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***ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE  
SECTION DE GENIE MECANIQUE  
6<sup>th</sup> & 8<sup>th</sup> Semester, Fall 2023***

***ME-435: Aeroelasticity & Fluid Structure Interaction***

***Chapter 1: Introduction***

*Dr Mohamed FARHAT*

*Assistants: Thomas Berger, Antoine Sache*

*Cavitation Research Group, EPFL SCI – STI – MF  
Av. de Cour 33 bis, 1007 Lausanne, Switzerland*

# About the course

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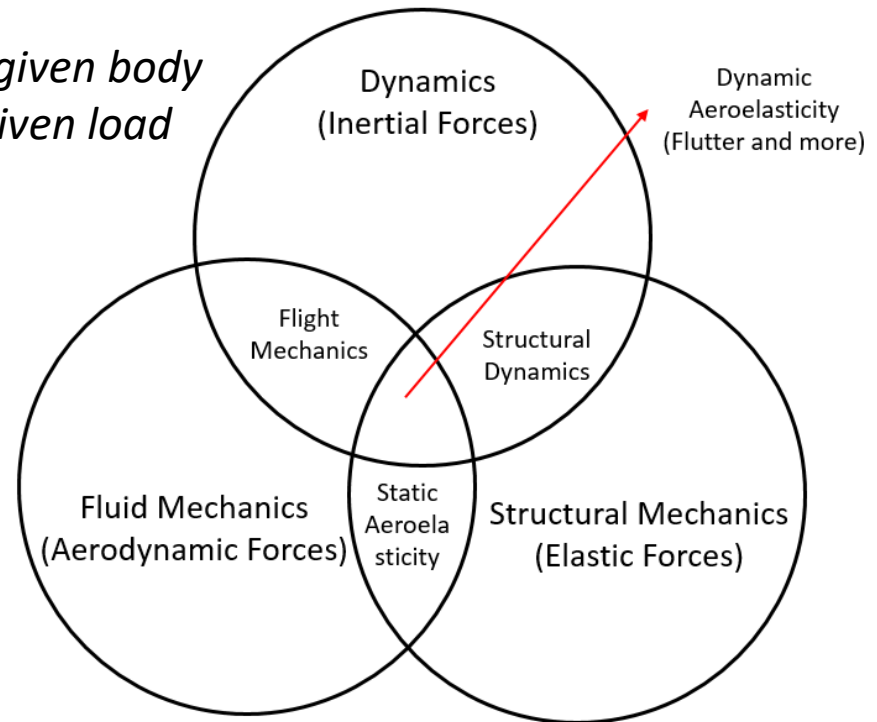
- **COURSE MATERIAL:**
  - *Slides (PDF, Moodle server)*
  - *Videos (Kaltura)*
- **EXAM**
  - *Written exam (3 hours)*  
*Without access to any written or digital document*

# DEFINITION

***Aeroelasticity*** is the interdisciplinary field of study concerned with the interaction between the deformation of an elastic structure in an airstream and the resulting aerodynamic forces

- **Dynamic Aeroelasticity:**

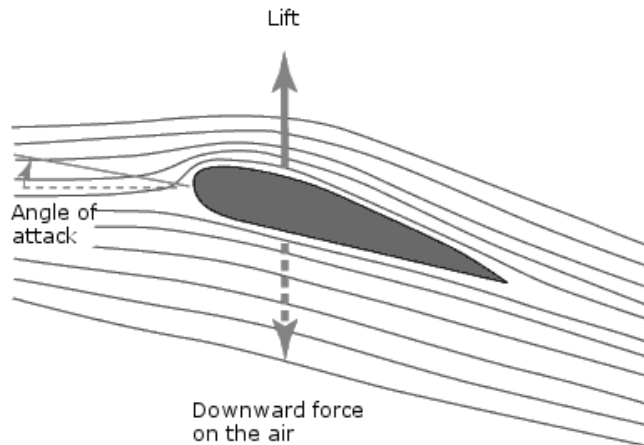
- *Fluid Mechanics* → Forces acting on a given body
- *Elasticity* → Shape of a body under a given load
- *Dynamics* → Effect of inertial forces



# DEFINITION

***Fluid-Solid Interaction (FSI)*** is a broader field of study concerned with the interaction between the deformation of an elastic structure in any stationary or flowing fluid and the resulting dynamic forces

*In classical fluid mechanics, the solid is simply seen as a boundary, which does not interact with the fluid*



*In classical solid mechanics, the fluid is simply seen as a load applied to the structure*



# Fluid dynamics framework

- **Continuum mechanics:**
  - Fluids are treated in macroscopic way as continuous mass:  
Kinematic of individual molecules is not considered
  - Fluid particle = Large number of molecules  
Within the particle: statistical variation of fluid properties negligible
  - Gas may be considered as continuous medium if the length scale is large compared to the mean free path (MFP):
    - Under normal conditions:
      - » The mean free path for a gas  $\sim 70$  nm
      - » Nb of molecules within  $1 \mu\text{m}^3 \sim 2.7 \cdot 10^7$
    - As the altitude is increased, the mean free path increases and becomes of the same order as a satellite diameter. In this case, the continuum mechanics hypothesis is no more valid.
  - For liquids, the hypothesis is always valid (smaller MFP)

# Fluid dynamics framework

- **Physical properties of liquids and gas:**

- Density:  $\rho = \frac{dM}{dV}$

- $dM$  = Mass of the molecules contained in a fluid particle of volume  $dV$
- Variation of the density: more likely to occur with a gas than with a liquid
- If the density of a moving fluid particle remains “constant”  
⇒ incompressible flow

- Viscosity

- Velocity gradient in a flowing fluid → shear forces
- Newtonian fluids: the shear stress (tangential force / unit surface) is proportional to the velocity gradient:

$$\tau = \mu \frac{\partial u}{\partial y} = \rho \nu \frac{\partial u}{\partial y} \quad \mu: \text{dynamic viscosity} \quad \nu: \text{kinematic viscosity}$$

- Shear forces small in comparison with pressure  
→ Ideal fluid, with no internal friction (inviscid flows)

# Fluid dynamics framework

- **Mass conservation (Continuity equation):**

- **Integral form (control volume  $V$  bound by a surface  $S$ ):**

$$\frac{\partial}{\partial t} \iiint_V \rho dV = - \oiint_S \rho \mathbf{v} \cdot \mathbf{n} dS$$

- **Differential form (using divergence theorem):**

$$0 = \frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = \frac{\partial \rho}{\partial t} + (\mathbf{v} \cdot \nabla) \rho + \rho(\nabla \cdot \mathbf{v}) = \frac{D\rho}{Dt} + \rho(\nabla \cdot \mathbf{v})$$

$$\text{where } \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

- **Incompressible flow:**

- Constant density of any fluid particle along its trajectory:  $\frac{D\rho}{Dt} = 0$

$$\rightarrow \nabla \cdot \mathbf{v} = 0$$

# Fluid dynamics framework

- **Momentum conservation (Navier Stokes equations):**

- Viscous and incompressible flows:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \rho \mathbf{f} + \mu \nabla^2 \mathbf{v}$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \rho \mathbf{f} + \mu \nabla^2 \mathbf{v}$$

- Inviscid and incompressible flow (Euler equation):

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \rho \mathbf{f}$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \rho \mathbf{f}$$

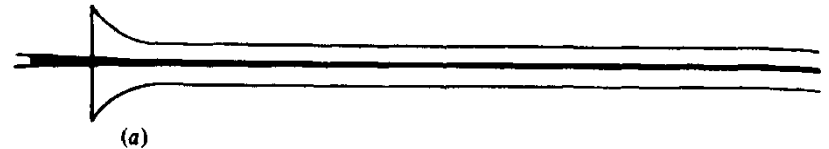
# Fluid dynamics framework

- **Reynolds number:**

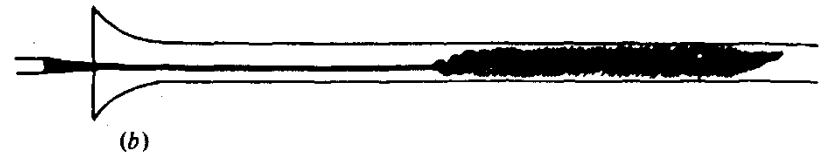
- Reynolds experiment:

- Flow visualization in a pipe of a diameter  $D$  (dye injection)
- $Re = \frac{U D}{\nu}$  , represents inertia over viscosity forces ratio
- Velocity gradually increased :

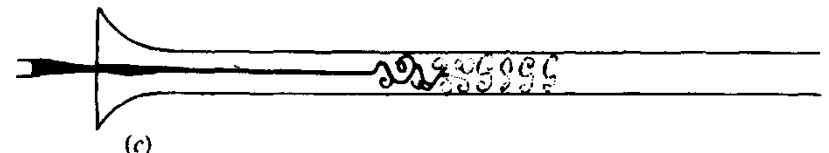
$Re < 2000$  : laminar flow



$2000 < Re < 4000$  :  
Transition to turbulence



$Re > 4000$  : Turbulent flow



**Fig. 9.2.** Reynolds's drawings of the flow in his dye experiment.

# Fluid dynamics framework

- **Reynolds number:**
  - Reynolds experiment:
    - Flow visualization in a pipe of a diameter  $D$  (dye injection)
    - $Re = \frac{U D}{\nu}$  , represents inertia over viscous forces ratio
    - Velocity gradually increased :

$D = .001 \text{ m}$  , Transition to turbulence for  $U = 2 \text{ m/s}$

$D = 1 \text{ m}$ ,      Transition to turbulence for  $U = 2 \text{ mm/s}$

# Fluid dynamics framework

- **Reynolds number:**

- Momentum conservation of viscous flow over a flat plate

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \rho \mathbf{f} + \mu \nabla^2 \mathbf{v} \quad \mathbf{v} = (u, v)$$

- Non dimensional form

$$u^* = \frac{u}{U_\infty} ; v^* = \frac{v}{U_\infty} ; p^* = \frac{p}{\rho U_\infty^2} ; x^* = \frac{x}{L} ; y^* = \frac{y}{L} ; t^* = \frac{t U_\infty}{L}$$

$$\frac{D\mathbf{v}^*}{Dt^*} = -\nabla p^* + \frac{1}{Re} \nabla^2 \mathbf{v}^*$$

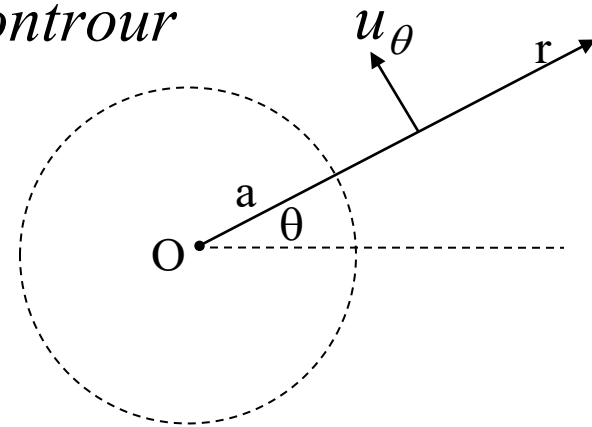
Reynolds number is the **similarity** parameter

## Vortices

- **Velocity field of a free vortex ( $\Gamma$ : vortex intensity)**

$$\vec{u} = (0, u_\theta, 0) \quad \text{avec} \quad u_\theta(r) = \frac{\Gamma}{2\pi r}$$

$$\Gamma = \int u_\theta(r) dl = \text{cste} \quad \forall C, \text{ closed contour}$$



- **Irrotational flow**

$$\text{rot}(\vec{u}) = \vec{\nabla} \times \vec{u} = \left( 0, 0, \frac{1}{r} \frac{\partial}{\partial r} (ru_\theta) \right) = (0, 0, 0) \quad \forall r > 0$$

## Vortices

- Momentum conservation (radial equilibrium):

$$\frac{\partial p}{\partial r} = \frac{\rho u_\theta^2}{r}$$

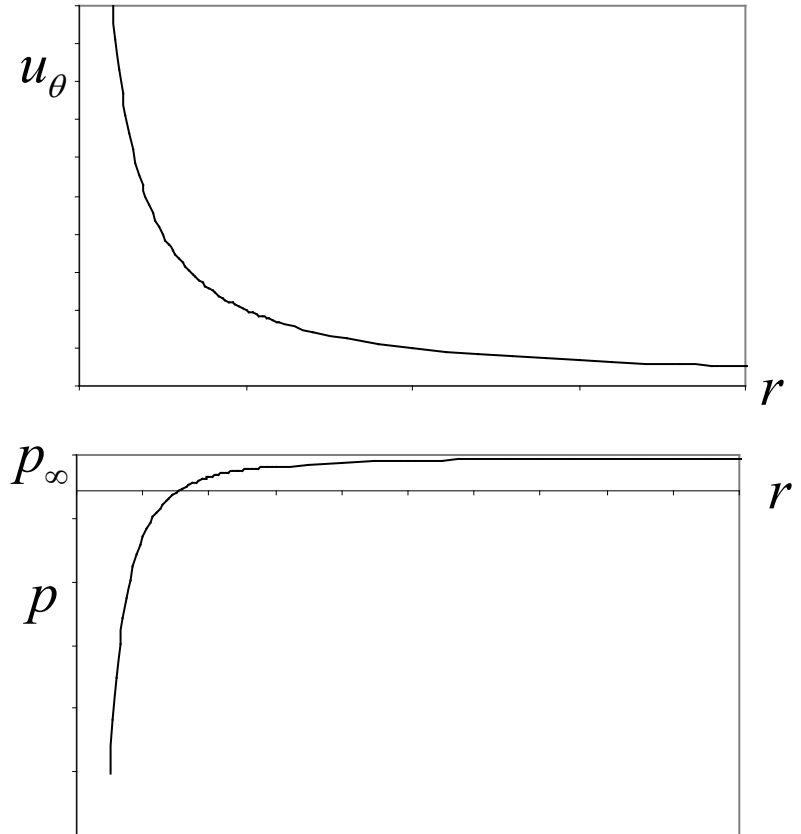
- Integration:

$$p(r) = p_\infty - \frac{\rho \Gamma^2}{8\pi^2 r^2}$$

$$r \rightarrow 0 \quad \Rightarrow \quad u_\theta(r) \rightarrow +\infty$$

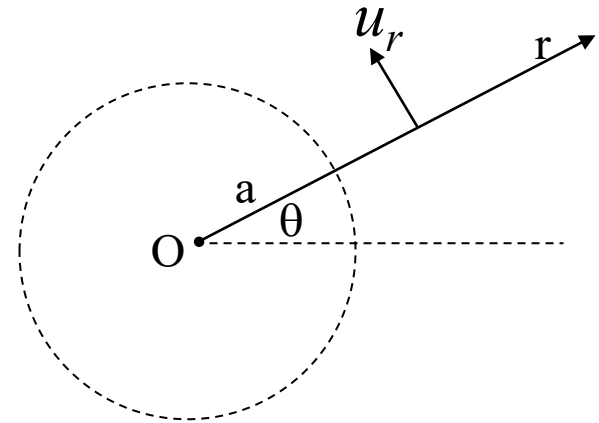
$$r \rightarrow 0 \quad \Rightarrow \quad p(r) \rightarrow -\infty$$

- Viscous effects ?



## Vortices

- **Rankine vortex model:**
  - **Free vortex zone (I)**
    - Inviscid flow
  - **Solid rotation zone (II)**
    - Viscosity term dominant (viscous core)



$$\text{Zone I } (r \geq a) \quad u_{\theta}(r) = \frac{\Gamma}{2\pi r} \quad (\Gamma = \text{cste})$$

$$\text{Zone II } (r \leq a) \quad u_{\theta}(r) = \omega r \quad (\omega = \text{cste})$$

- **Continuity of the velocity at  $r = a$ :**

$$\omega a = \frac{\Gamma}{2\pi a} \quad \Rightarrow \quad \Gamma = 2\pi\omega a^2$$

## Vortices

- Rankine vortex model:
  - Momentum conservation (radial equilibrium):

$$\frac{\partial p}{\partial r} = \frac{\rho u_{\theta}^2}{r}$$

- Solution:

$$\begin{aligned} r \geq a \quad p(r) &= p_{\infty} - \frac{\rho \Gamma^2}{8\pi^2 r^2} \\ r \leq a \quad p(r) &= p_{\infty} - \frac{\rho \Gamma^2}{8\pi^2 a^2} \left( 2 - \frac{r^2}{a^2} \right) \end{aligned}$$

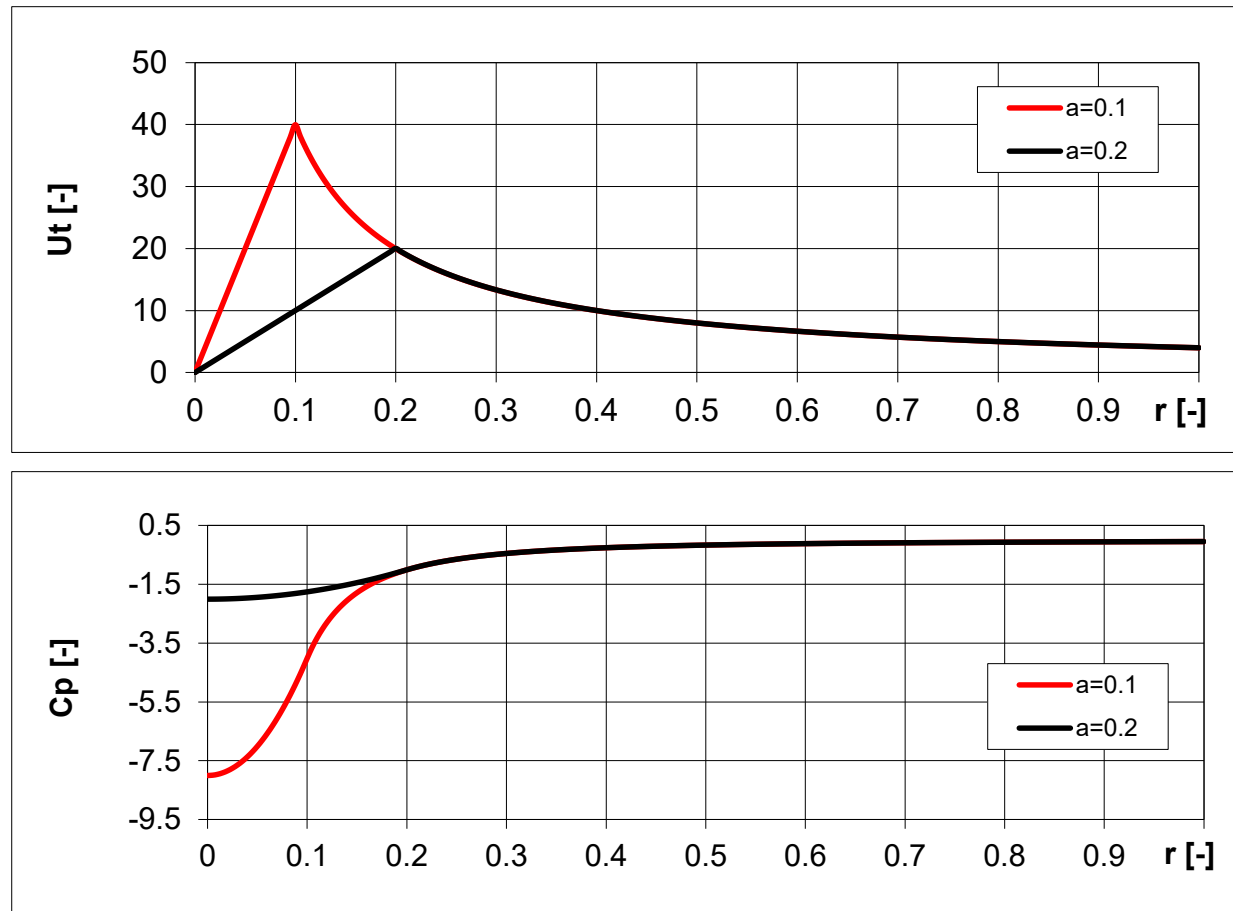
## Vortices

- **Rankine vortex model:**
  - **Non-dimensional form:**
    - **Case of a vortex produced at the tip of a foil**  
( $U_\infty$  is the upstream velocity)

$$\begin{aligned} r \geq a \quad c_p(r) &= -\frac{\Gamma^2}{4\pi^2 r^2 U_\infty^2} \\ r \leq a \quad c_p(r) &= -\frac{\Gamma^2 \left(2 - \frac{r^2}{a^2}\right)}{4\pi^2 a^2 U_\infty^2} \end{aligned}$$

## Vortices

- Rankine vortex model:
  - Effect of the viscous core size



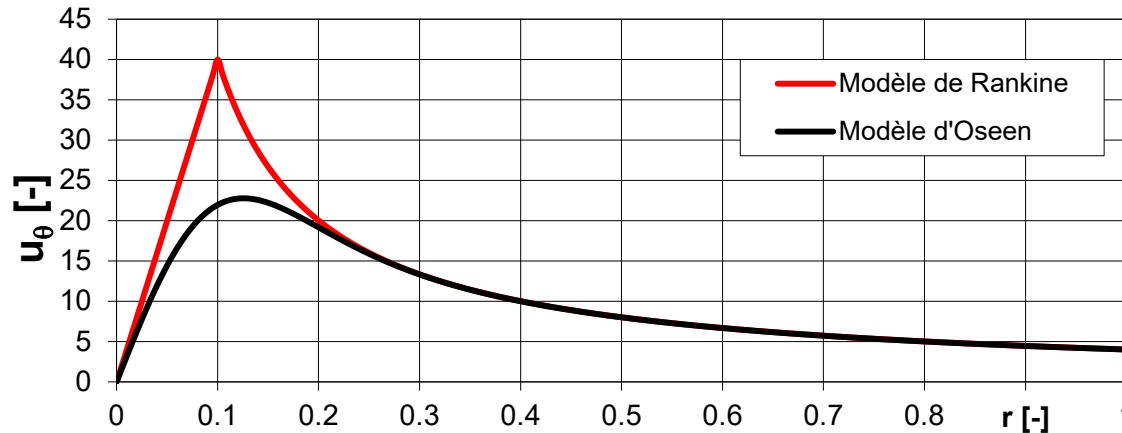
## Vortices

- Rankine model incompatible with the physics :
  - Discontinuity of the velocity derivative at  $r = a$
- Oseen Model (more realistic)

$$u_{\theta}(r) = \frac{\Gamma}{2\pi r} (1 - e^{-(r/\lambda)^2}) \quad \forall r \geq 0 \quad \lambda = \frac{a}{1.12}$$

- $a$ : Location of maximum tangential velocity (viscous core size)

$$r \rightarrow +\infty \Rightarrow u_{\theta}(r) \approx \frac{\Gamma}{2\pi r} \text{ (Free vortex) and } u_{\theta}(0) = 0$$



# Fluid dynamics framework

Tip vortices are commonly observed in a variety of industrial applications

- Airplane wings, wind turbines, compressors, ...
- Difficult to predict (trajectory, intensity, viscous core size)

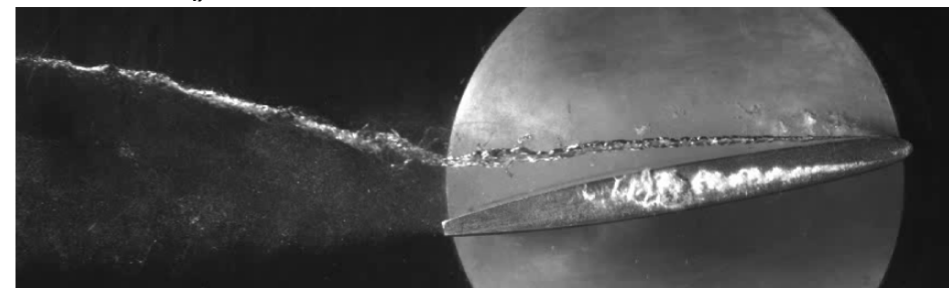
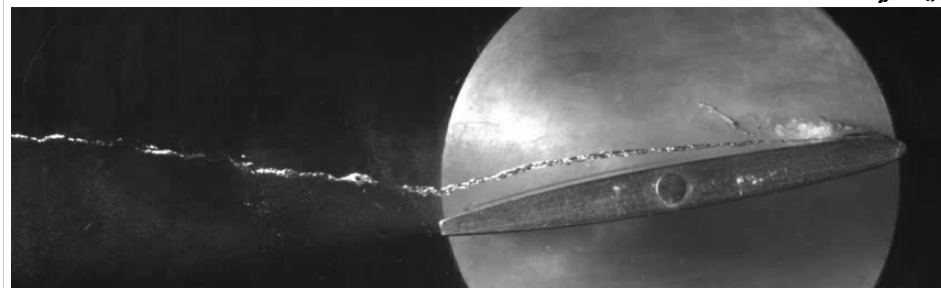
**Influence of the gap width on tip leakage vortex in EPFL cavitation tunnel**

20'000 frames/s, inlet velocity = 10 m/s, incidence=10°, inlet pressure=1 bar

*gap=15 [mm]*

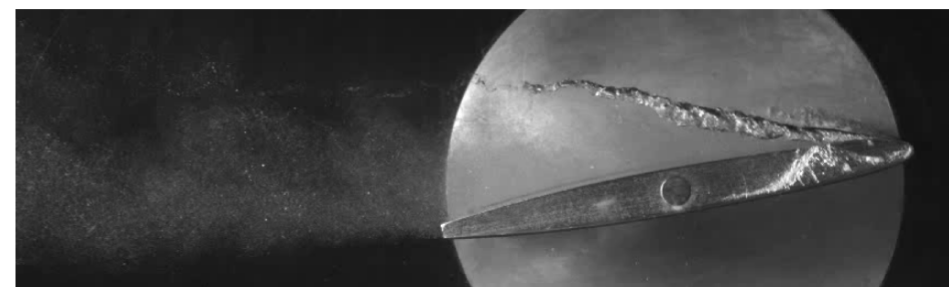
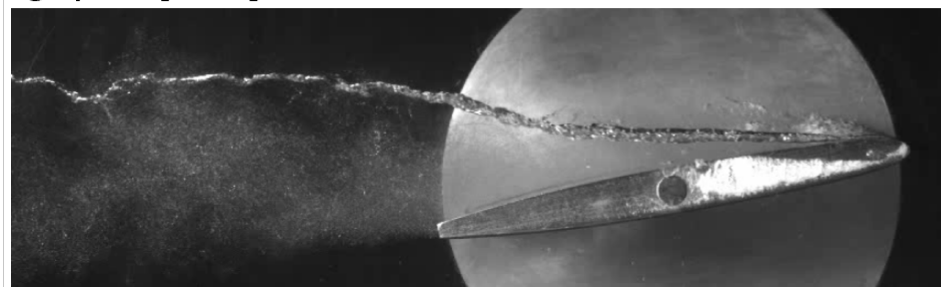
*Matthieu Dreyer, PhD student, EPFL 2013*

*gap=7 [mm]*



*gap=4 [mm]*

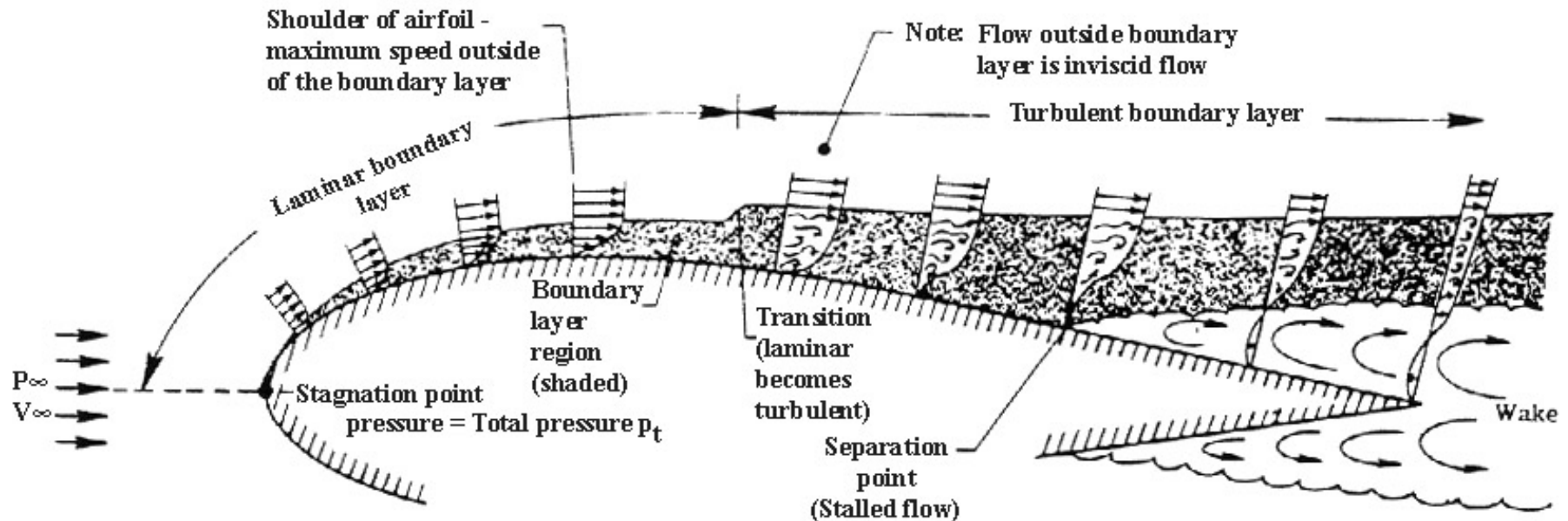
*gap=2 [mm]*



# Fluid dynamics framework

- **Boundary layer:**

- **Definition:** the fluid layer, next to a solid surface, where the viscous forces are dominant
  - Velocity at the wall = 0 (No slip condition)
  - The flow outside the boundary layer is inviscid

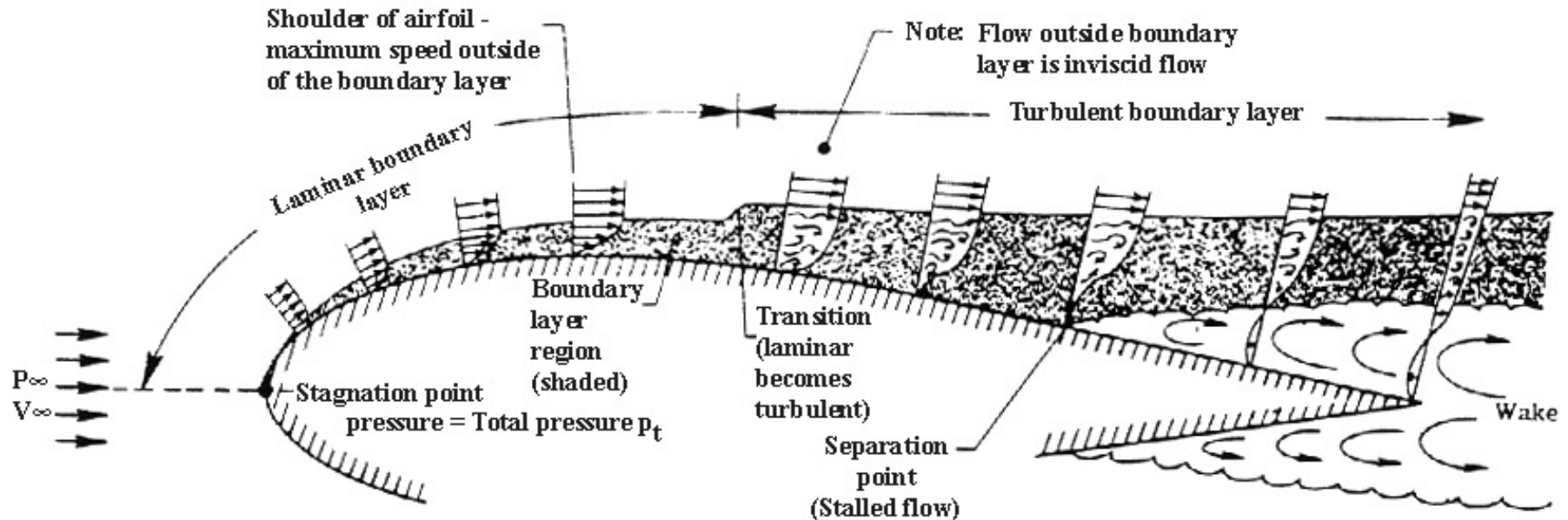


# Fluid dynamics framework

- **Boundary layer:**

- **Boundary layer states:**

- Laminar : fluid layers slide on each other without being mixed
- Transition to turbulence
- Turbulent : chaotic motion of the fluid within the BL
- Separated : due to adverse pressure gradient (decelerated flow)



# Fluid dynamics framework

- **Boundary layer:**
  - **Boundary layer thickness:**

$$\textit{Thickness} : \delta = y|_{u=0.99U_e}$$

$$\textit{Displacement thickness} : \delta_1 = \int_0^\delta \left(1 - \frac{u}{U_e}\right) dy$$

$$\textit{Momentum thickness} : \delta_2 = \int_0^\delta \left(1 - \frac{u}{U_e}\right) \frac{u}{U_e} dy$$

$$\textit{Form factor} : H_{12} = \frac{\delta_1}{\delta_2}$$

# Fluid dynamics framework

- **Boundary layer:**

- **Wall shear stress:**

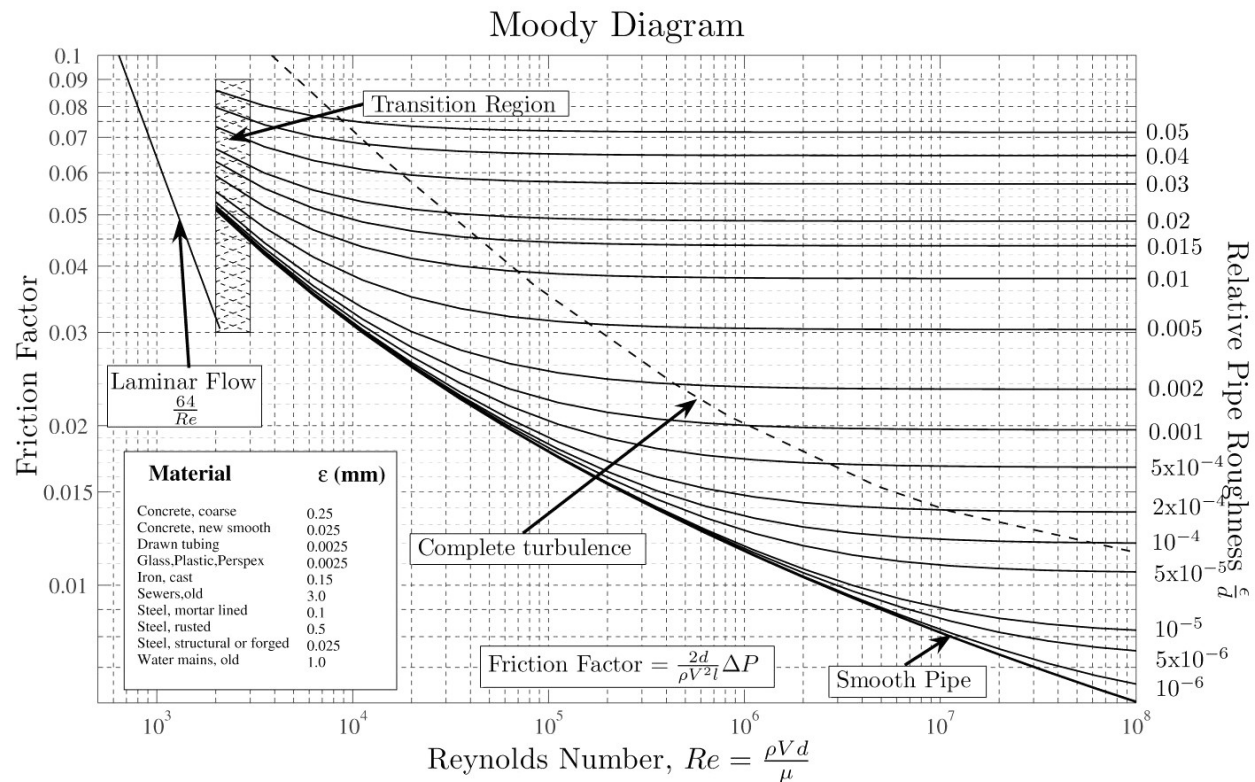
$$\tau_w(x) = \mu \left( \frac{\partial u}{\partial y} \right) \bigg|_{y=0}$$

- **Skin Friction Coefficient:**

$$C_f(x) = \frac{\tau_w(x)}{\frac{1}{2} \rho U_\infty^2}$$

# Fluid dynamics framework

- **Boundary layer:**
  - **Friction factor of a flow in a pipe (Moody's diagram):**
    - Mainly dependent on Reynolds number and relative roughness
    - Fully turbulent regime: Friction no more depends on Re number



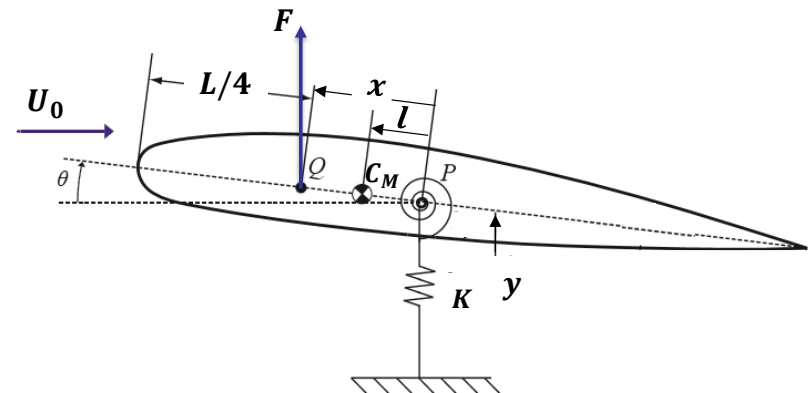
# Examples of Fluid-Structure Interaction

## *Dynamic instability of a wing : Flutter*

***Flutter: Large vibration involving 2 vibration modes of a foil in a flow***



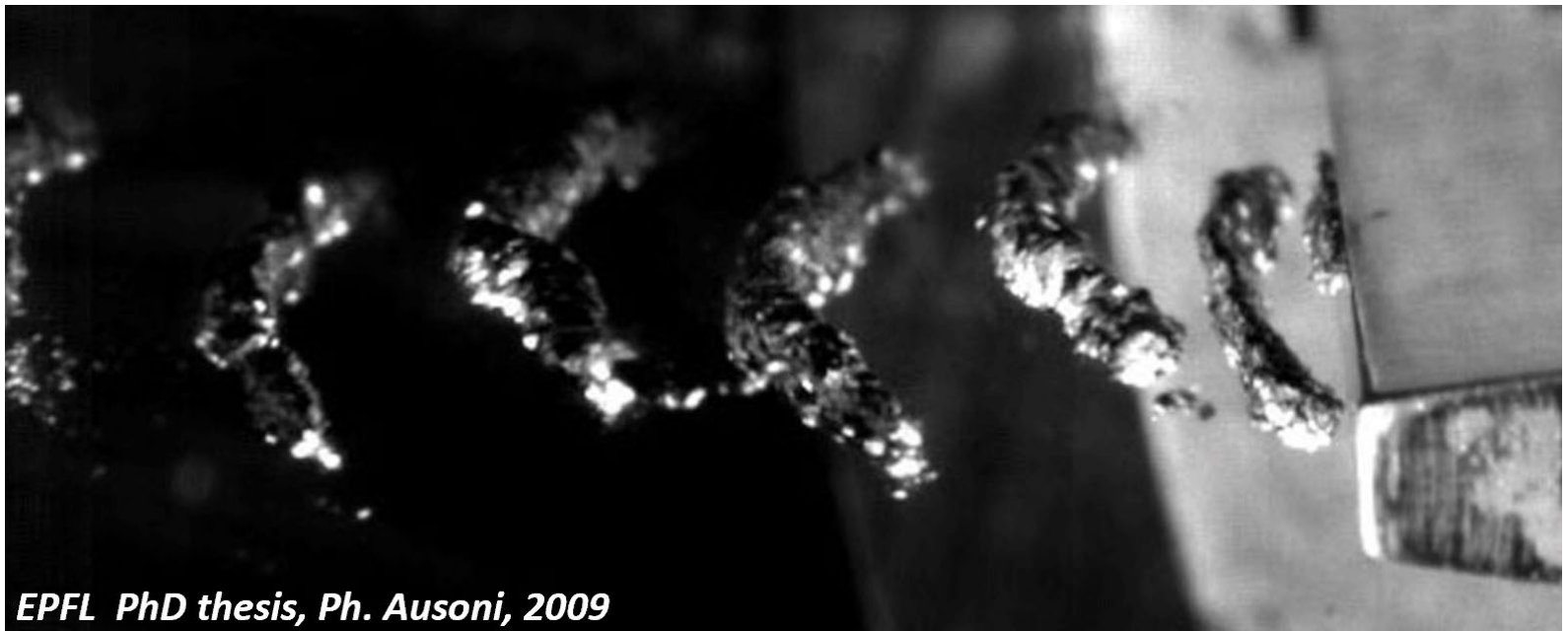
***Twist and plunge modes approximation***



# Examples of Fluid-Structure Interaction

## *Vibration induced by wake instabilities*

- *Example: Blunt trailing edge hydrofoil in water stream*
  - *Naca009, 100 mm chord, 150 mm span, 3.2 mm trailing edge thickness*
  - *Blunt trailing edge → Alternate vortex shedding (Kàrmàn)*
  - *Excitation of the 1<sup>st</sup> torsional mode*
  - *Lock-in: shedding frequency locked on resonance frequency in a wide range of flow velocities (!)*

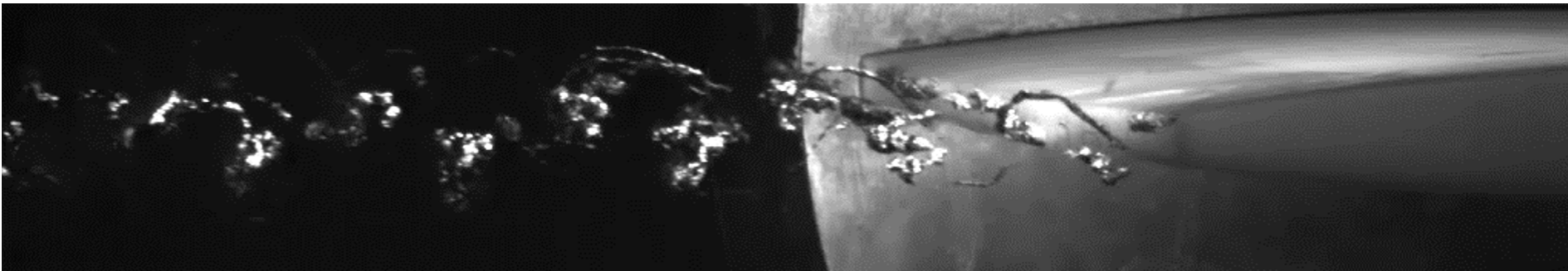


*EPFL PhD thesis, Ph. Ausoni, 2009*

# Examples of Fluid-Structure Interaction

## *Vibration induced by wake instabilities*

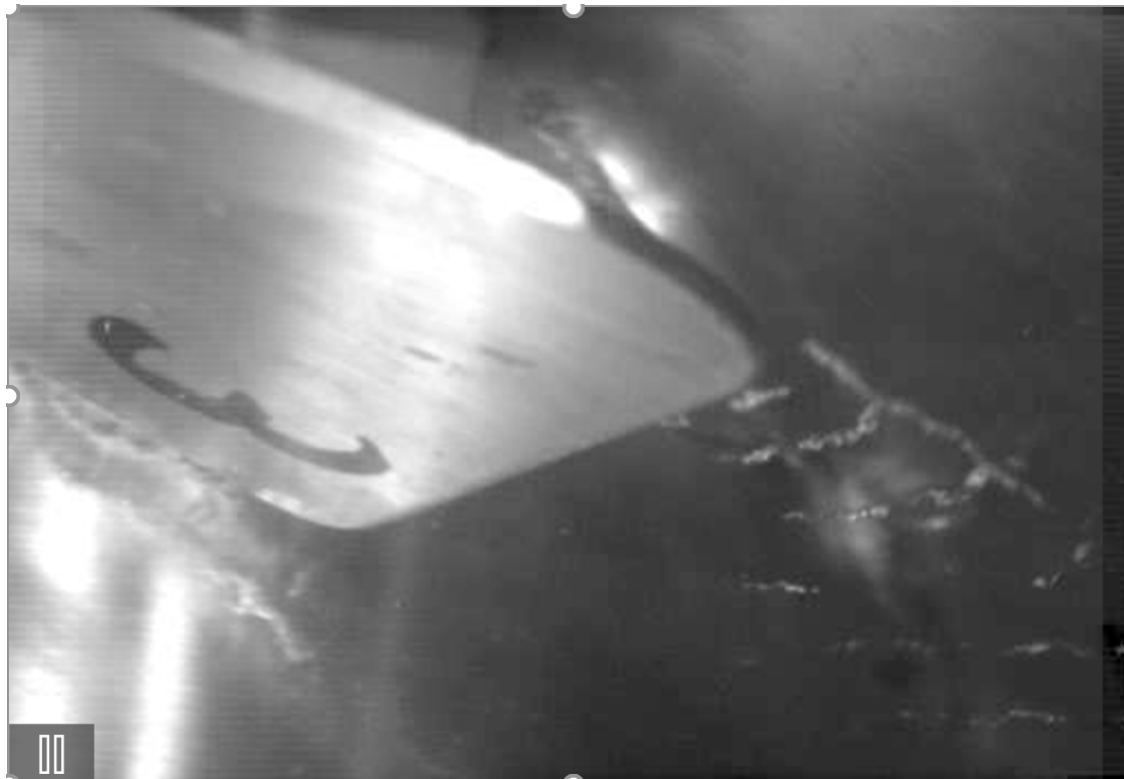
- *Example: Deformable hydrofoil (POM) in water stream*
  - *Naca009, 100 mm chord, 150 mm span, 3.2 mm trailing edge thickness*
  - *Blunt trailing edge → Alternate vortex shedding (Kàrmàn)*
  - *Excitation of the 2<sup>nd</sup> bending mode → alteration of shedding mechanism*



# Examples of Fluid-Structure Interaction

## *Vibration induced by wake instabilities*

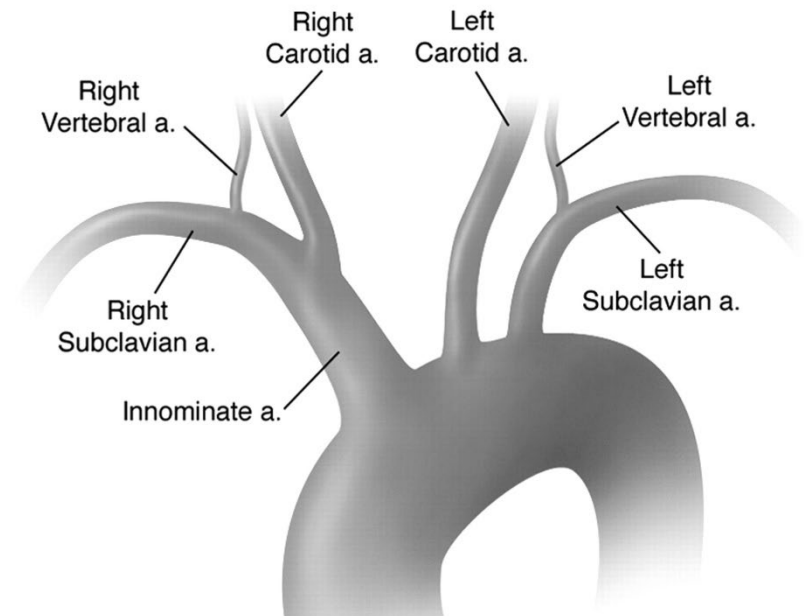
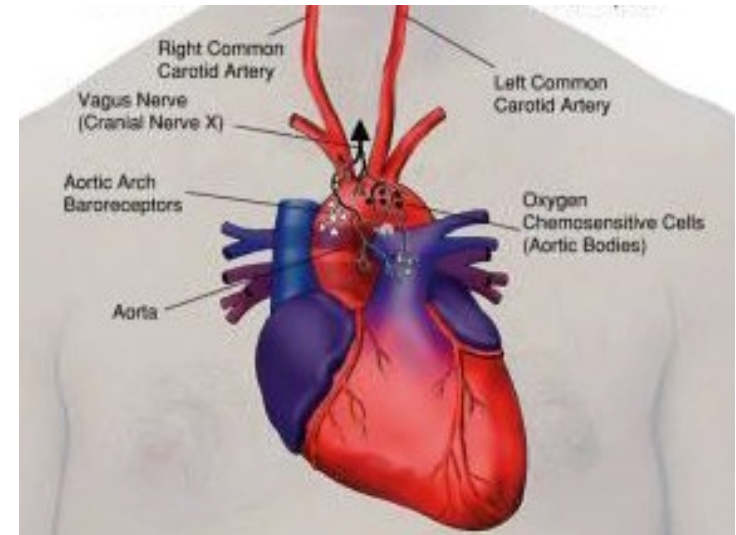
- ***Example: Kaplan Turbine Model***
  - ***Alternate vortex shedding (Kàrmàn)***
  - ***Possible excitation of the blades → risk of premature cracks***
  - ***Lack of scale-up rules for fluid-interaction interaction in turbines***



# Examples of Fluid-Structure Interaction

## ***Blood flow in a human aorta***

- ***Flow visualization in a rigid aorta model***
  - ***Reverse flow in the arteries is due to the rigidity of the aorta***
  - ***In reality aorta is flexible and adapts its shape to avoid reverse flows***



# Examples of Fluid-Structure Interaction

## Collapse of Tacoma Narrows bridge

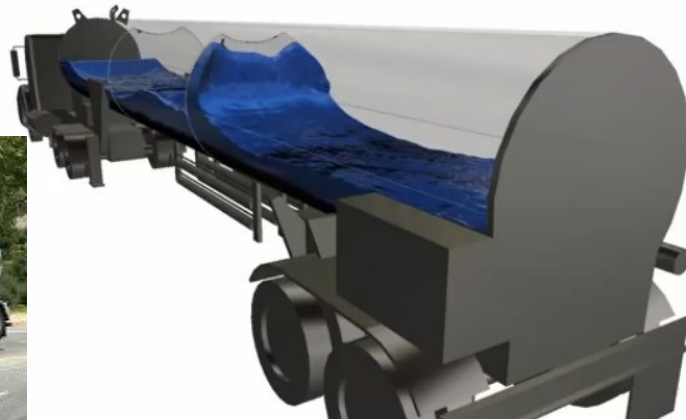
- Nov. 7<sup>th</sup> 1940 (after 4 months of operation)
- 65 km/h wind speed
- Root cause: Aero-elastic coupling
  - Vortex shedding matches resonance



# Examples of Fluid-Structure Interaction

- ***Sloshing Dynamics***

- *Oscillating motion of the free surface of a liquid within a partially filled container*
- *Variety of examples:*
  - *Liquid transportation (trucks and tankers)*
  - *Ballast, or fuel tanks of large ships*
  - *Aircraft and rocket fuel tanks*
  - *Food processing*
  - *Bioreactors*
  - ...



# Sloshing Dynamics

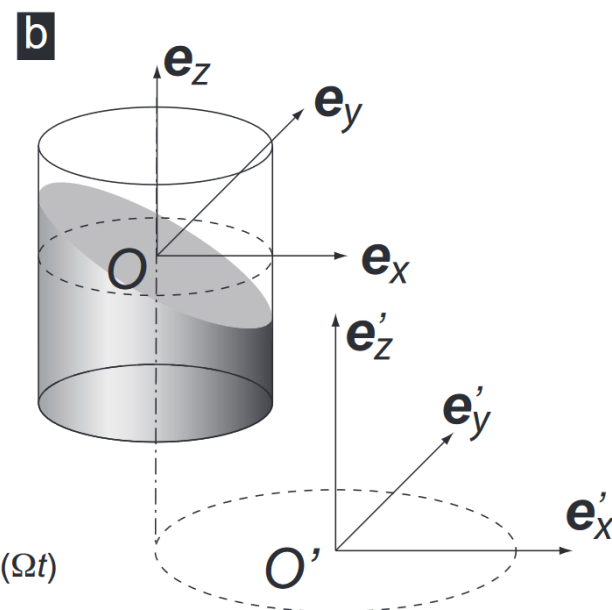
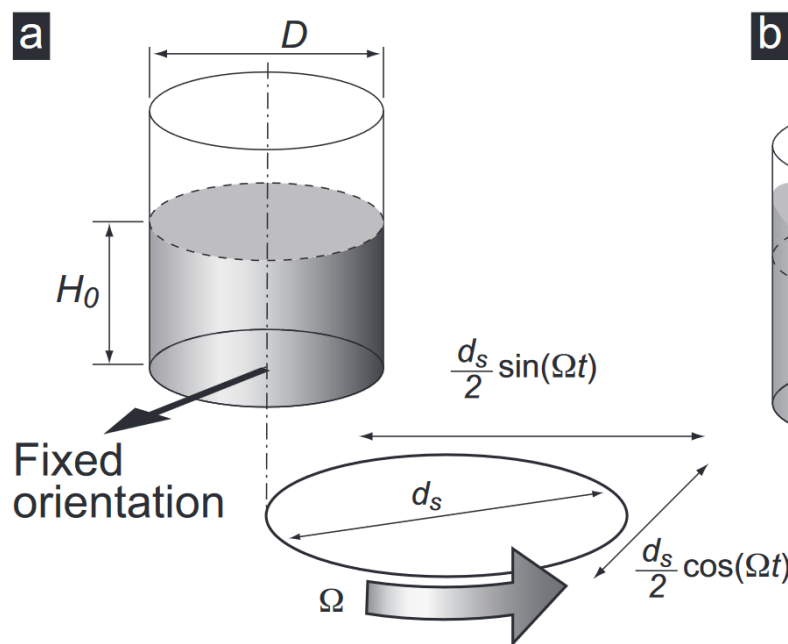
- **Sloshing Dynamics:**
  - *Orbital sloshing as a tool to improve mixing and gas exchange*
    - *Bioreactors for cell cultivation*
    - *Food processing*



# Sloshing Dynamics

- **Orbital sloshing dynamics in a cylindrical container - Assumptions :**
  - **Upright rigid cylinder**
    - Diameter:  $D$ , liquid height at rest:  $H_0$ , Excentricity:  $d_s$ , Rotational speed:  $\Omega$ , Interface position:  $\xi$
  - **Incompressible, inviscid and irrotational liquid flow**
  - **Small oscillation amplitude of the interface**
- **Non-dimensional numbers:**

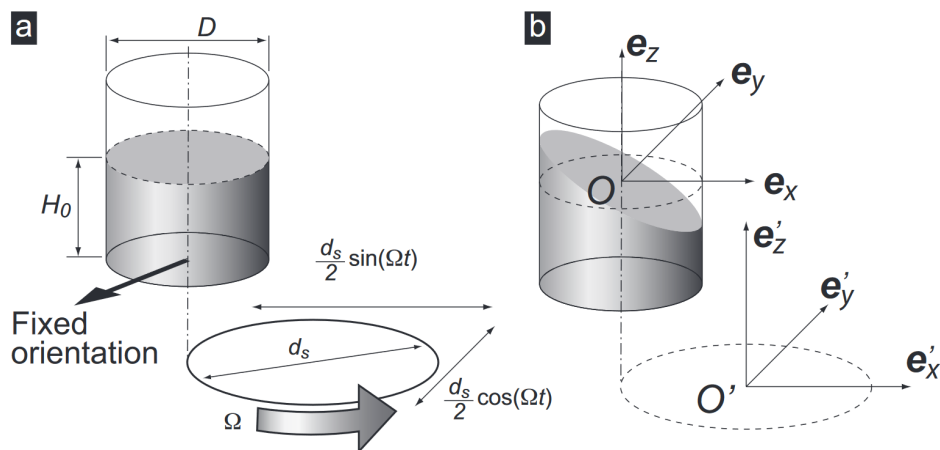
$$\left\{ \begin{array}{l} \tilde{d}_s = \frac{d_s}{D} \\ \tilde{H}_0 = \frac{H_0}{D} \\ Fr = \sqrt{\frac{d_s \Omega^2}{g}} \\ \tilde{\xi} = \frac{\xi}{D} \end{array} \right.$$



# Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container***
  - ***Rich variety of wave shapes***
  - ***Complex flow dynamics induced by the rotating wave***

***M. Reclari, EPFL PhD Thesis, 2014***



27/11/11

Why swirling wine in a glass makes it taste better - Telegraph

**The Telegraph**

Why swirling wine in a glass makes it taste better

Wine buffs who swirl their drink in a glass are using the sophisticated physics of wave technology to unleash the flavour, scientists say.



Swirling a glass of wine churns the liquid as it travels, drawing in oxygen from the air and intensifying the smell Photo: ALAMY

# Sloshing Dynamics

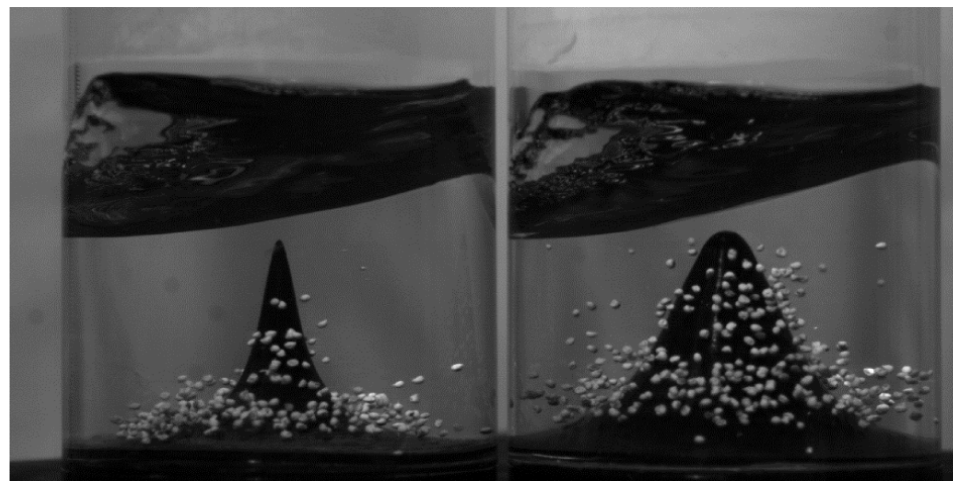
- ***Orbital sloshing dynamics in a cylindrical container***
  - ***Effect of container shape ?***
    - ***Semester projects, M. Moosavi 2015, S. Eghbali 2016, Rossi 2018***

*Visualisation of mixing improvements with the bump bottom using solid particles in suspension*

$\Omega=135$  RPM,  $d_S=44$  mm,  $H_0=84$  mm



$\Omega=135$  RPM,  $d_S=44$  mm,  $H_0=84$  mm



# Examples of Fluid-Structure Interaction

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## The Roberto Carlos Spiral

# Examples of Fluid-Structure Interaction

## The Roberto Carlos Spiral



# Examples of Fluid-Structure Interaction

## Roberto Carlos Spiral



# Examples of Fluid-Structure Interaction

## *The Roberto Carlos Spiral (Aerodynamics related to soccer ball)*

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### Football curves

Guillaume Dupeux<sup>a,b</sup>, Caroline Cohen<sup>a,b</sup>, Anne Le Goff<sup>a,b</sup>, David Quéré<sup>a,b</sup>,  
Christophe Clanet<sup>a,b,\*</sup>

<sup>a</sup> LadHyX, UMR7646 du CNRS, Ecole Polytechnique, 91128 Palaiseau, France

<sup>b</sup> PMMH, UMR7636 du CNRS, ESPCI, 10 rue Vauquelin, 75005 Paris, France

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#### ABSTRACT

Straight lines, zigzag, parabolas (possibly truncated), circles and spirals are the main curves which can be observed in football (in the European sense, soccer elsewhere). They are, respectively, associated to heavy kick, knuckleball, lob and banana kicks. We discuss their physical origin and determine their respective domain of existence.

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# Examples of Fluid-Structure Interaction

## The Roberto Carlos Spiral

