pressure = the location where the resultant of a distributed load effectively acts on the body

General: $\boxed{M_{\mathrm{x}} = M_{\mathrm{a}} - (a - x)N}$



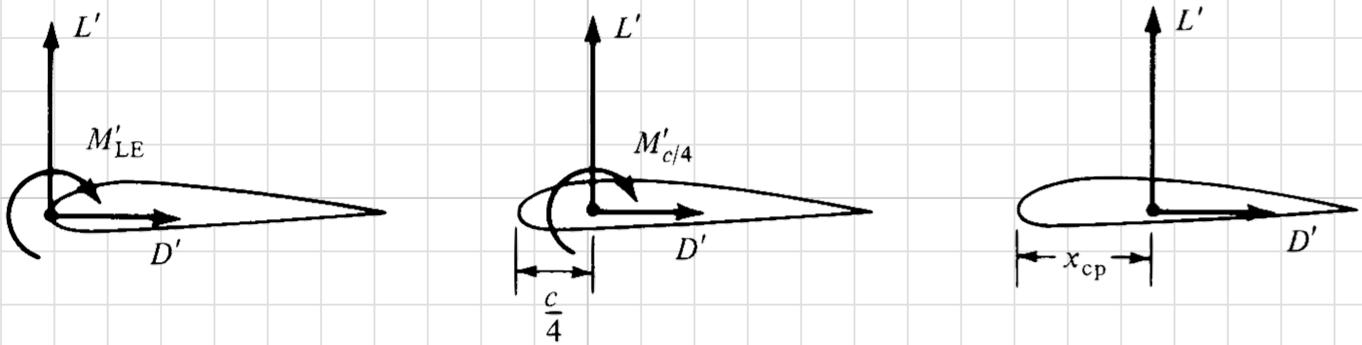
Special case: a=0, x=x_{cp}

$$M_{\mathrm{x}} = 0 = M_{\mathrm{LE}} + x_{\mathrm{cp}} N$$

$$\boxed{x_{\mathrm{cp}} = -\frac{M_{\mathrm{LE}}}{N}}$$

Centre of pressure

Centre of pressure = the location where the resultant of a distributed load effectively acts on the body



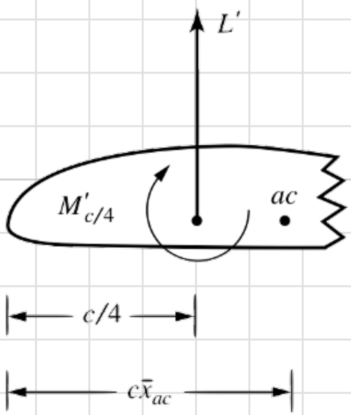
$$c_{m,x} = c_{m,a} - (c_l \cos\alpha + c_d \sin\alpha)\left(\frac{a}{c} - \frac{x}{c}\right)$$

$$x_{cp} = -\frac{M_{LE}}{N}$$

Centre of pressure = the location on the body about which the aerodynamic moment is zero.

Aerodynamic centre

Aerodynamic centre = location on the airfoil about which the aerodynamic moment is independent of angle of attack.



General: $$\boxed{c_{\mathrm{m,x}} = c_{\mathrm{m,a}} - \left(\frac{a}{c} - \frac{x}{c}\right) c_{\mathrm{n}}}$$

Now take: $a/c = 0, \quad x/c = \mathrm{ac}$

$$\Rightarrow \quad c_{\mathrm{m,ac}} = c_{\mathrm{m,LE}} + ac\, c_{\mathrm{n}}$$

$$\Rightarrow \quad 0 = \frac{\mathrm{d}c_{\mathrm{m,ac}}}{\mathrm{d}\alpha} = \frac{\mathrm{d}c_{\mathrm{m,LE}}}{\mathrm{d}\alpha} + ac\,\frac{\mathrm{d}c_{\mathrm{n}}}{\mathrm{d}\alpha}$$

$$\Rightarrow \quad ac = \frac{-\frac{\mathrm{d}c_{\mathrm{m,LE}}}{\mathrm{d}\alpha}}{\frac{\mathrm{d}c_{\mathrm{n}}}{\mathrm{d}\alpha}}$$