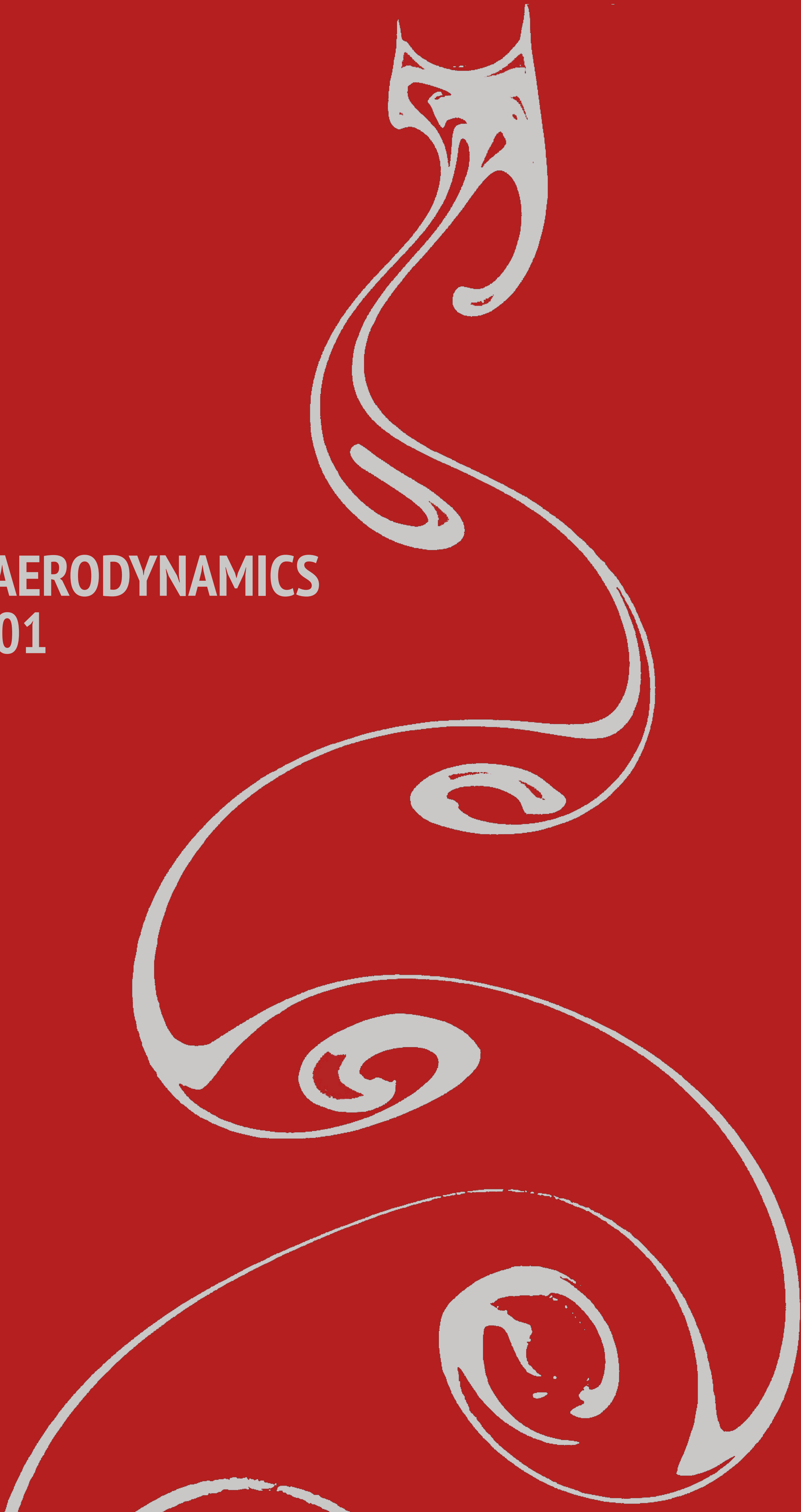


ME-445 AERODYNAMICS
Exercise 01
Week 1

Formula sheet

Cylindrical coordinates

$$\nabla \vec{u} = \left(\frac{\partial v_r}{\partial r}, \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}, 0 \right)$$

$$\nabla \cdot \vec{u} = \frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}$$

$$\nabla \times \vec{u} = \left(0, 0, \frac{1}{r}\left[\frac{\partial (r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta} \right] \right)$$

Potential flow

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

Uniform parallel flow $\quad w = \phi + i\psi = U_\infty e^{-i\alpha} z$

Potential vortex in z_0 $\quad w = -\dfrac{i\gamma}{2\pi} \ln(z - z_0)$

Point source or sink in z_0 $\quad w = \dfrac{Q}{2\pi} \ln(z - z_0)$

Source-sink doublet in z_0 $\quad w = \dfrac{\mu}{2\pi(z - z_0)}$

$$\frac{\mathrm{d}w}{\mathrm{d}z} = u - iv$$

Milne-Thomson circle theorem:

$$g(z) = w(z) + \overline{w\left(\frac{a^2}{\bar{z}}\right)}$$

Thin airfoil theory

For a camber line with:

$$\frac{\mathrm{d}y_c}{\mathrm{d}x} = A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta$$

$$\frac{x}{c} = \frac{(1 - \cos\theta)}{2}$$

we know:

$$k = 2U_\infty \left[(\alpha - A_0)\frac{\cos\theta + 1}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta \right]$$

$$A_0 = \frac{1}{\pi}\int_0^\pi \frac{\mathrm{d}y_c}{\mathrm{d}x}\mathrm{d}\theta$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{\mathrm{d}y_c}{\mathrm{d}x}\cos n\theta \mathrm{d}\theta$$

$$C_l = 2\pi\alpha + \pi(A_1 - 2A_0)$$

$$C_{m,1/4} = -\frac{\pi}{4}(A_1 - A_2)$$

$$x_{cp} = \frac{1}{4} + \frac{\pi}{4C_l}(A_1 - A_2)$$

Finite wings with AR=b^2/S

Sign convention:

if induced velocity points downward: $w(y) > 0, \alpha_i(y) > 0$

if induced velocity points upward: $w < 0, \alpha_i < 0$

Prandtl's lifting-line theory

$$U_\infty \alpha_i(y_0) = w(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2} \frac{(\mathrm{d}\Gamma/\mathrm{d}y)}{y - y_0}\mathrm{d}y$$

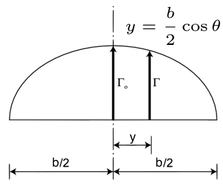
$$\alpha(y_0) = \alpha_{\text{eff}}(y_0) + \alpha_i(y_0)$$

Elliptical loading $\Gamma(y) = \Gamma_0\sqrt{1 - \left(\dfrac{2y}{b}\right)^2}$

$$w = \frac{\Gamma_0}{2b}$$

$$\alpha_i = \frac{C_L}{\pi \text{AR}}$$

$$C_{D,i} = \frac{C_L^2}{\pi \text{AR}}$$



General loading $\Gamma(\theta) = 2bU_\infty \sum_{n=1}^{\infty} A_n \sin n\theta$

$$w(\theta) = U_\infty \sum_{n=1}^{\infty} n A_n \frac{\sin n\theta}{\sin\theta}$$

$$C_L = \pi A_1 \text{AR}$$

$$C_{D,i} = \frac{C_L^2}{\pi \text{AR}}(1 + \delta) \text{ with } \delta = \sum_{n=2}^{\infty} n\,(A_n/A_1)^2$$

Boundary Layer

Flat plate **laminar** boundary layer

$\dfrac{\delta}{x} = \dfrac{5}{\sqrt{Re_x}}$ boundary layer growth

$C_f = \dfrac{1.328}{\sqrt{Re_x}}$ skin friction drag coefficient

Flat plate **turbulent** boundary layer

$\dfrac{\delta}{x} = \dfrac{0.37}{Re_x^{1/5}}$ boundary layer growth

$C_f = \dfrac{0.074}{Re_x^{1/5}}$ skin friction drag coefficient

Miscellanous

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

water

kinematic viscosity $\nu = 1 \times 10^{-6}\,\mathrm{m}^2\,\mathrm{s}^{-1}$

density $\rho = 1000\,\mathrm{kg}\,\mathrm{m}^{-3}$

air

kinematic viscosity $\nu = 1.5 \times 10^{-5}\,\mathrm{m}^2\,\mathrm{s}^{-1}$

density $\rho = 1.2\,\mathrm{kg}\,\mathrm{m}^{-3}$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$
$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\int_0^\pi \cos\theta \mathrm{d}\theta = 0$$
$$\int_0^\pi \sin\theta \mathrm{d}\theta = 2$$
$$\int_0^\pi \cos^2\theta \mathrm{d}\theta = \int_0^\pi \sin^2\theta \mathrm{d}\theta = \frac{\pi}{2}$$

$$\int_0^\pi \frac{\cos n\theta}{\cos\theta - \cos\theta_1}\mathrm{d}\theta = \pi\frac{\sin n\theta_1}{\sin\theta_1} \qquad n = 0, 1, 2, \ldots$$
$$\int_0^\pi \frac{\sin n\theta \sin\theta}{\cos\theta - \cos\theta_1}\mathrm{d}\theta = -\pi\cos n\theta_1 \qquad n = 1, 2, 3, \ldots$$

1. A vertical axis wind turbine (VAWT) model has to be designed to be tested in a water channel. The VAWT has a radius R and N_b blades with a chord length c. The model will be tested in a uniform flow of density ρ_{m} and free stream velocity $\mathrm{U}_{\infty\mathrm{m}}$ and reach a rotational velocity Ω_{m}. The goal of the model VAWT experiments is to measure its aerodynamic power P.

Use dimensional analysis to formulate the problem using non-dimensional parameters only.

(a) Identify the relevant dimensional variables influencing the aerodynamic power P produced by a VAWT.

(b) Outline the dimensions for each variable. How many non-dimensional groups (Π-groups) do you need to reformulate the problem?

(c) Find all the non-dimensional groups you need and name the ones you recognise. Use (ρ, U_∞, R) as the basis set.

(d) The number of blades is an important parameter in the design of VAWT and propellors and rotors in general. The effect of the number of blades on the aerodynamic performance will depend on the size of the blades and the rotor itself. What matters in not solely the number of blades but the ratio between the area covered by the blades and the overall rotor area. This is expressed by the rotor solidity σ. Based on this explanation, how would you define the rotor solidity?

(e) The tip-speed ratio $\lambda = \dfrac{\omega R}{\mathrm{U}_\infty}$ strongly influences the VAWT power performance. We wish to test the power performance of a 3-bladed VAWT of radius $R_o = 5\,\mathrm{m}$ and $\sigma_o = 0.36$ which will be commissioned in an area where wind blows on average at $5\,\mathrm{m\,s^{-1}}$ and $15°$ Celsius. The test will be carried out in a water channel using a $1/10$ scaled down 3-bladed model operating at optimal power production conditions where $\lambda_m = 3$.

Based on the data given below, what should be the water channel testing flow velocity $U_{\infty,\mathrm{m}}$?

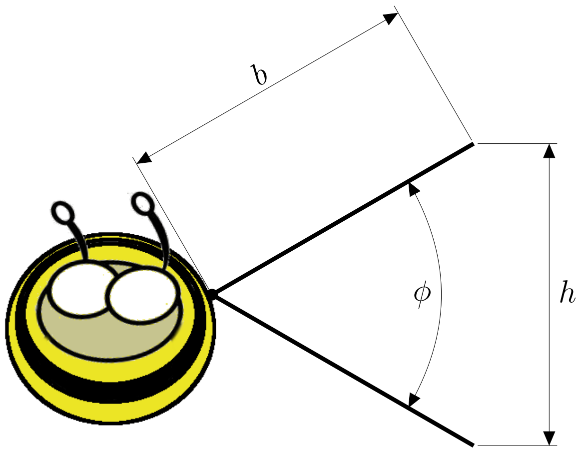
	Air	Water
Temperature	$15°$ Celsius	$20°$ Celsius
Density	$1.225\,\mathrm{kg\,m^{-3}}$	$998\,\mathrm{kg\,m^{-3}}$
Dynamic viscosity	$1.80 \times 10^{-5}\,\mathrm{kg\,m^{-1}\,s^{-1}}$	$1.02 \times 10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}$

(f) Based on the data given above, what should be the testing VAWT rotational speed?

(g) The same $1/10$ scaled model is now to be tested in an wind tunnel operating at T = 15°C. What should be the wind tunnel testing flow velocity and rotational speed?

(h) What are the advantages and/or disadvantages of using a water channel over a wind tunnel?

2. You want to build a flapping wing micro air vehicle modeled after the honey bee. Its wings can be described by a span b and a mean chord c, flapping at a frequency f with an angular amplitude ϕ or peak-to-peak amplitude h. The forward flight velocity U_∞ is determined by the resulting aerodynamic forces F acting on the bee and are dependent on the fluid characteristic parameters, such as the density ρ and dynamic viscosity μ. To assess the aerodynamic performance of the flapping wing system the force coefficient C_F has to be calculated.



(a) Determine the dimensional parameters describing the flapping wing system of the honey bee.

(b) How many non dimensional groups are needed to define the system?

(c) Find all the non-dimensional groups you need and name the ones you recognise. Use (ρ, U_∞, c) as the basis set.

(d) After studying the flight of multiple specimen you have determined the geometry and flight kinematics of the insect. The geometry of the bee's wings is given by the chord length $c_\text{o} = 3\,\text{mm}$ and the span $b_\text{o} = 10\,\text{mm}$. Observing the flapping wings you determine its flapping frequency $f_\text{o} = 240\,\text{Hz}$ and stroke amplitude $\phi_\text{o} = 90°$. The honey bee is flying forward with an average velocity $U_{\infty,o} = 8\,\text{m}\,\text{s}^{-1}$ producing a total force of $F_\text{o} = 1.38 \times 10^{-3}\,\text{N}$. To study the aerodynamic forces on the flapping wings you want to design an experiment in a water channel which lets you further observe its flight capabilities.

For the new scaled wing you chose a chord length $c_\text{m} = 0.012\,\text{m}$. Calculate the new span b_m from the corresponding Π-group of the observed honey bee dimensions.

(e) Calculate the new peak-to-peak amplitude h_m of the model.

(f) What needs to be the velocity $U_{\infty,\mathrm{m}}$ for the water channel?

(g) What is the new flapping frequency f_{m}?

(h) What trend in the dimensional values of the scaled system can you observe? What could be the advantage of conducting the experiment in water instead of air?

(i) You have the brilliant idea to design robotic bees to explore the Mars surface. To achieve the minimum amount of lift the wings need to have a span of at least $b_{\text{Mars}} = 0.1\,\text{m}$. Assuming the Mars atmosphere near the surface with $\rho_{\text{Mars}} = 1.50 \times 10^{-2}\,\text{kg}\,\text{m}^{-3}$ and $\mu_{\text{Mars}} = 1.422 \times 10^{-5}\,\text{kg}\,\text{m}^{-1}\,\text{s}$, what is the forward flight velocity $U_{\infty,\text{Mars}}$ your robot bees can reach while respecting the same characteristic flow parameters?