Exercise sheet - Reminder Mechanical Vibrations

1 Airfoil on two springs

Consider an airfoil of mass m and radius of gyration r. In order to test it in a wind tunnel, it is mounted on two springs of constant rigidity k_1 and k_2, located at distance l_1 and l_2 from the vertical axis passing through the center of rotation of the airfoil O (see figure 1 b elow). Only small oscillations are considered.

1. Write the equations of motion of the airfoil in matrix form.
2. Determine the condition that must be satisfied so that the translation and rotation motion are decoupled.
3. Find the natural frequncies, ω_1 and ω_2.
4. Optional: Using the state space representation, write a script that can simulate the motion of the airfoil given any set of initial conditions (within the small oscillation assumption). Use the following parametrs: $m = 25$ kg, $r = 0.08$ m, $k_1 = 7200$ N/m, $k_2 = 10800$ N/m, $l_1 = 0.3$ m, $l_{2,decoupled} = 0.2$ m, $l_{2,coupled} = 0.05$ m.

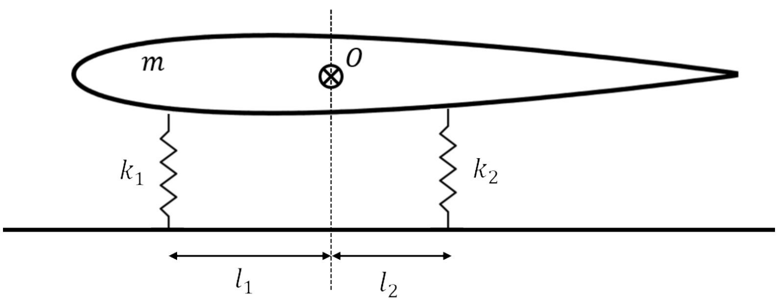

Figure 1: Airfoil supported by two springs.

2 Tuned Mass Damper

Tuned mass dampers (TMD) are used to dampen the horizontal vibrations of tall structures (often due to the vortex shedding in strong wind situations). In their simplest form, they consist of a mass placed on top of the structure to be dampened. This mass is usually connected to the structure by a system of springs and dampers. TMDs are tuned to swing at approximately the same rate as the structure's natural frequency. An equivalent mechanical system is presented on figure 2. The subscript s indicates the properties related to the structure and the subscript d the ones related to the damper.

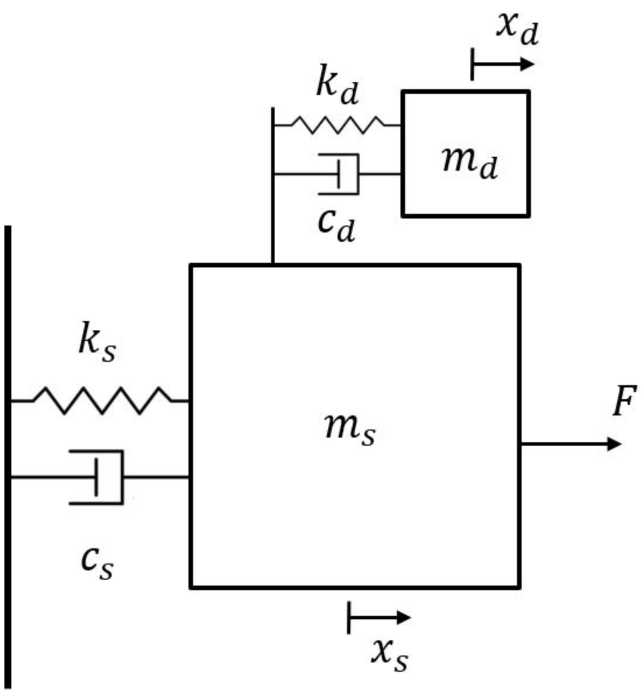

Figure 2: Structure with tuned mass damper.

We consider that the structure is excited by a harmonic force of amplitude A and frequency ω.

1. Write the equations of motion of the system.
2. Compute the amplitude of the motion of the structure, X_s, for values of ω ranging between $0.9\omega_s$ and $1.1\omega_s$. Compare the results to the case without the tuned mass damper. Use the following parameters: $m_s = 24000$ tons, $f_s = 0.5$ Hz, $\xi_s = \frac{c_s}{2\omega_s m_s} = 0.02$, $\frac{m_d}{m_s} = \mu = 0.05$, $\xi_d = \frac{c_d}{2\omega_d m_d} = 0.1$, $f_d = \frac{f_s}{1+\mu}$ and $A = 1 \cdot 10^5$ N.
3. How does the damper effectiveness change if its mass is modified (keep every other parameter unchanged)? Explain.