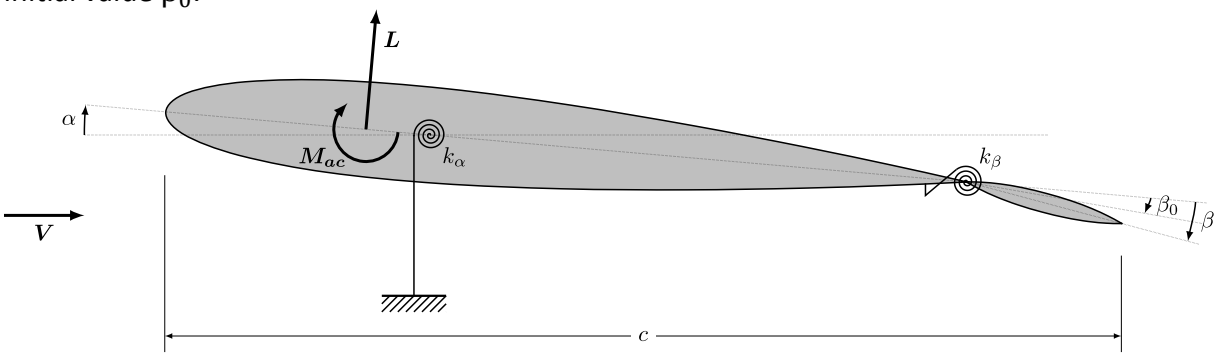
Exercise – Serie 6 – Static instability

Exercise 1 - Loads on an airfoil

An airfoil of chord c is mounted in a wind tunnel such that it can only move in pitch against a spring of stiffness k_α. The airfoil is fitted with a flap with a spring restraint of stiffness k_β that creates a restoring moment on the hinge proportional to the incremental deflection from an initial value β_0.



Assume known dynamic pressure q and wing area S. Assume known (non-zero) values of the following aerodynamic derivatives:

$C_{L,\alpha}\left(=\frac{\partial C_L}{\partial \alpha}\right)$, $C_{L,\beta}$, $C_{M_{ac},\beta}$, $C_{H,\alpha}$, $C_{H,\beta}$ Note that $C_{H,\beta}$ is typically negative.

These coefficients are defined as

$$C_L = \frac{L}{qS} \qquad C_{M_{ac}} = \frac{M_{ac}}{qSc} \qquad C_H = \frac{H}{qSc}$$

where L and M_{ac} are the total lift and moment at the aerodynamic center, respectively, and H is the aerodynamic moment of the flap around the hinge. The distance between the aerodynamic center (the point where the lift and moment are applied) and the elastic axis (the point where the torsional spring is attached) is e, with e positive if the aerodynamic axis is ahead of the elastic axis. For simplicity, we ignore the effects of the airfoil weight and assume small angles.

We can approximate the changes in lift and moment coefficients with linear aerodynamics as:

$$C_L = C_{L,\alpha}\alpha + C_{L,\beta}\beta, \qquad C_{M_{ac}} = C_{M_{ac},\beta}\beta, \qquad C_H = C_{H,\alpha}\alpha + C_{H,\beta}\beta$$

a) Show that the static equilibrium equation, in matrix form, will be written as

$$\begin{bmatrix} qSeC_{L,\alpha} - k_\alpha & qSeC_{L,\beta} + qScC_{M_{ac},\beta} \\ qScC_{H,\alpha} & qScC_{H,\beta} - k_\beta \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ -k_\beta \beta_0 \end{bmatrix}$$

b) From the first equation find the control reversal speed. What happens when $k_\beta \to \infty$.

c) Solve the system of question a) to express α and β (Hint: use Cramer's rule).

d) Compute the divergence speed.

e) For $k_\beta \to \infty$, how is the control reversal speed related to the divergence speed? Comment on the results.