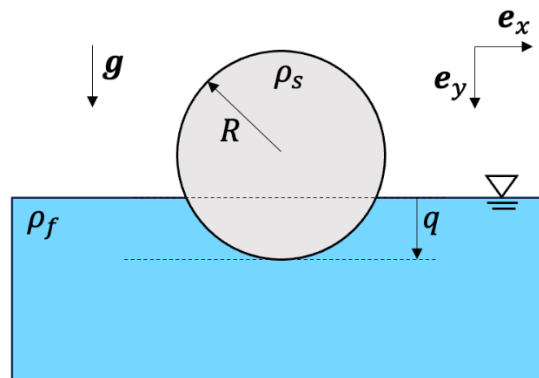


Exercises – Serie 2 – Oscillations of a floating sphere

We want to study the oscillatory motion of a solid sphere of density ρ_s and radius R which is partially immersed under water (density $\rho_f = 1000 \text{ kg/m}^3$).

We suppose that the sphere only moves in the vertical direction and we define the parameter $q \in [0, 2R]$ as the distance between the free surface and the bottom of the sphere. $\mathbf{g} = g\mathbf{e}_y$ is the gravitational acceleration constant equal to 9.81 m/s^2 . The density of the sphere is considered smaller than the one of water and we neglect viscous and added mass effects.



- 1) Find the expression of the immersed volume $V_f(q)$.
- 2) Using the Newton's second law of motion, write the ordinary differential equation (ODE) characterizing the motion of the sphere $\ddot{q} = f(q)$. Why does it differ from the case of a rectangular floating object?
- 3) Express the algebraic equation at the equilibrium point $0 = f(\bar{q})$.
- 4) Solve this ODE numerically using *Matlab's* 'ode45' function for various values of R and ρ_s and different initial conditions $[q(0), \dot{q}(0)]$. Can you find a set of parameters for which the solution differs from a simple harmonic motion?
- 5) Linearize the ODE around the equilibrium point $q = \bar{q} + q' \Rightarrow \ddot{q}' = f(q', \bar{q})$ (with $q' \ll 1$) and find the expression of the natural frequency of the sphere $\omega_0(\bar{q})$ around this point.
- 6) For a given set of parameters, compare the oscillation frequency found from the numerical solution with the natural frequency obtained in question 5. You can use *Matlab's* 'fsolve' function to find the equilibrium point $\bar{q}$.
- 7) At home: Set up a basic experiment using any common floating object (e.g. an ice cube, an apple, a half-filled water bottle, or any floating object you can think of). Record the experiment with your phone, measure the oscillation frequency of the object, and compare it with the frequencies predicted by your numerical findings.