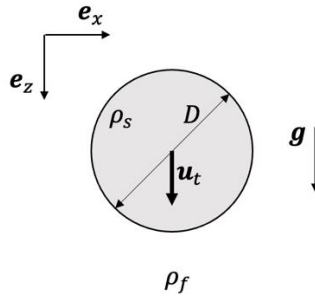


Correction – Serie 3 – Added damping

Exercise 1 - Drag of a free-falling sphere in water

Consider a smooth sphere of diameter $D = 2\text{cm}$, made of aluminum (density $\rho_s = 2700 \frac{\text{kg}}{\text{m}^3}$) falling vertically into a quiet and unbounded volume of water ($\rho_f = 1000 \frac{\text{kg}}{\text{m}^3}$, $\mu = 1 \times 10^{-3} \text{Pa} \cdot \text{s}$). The sphere is assumed to have reached its terminal velocity $\mathbf{u}_t = u_t \mathbf{e}_z$. The gravitational acceleration is equal to $g = 9.81 \text{ m/s}^2$.



Recall that, for a given object, the drag force is given by:

$$\mathbf{F}_D = -\frac{1}{2} \rho_f C_D(Re) A |\mathbf{u}_o - \mathbf{u}_f|^2 \hat{\mathbf{u}}, \quad Re = \frac{\rho_f |\mathbf{u}_o - \mathbf{u}_f| D_e}{\mu}$$

$\mathbf{u}_o$ is the velocity of the object and $\mathbf{u}_f$ is the far-field fluid velocity.

$\hat{\mathbf{u}} = \frac{(\mathbf{u}_o - \mathbf{u}_f)}{|\mathbf{u}_o - \mathbf{u}_f|}$ is a dimensionless unit vector pointing in the direction of $\mathbf{u}_o - \mathbf{u}_f$.

A is the reference area of the object and D_e is the equivalent diameter of the object.

For a sphere, the drag coefficient C_D in different flow regimes is:

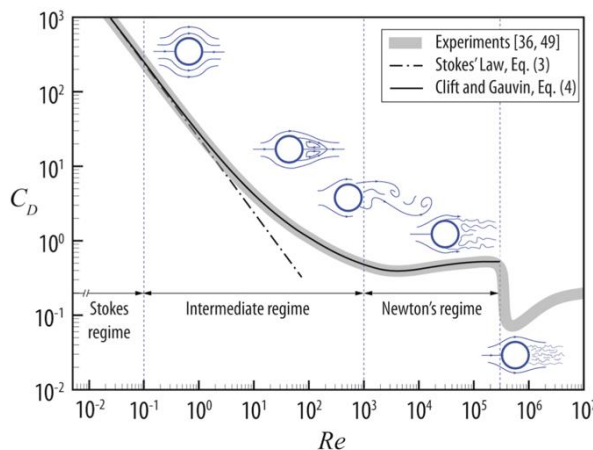


Figure 1: Dependency of C_D on Re for sphere. Streamlines around sphere at various Re are also shown in the plot. From Bagheri et al. 2018.

$$C_D(Re) = \begin{cases} \frac{24}{Re} & Re < 0.1 \text{ (Stokes regime, laminar)} \\ f(Re) & 0.1 < Re < 1000 \text{ (Intermediate regime)} \\ \sim 0.45 & 1000 < Re < 3.5 \times 10^5 \text{ (Newton's regime, turbulent)} \end{cases}$$

- a) Why should the added mass not be taken into account in the previous hypothesis?
- b) Give the expression of the drag force in the Stokes and Newton's regimes.
- c) What is the expression of the terminal velocity u_t of the sphere as a function of C_D ?
Then, deduce its expression in the Stokes and Newton's regimes.
- d) Which flow regime is correct in this particular case?
- e) In this flow regime, what phenomenon not considered here can make the flow unsteady?

Exercise 2

Structure loaded by a turbulent air flow

Consider a metallic structure dynamically loaded by a turbulent air flow. The density of the structure is assumed to be much larger than that of air.

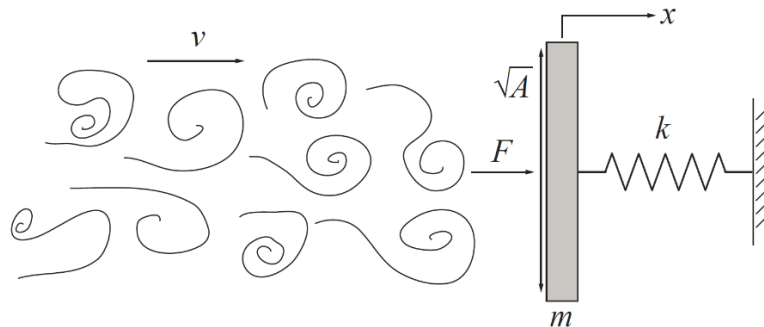


Figure 2: Fluidodynamic forces acting on a deformable structure.

The known parameters are the drag coefficient C_D , the fluid density ρ , reference area A , the mass m and the stiffness k . v is the fluid velocity and x the displacement of the structure. The objective of this problem is to familiarize yourself with deriving linearized equations of motion describing fluctuations about the steady-state values.

- What type of force is F in the system shown in Figure 2, give its expression? The drag coefficient given above is considered as a constant and does not depend on relative velocity. Explain the reasoning behind this assumption.
- Derive the equation of motion for the system shown in figure given the fluid loading F as expressed in point a).
- The fluid velocity can be expressed as the sum of the mean fluid velocity $\bar{v}$ plus a perturbation v' , as $v(t) = \bar{v} + v'(t)$. This leads to the same decomposition for the displacement of the structure: $x = \bar{x} + x'(t)$. Since we intend to use a linear model for the structural response, we consider small perturbations with $v' \ll \bar{v}$, and $x' \ll \bar{x}$. This means that in your linearization, you can neglect second order terms.
- From your equation of motion, what is the structure's mean displacement $\bar{x}$?
- What is the equation of motion that governs the fluctuations only?
- From this linear differential equation, find the natural frequency of the system and compare its value with the no-flow case $F = 0$.