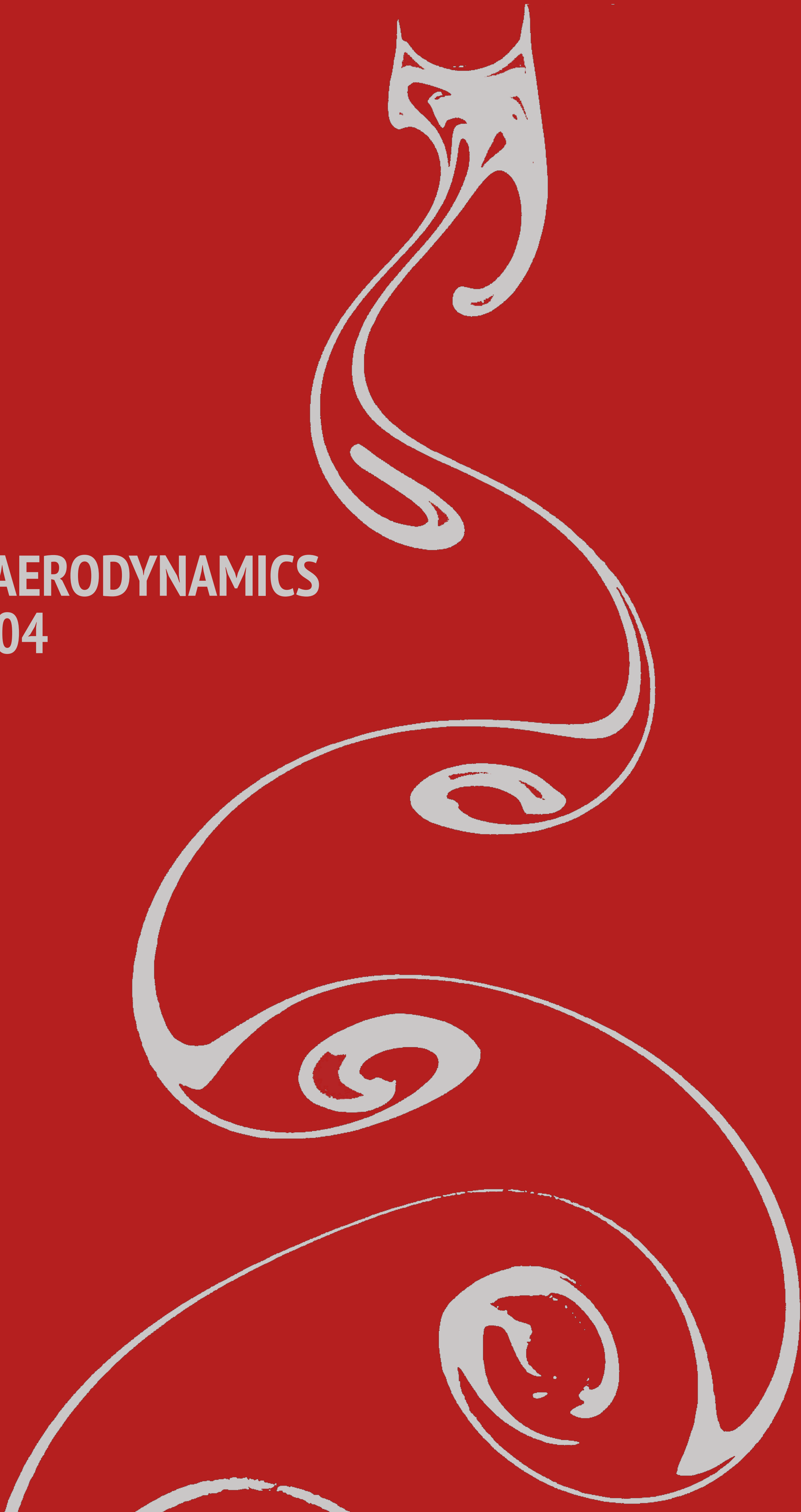


ME-445 AERODYNAMICS
Exercise 04
Week 3

Formula sheet

Cylindrical coordinates

$$\nabla \vec{u} = \left(\frac{\partial v_r}{\partial r}, \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}, 0 \right)$$

$$\nabla \cdot \vec{u} = \frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}$$

$$\nabla \times \vec{u} = \left(0, 0, \frac{1}{r}\left[\frac{\partial (r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta} \right] \right)$$

Potential flow

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

Uniform parallel flow $\quad w = \phi + i\psi = \mathrm{U}_\infty e^{-i\alpha} z$

Potential vortex in z_0 $\quad w = -\frac{i\gamma}{2\pi}\ln(z - z_0)$

Point source or sink in z_0 $\quad w = \frac{Q}{2\pi}\ln(z - z_0)$

Source-sink doublet in z_0 $\quad w = \frac{\mu}{2\pi(z - z_0)}$

$$\frac{\mathrm{d}w}{\mathrm{d}z} = u - iv$$

Milne-Thomson circle theorem:

$$g(z) = w(z) + \overline{w\left(\frac{a^2}{\bar{z}}\right)}$$

Thin airfoil theory

For a camber line with:

$$\frac{\mathrm{d}y_c}{\mathrm{d}x} = A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta$$

$$\frac{x}{c} = \frac{(1 - \cos\theta)}{2}$$

we know:

$$k = 2\mathrm{U}_\infty \left[(\alpha - A_0)\frac{\cos\theta + 1}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta \right]$$

$$A_0 = \frac{1}{\pi}\int_0^\pi \frac{\mathrm{d}y_c}{\mathrm{d}x}\mathrm{d}\theta$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{\mathrm{d}y_c}{\mathrm{d}x}\cos n\theta \mathrm{d}\theta$$

$$C_l = 2\pi\alpha + \pi(A_1 - 2A_0)$$

$$C_{m,1/4} = -\frac{\pi}{4}(A_1 - A_2)$$

$$x_{cp} = \frac{1}{4} + \frac{\pi}{4C_l}(A_1 - A_2)$$

Finite wings with AR=b^2/S

Sign convention:

if induced velocity points downward: $w(y) > 0$, $\alpha_i(y) > 0$

if induced velocity points upward: $w < 0$, $\alpha_i < 0$

Prandtl's lifting-line theory

$$\mathrm{U}_\infty \alpha_i(y_0) = w(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2} \frac{(\mathrm{d}\Gamma/\mathrm{d}y)}{y - y_0}\mathrm{d}y$$

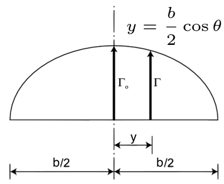
$$\alpha(y_0) = \alpha_{\text{eff}}(y_0) + \alpha_i(y_0)$$

Elliptical loading $\Gamma(y) = \Gamma_0\sqrt{1 - \left(\frac{2y}{b}\right)^2}$

$$w = \frac{\Gamma_0}{2b}$$

$$\alpha_i = \frac{C_L}{\pi \mathrm{AR}}$$

$$C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}$$



General loading $\Gamma(\theta) = 2b\mathrm{U}_\infty \sum_{n=1}^{\infty} A_n \sin n\theta$

$$w(\theta) = \mathrm{U}_\infty \sum_{n=1}^{\infty} nA_n \frac{\sin n\theta}{\sin\theta}$$

$$C_L = \pi A_1 \mathrm{AR}$$

$$C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}(1 + \delta) \text{ with } \delta = \sum_{n=2}^{\infty} n\left(A_n/A_1\right)^2$$

Boundary Layer

Flat plate **laminar** boundary layer

$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$ boundary layer growth

$C_f = \frac{1.328}{\sqrt{Re_x}}$ skin friction drag coefficient

Flat plate **turbulent** boundary layer

$\frac{\delta}{x} = \frac{0.37}{Re_x^{1/5}}$ boundary layer growth

$C_f = \frac{0.074}{Re_x^{1/5}}$ skin friction drag coefficient

Miscellanous

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

water

kinematic viscosity	$\nu = 1 \times 10^{-6}\,\mathrm{m^2\,s^{-1}}$
density	$\rho = 1000\,\mathrm{kg\,m^{-3}}$

air

kinematic viscosity	$\nu = 1.5 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$
density	$\rho = 1.2\,\mathrm{kg\,m^{-3}}$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$
$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\int_0^\pi \cos\theta \mathrm{d}\theta = 0$$
$$\int_0^\pi \sin\theta \mathrm{d}\theta = 2$$
$$\int_0^\pi \cos^2\theta \mathrm{d}\theta = \int_0^\pi \sin^2\theta \mathrm{d}\theta = \frac{\pi}{2}$$

$$\int_0^\pi \frac{\cos n\theta}{\cos\theta - \cos\theta_1}\mathrm{d}\theta = \pi\frac{\sin n\theta_1}{\sin\theta_1} \qquad n = 0, 1, 2, \ldots$$
$$\int_0^\pi \frac{\sin n\theta \sin\theta}{\cos\theta - \cos\theta_1}\mathrm{d}\theta = -\pi\cos n\theta_1 \qquad n = 1, 2, 3, \ldots$$

1. During the design and development of the McDonnell Douglas F/A-18 Hornet (Figure 1), several wind tunnel tests were conducted. A $16\,\%$ scaled down model was tested at low speed ($\mathrm{Ma}_\infty = 0.08$), providing measurements for the lift coefficient C_L as a function of the angle of attack α, the lift versus drag coefficient, and the coefficient of moment $C_{M,c/4}$ with respect to the $25\,\%$ mean chord as a function of α (Figure 2). Consider the final version of the airplane in horizontal flight with constant velocity U_∞ and assume that the model was properly scaled and dynamic similarity is assured. The F/A-18 has a maximum take-off mass $m = 23\,500\,\mathrm{kg}$, a wing area $S = 38\,\mathrm{m}^2$, a wingspan $b = 12.3\,\mathrm{m}$, a length $l = 17.1\,\mathrm{m}$, and each of its two engines provide a maximum thrust $T = 79.2\,\mathrm{kN}$. The $25\,\%$ mean chord is located at a distance from the front of $60\,\%$ of the airplane length. Assume that the air density at the flight altitude is $\rho = 1.23\,\mathrm{kg\,m^{-3}}$.

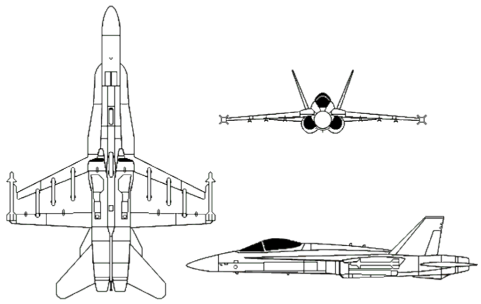


Figure 1: Schematic representation of the McDonnell Douglas F/A-18 Hornet

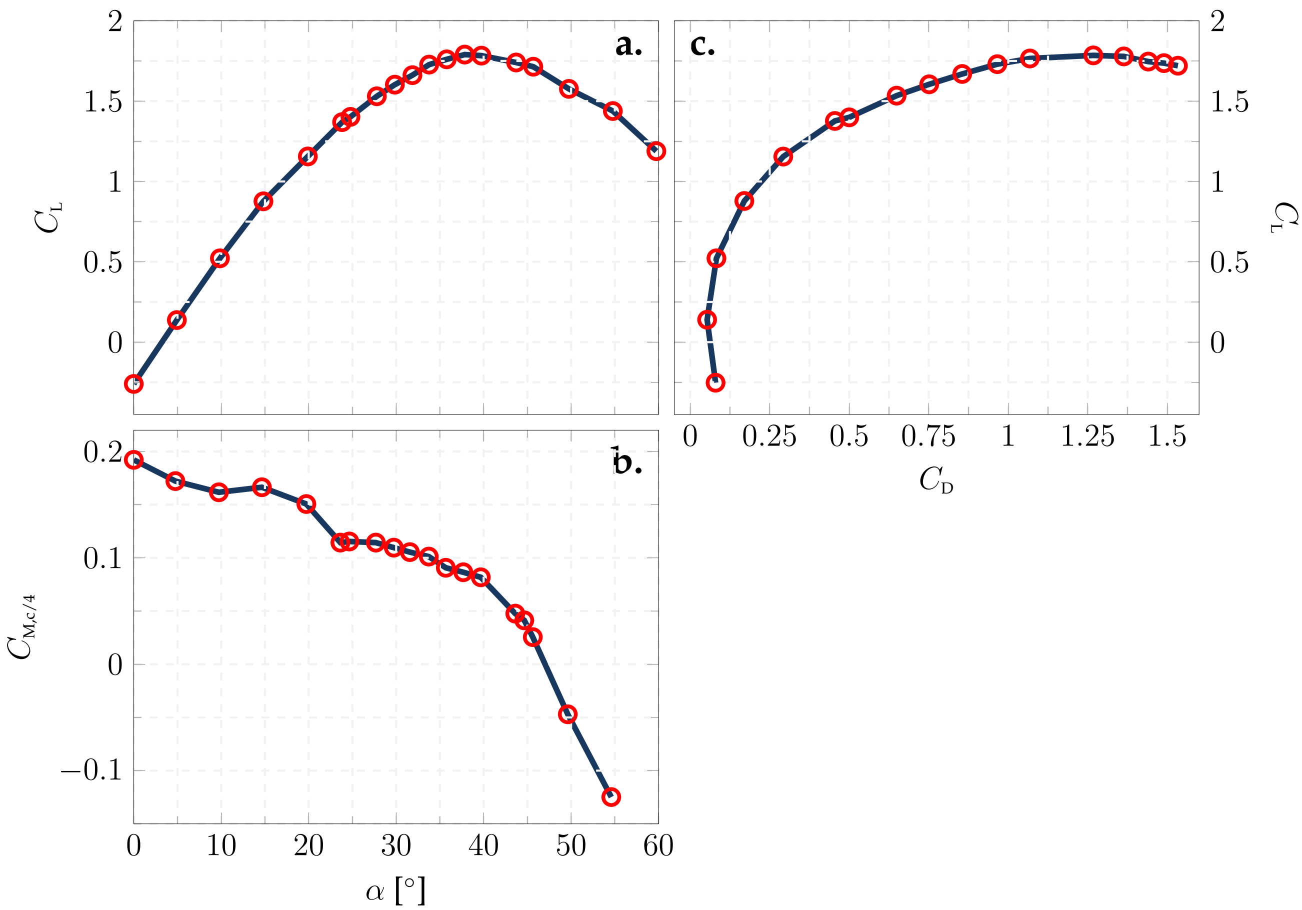


Figure 2: a.) Lift coefficient in function of the angle of attack, b.) pitching moment coefficient with respect to the $25\,\%$ mean chord, c.) lift versus drag coefficient.

(a) Retrieve the data from the graphs in Figure 2 and load them into Matlab.
Note: You can use `image_digitizer.m` *to digitize the data points from the plots or use the file* `F18Hornet.mat` *from Moodle directly.*

(b) Compute and plot drag coefficient C_D as a function of the angle of attack α. What is the highest C_D of the airfoil?
Note: The data points are spaced unequally and need to be interpolated onto the same α spacing before they can be multiplied with one another. Use `interp1(x,y,xq,'linear','extrap')` *to convert between the different graphs.*

(c) Compute and plot the aerodynamic efficiency η_{aero} in terms of the lift-to-drag ratio C_L/C_D. At which angle does the airplane have the highest C_L/C_D-ratio?

(d) Compute and plot the center of pressure relative to the length of the airplane x_{cp}/l as a function of the angle of attack α. Where is the center of pressure located for $\alpha = 0°$?

(e) Calculate the minimum steady flight speed, the stalling velocity u_{stall}, of the airplane. What is the value of the stall angle α_{stall}? Calculate the thrust needed to maintain flight at these conditions.

(f) Calculate the maximum velocity of the airplane. Determine the Mach number Ma, the angle of attack α, and the lift L at these flight conditions? Can you see any problems arising from these results?

(g) What is the angle of attack that maximises C_L/C_D? Calculate the lift-to-drag ratio, velocity, lift L, and drag D of the airplane.

(h) At an altitude of $h = 4000\,\mathrm{m}$ both engines are turned off ($T = 0\,\mathrm{N}$) and the airplane glides to the ground. Calculate the distance that the airplane is able to glide at maximum lift-to-drag ratio.

2. To analyse the aerodynamic performance of an airfoil wind tunnel experiments are conducted and pressure measurements for a range of angles α at different chordwise positions taken. A total of 36 pressure sensors is distributed over the surface of the airfoil (see Figure 3). The measurements were carried out for 52 angular positions of $\alpha = [-21°, 21°]$. The airfoil is a NACA0015 with a chord length of $c = 0.3\,\text{m}$. The incoming flow has a velocity of $U_\infty = 30\,\text{m}\,\text{s}^{-1}$ with a density of $\rho = 1.2\,\text{kg}\,\text{m}^{-3}$.

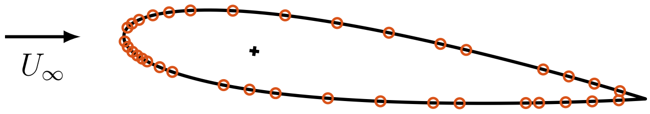


Figure 3: NACA0015 airfoil with integrated pressure sensors in red

(a) Load the file `NACA0015.mat` into Matlab containing the 36 chordwise positions of the pressure sensors of the lower and upper side of the airfoil, the angle of attack variations, and the recorded pressure data for each sensors and angle. Compute and plot the resulting normal force coefficient C_n as a function of α based on the pressure data. What is the slope $\partial C_n/\partial\alpha$ of the linear part of the curve?

(b) Compute and plot the lift coefficient polar. What is the maximal lift coefficient of this airfoil and at what angle of attack does it occur?

(c) Compute and plot the pitching moment coefficient at quarter chord $C_{m,c/4}$. What is the slope $\partial C_m/\partial\alpha$ of the linear part of the curve?

(d) Determine the aerodynamic centre of the airfoil.