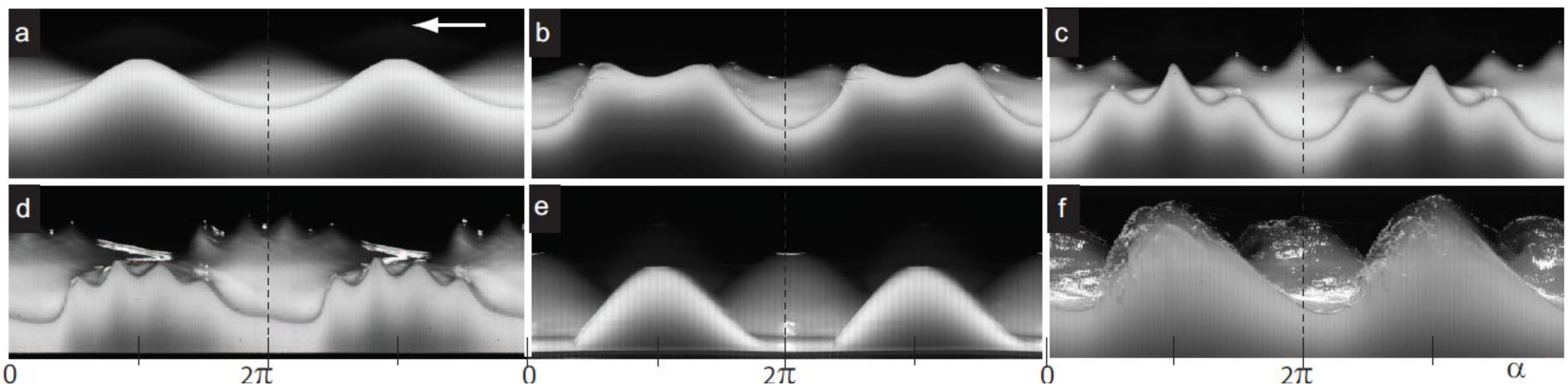
AEROELASTICITY AND FLUID-STRUCTURE INTERACTION



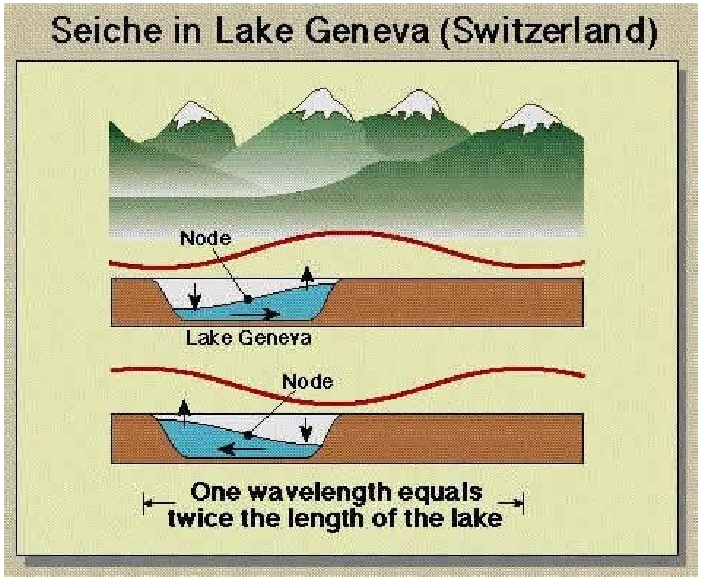
Chapter 8: Sloshing Dynamics

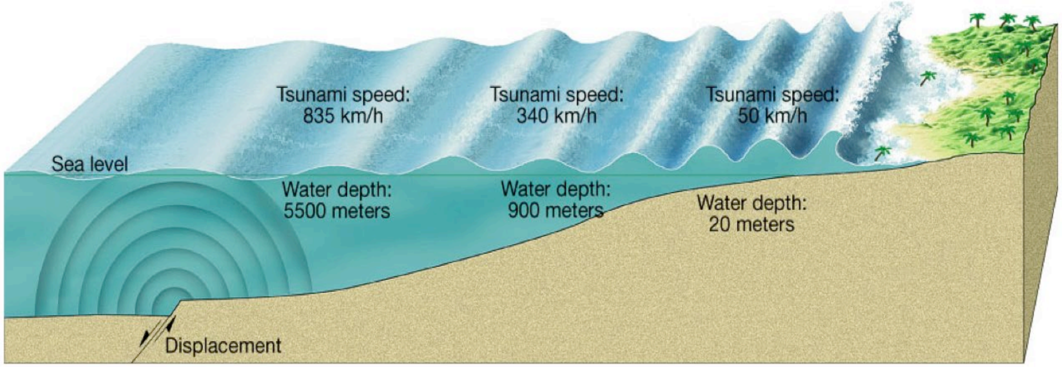
Sloshing Dynamics

- ***Definition: Sloshing is the oscillating motion of the free surface of a liquid within a partially filled container***
- ***Main causes: Acceleration of the container, air flow (wind), …***
- ***Variety of examples:***
  - ***Liquid transportation (trucks and tankers)***
  - ***Ballast, or fuel tanks of large ships***
  - ***Aircraft and rocket fuel tanks***

Sloshing Dynamics

- ***Variety of examples:***
  - ***Large waves in oceans, lakes, harbors and storage tanks due to earthquakes (Seiche and tsunamis)***





"Seiche is a Swiss French dialect word meaning to sway back and forth and is used by hydrologists to describe an oscillating wave form found in enclosed or partially enclosed bodies of water. The term was picked up and promoted by François-Alphonse Forel, who made the first scientific observations of them on Switzerland's Lake Geneva in the 1890s"

Sloshing Dynamics

- ***Variety of examples:***

NATURE | NEWS

Ancient tsunami devastated Lake Geneva shoreline

Sediments suggest wave was triggered by massive rock fall, highlighting risk to modern lakeside communities.

Jessica Marshall

28 October 2012

In ad 563, more than a century after the Romans gave up control of what is now Geneva, Switzerland, a deadly tsunami on Lake Geneva poured over the city walls. Originating from a rock fall where the River Rhône enters at the opposite end of the lake to Geneva, the tsunami destroyed surrounding villages, people and livestock, according to two known historical accounts.

Researchers now report the first geological evidence from the lake to support these ancient accounts. The findings, published online in *Nature Geoscience*, suggest that the region would be wise to evaluate the risk today, with more than one million inhabitants living on the lake's shores, including 200,000 people in Geneva alone[1].

“It’s certainly happened before and I think we can expect that it will probably happen again sometime,” says geologist Guy Simpson, from the University of Geneva, one of the researchers behind the project.

Radius Images/Alamy

The Swiss city of Geneva could be at risk of flooding should a major rock fall trigger a tsunami on Lake Geneva, as seems to have happened in ad 563.

Sloshing Dynamics

- ***Variety of examples:***

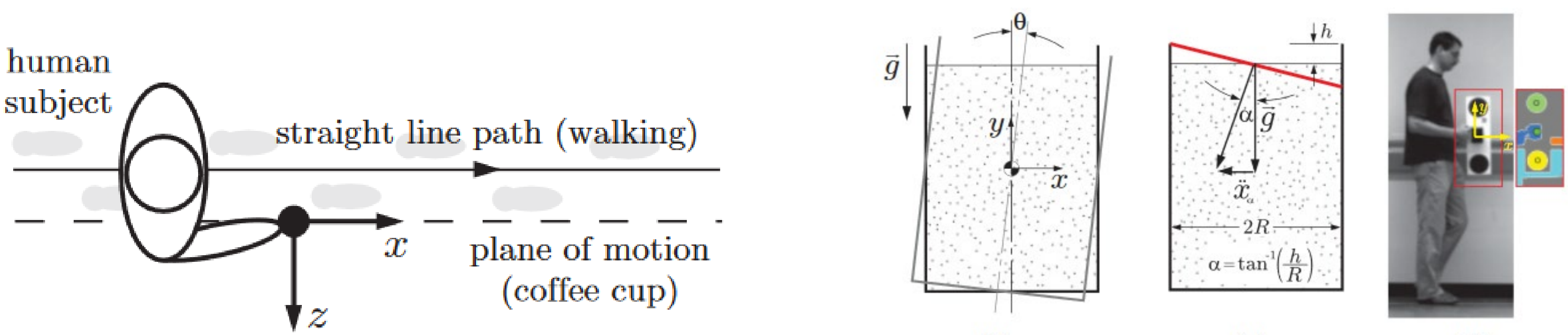
PHYSICAL REVIEW E **85**, 046117 (2012)

Walking with coffee: Why does it spill?

H. C. Mayer and R. Krechetnikov
Department of Mechanical Engineering, University of California, Santa Barbara, California 93106, USA
(Received 23 December 2011; published 26 April 2012)

In our busy lives, almost all of us have to walk with a cup of coffee. While often we spill the drink, this familiar phenomenon has never been explored systematically. Here we report on the results of an experimental study of the conditions under which coffee spills for various walking speeds and initial liquid levels in the cup. These observations are analyzed from the dynamical systems and fluid mechanics viewpoints as well as with the help of a model developed here. Particularities of the common cup sizes, the coffee properties, and the biomechanics of walking proved to be responsible for the spilling phenomenon. The studied problem represents an example of the interplay between the complex motion of a cup, due to the biomechanics of a walking individual, and the low-viscosity-liquid dynamics in it.

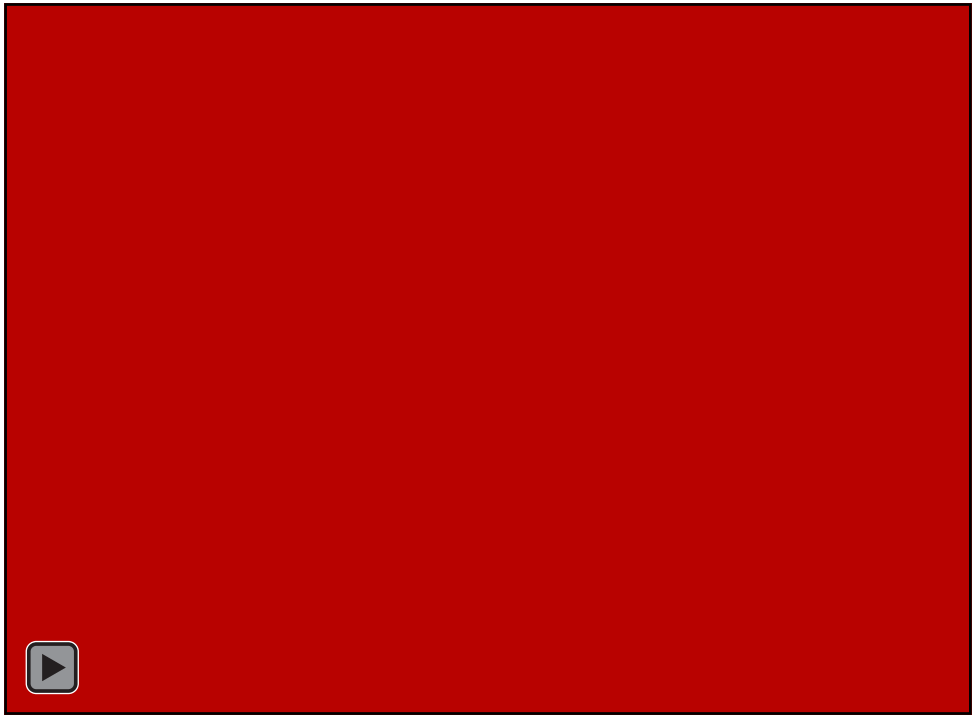
DOI: 10.1103/PhysRevE.85.046117 PACS number(s): 89.90.+n, 87.85.G−, 47.10.Fg, 47.20.Cq



(b) (c) (d)

Sloshing Dynamics

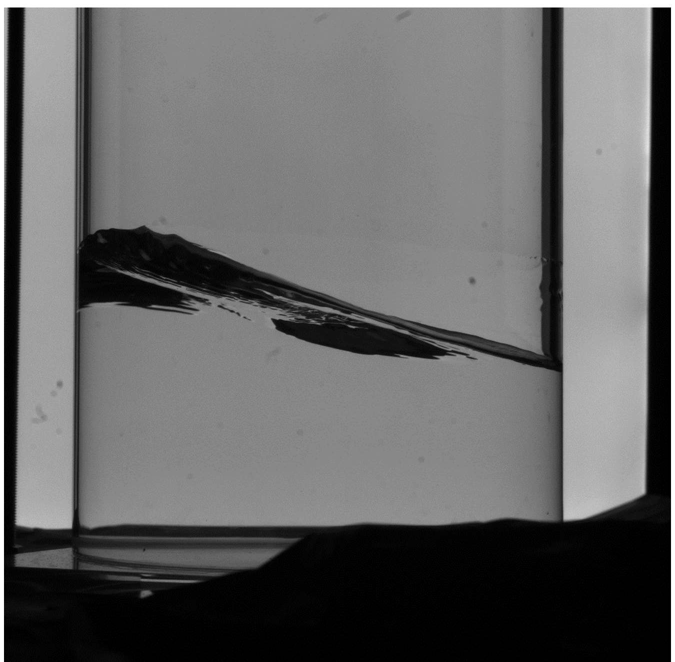
- *Variety of examples:*
  - *Damping for large civil structures*



Tuned liquid "sloshing" mass damper
Highcliff Apartments, Hong Kong, 1995.

Sloshing Dynamics

- ***Variety of examples:***
  - ***Orbital sloshing as a tool to improve mixing and gas exchange***
    - ***Bioreactors for cell cultivation***
    - ***Food processing***



Sloshing Dynamics

- ***Variety of examples: Wine swirling***
  - ***M. Reclari, EPFL 2011***

The Telegraph

Why swirling wine in a glass makes it taste better

Wine buffs who swirl their drink in a glass are using the sophisticated physics of wav
technology to unleash the flavour, scientists say.

Swirling a glass of wine churns the liquid as it travels, drawing in oxygen from the air and intensifying the smell Photo: ALAMY

La science fait danser le vin dans les verres

LE FIGARO

Par Cyrille Vanlerberghe

Publié le 23 novembre 2011 à 19h14, mis à jour le 24 novembre 2011 à 13h02

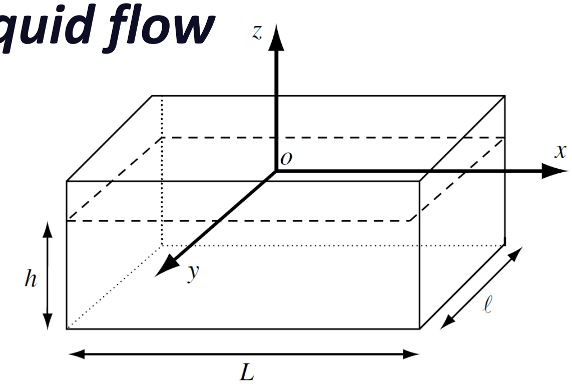
Copier le lien

La technique « d'agitation orbitale» d'un verre de vin pourrait s'appliquer à une cuve de plusieurs milliers de litres servant à la culture de cellules. *ALEJANDRO PAGNI/AFP*

VIDÉO - Des chercheurs suisses tentent de déterminer les meilleurs paramètres de rotation pour libérer les arômes d'un grand cru.

Sloshing Dynamics

- ***Linear sloshing dynamics in a 2D container - Assumptions :***
  - ***2D rectangular rigid container ($l \gg L$)***
  - ***Incompressible, inviscid and irrotational liquid flow***
- ***Equations of motion (Euler):***



$$\begin{cases} \nabla u = 0 \\ \dfrac{\partial u}{\partial t} + (u.\nabla)u = -\dfrac{1}{\rho}\nabla p - \nabla(gz) \end{cases}$$

Irrotational → $\nabla \times u = 0$ → $u = -\nabla\varphi$ ***(φ : velocity potential function)***

→ $(u.\nabla)u = \frac{1}{2}\nabla u^2 - u \times (\nabla \times u) = \frac{1}{2}\nabla u^2$

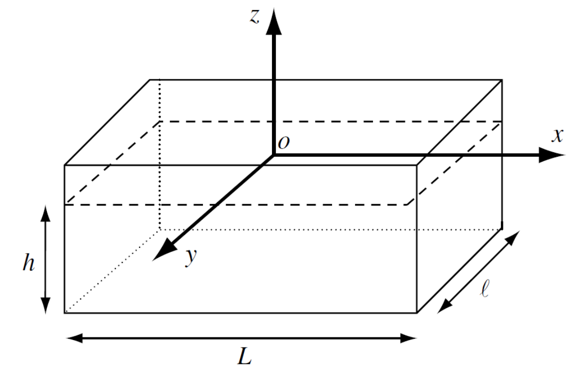
→ $\nabla\left(\dfrac{p}{\rho} + \dfrac{u^2}{2} + gz - \dfrac{\partial\varphi}{\partial t}\right) = 0$ $\xrightarrow{\text{integration}}$ $\dfrac{p}{\rho} + \dfrac{u^2}{2} + gz - \dfrac{\partial\varphi}{\partial t} = C(t)$

C(t) is an arbitrary function of time

Incompressible → $\nabla u = 0$ → $\nabla^2\varphi = 0$ ***(Laplace equation)***

Sloshing Dynamics

- ***Linear sloshing dynamics in a 2D container:***
  - ***A Laplace problem:*** $\nabla^2\varphi = 0$
    with the boundary conditions:



$$\left.\frac{\partial\varphi}{\partial x}\right|_{x=\pm L/2} = 0 \qquad \left.\frac{\partial\varphi}{\partial z}\right|_{z=-h} = 0$$

  - ***… The solution reads:***

$$\varphi(x,y,z,t) = \sum_{m=1}^{\infty}[\alpha_m(t)cos(k_m x) + \beta_m(t)sin(k_m x)]cosh[k_m(z+h)]$$

where $k_m = (2m-1)\pi/L$ ***for asymmetric modes***
and $k_m = 2m\pi/L$ ***for symmetric modes***

Sloshing Dynamics

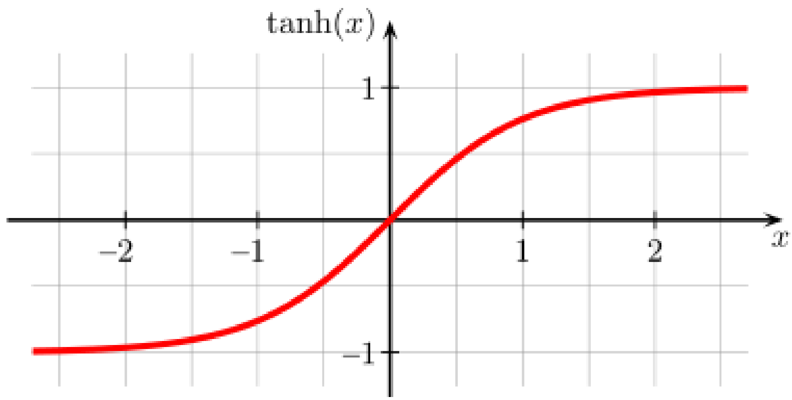
- ***Linear sloshing dynamics in a 2D container:***
  - ***The natural frequencies of the free surface are given by :***

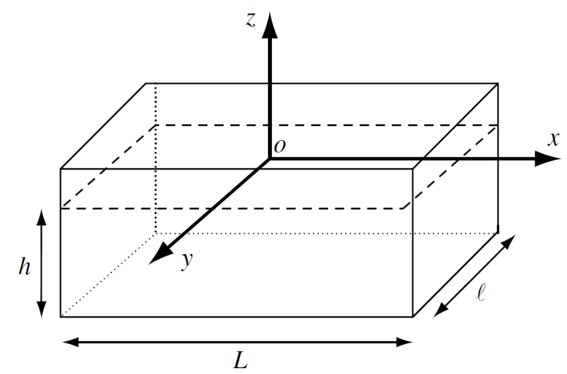
$$\omega_m^2 = g k_m \tanh(k_m h)$$

  - ***For deep liquids*** $(h \gg 2L)$ ***:*** $\tanh(k_m h) \sim 1 \;\rightarrow\; \omega_m^2 \approx g k_m$

where $k_m = (2m-1)\pi/L$ ***for asymmetric modes and*** $k_m = 2m\pi/L$ ***for symmetric modes***

First asymmetric mode $(m = 1)$: $\omega_1^2 \approx \dfrac{2\pi g}{L}$

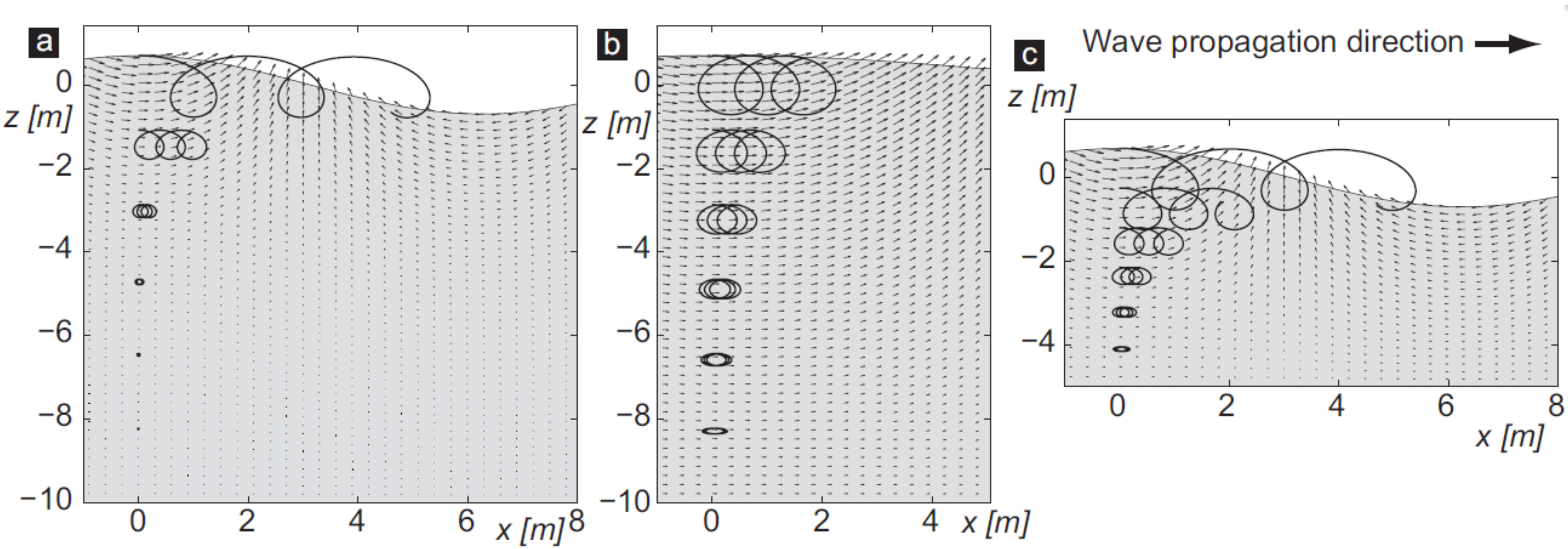




Sloshing Dynamics

- ***Linear sloshing dynamics in a large container:***
  - ***Liquid motion due to a non-breaking wave : Stokes drift***

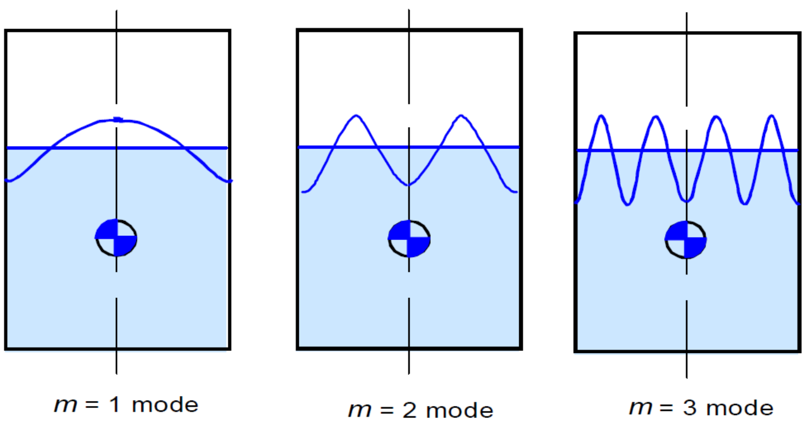
Linear wave velocity fields and trajectories followed by groups of particles released at x_0=0 and various depths (z_0) during 3 periods, for waves with different characteristics. The free surface height and the velocity fields are depicted at t =3T .
a: H_0=10m, k =0.5, a =0.7m. **b:** H_0=10m, k =0.2, a =0.7m.
c: H_0=2m, k =0.5, a =0.7m. (k =2π/λ) is the wave number, λ being the wavelength)



Sloshing Dynamics

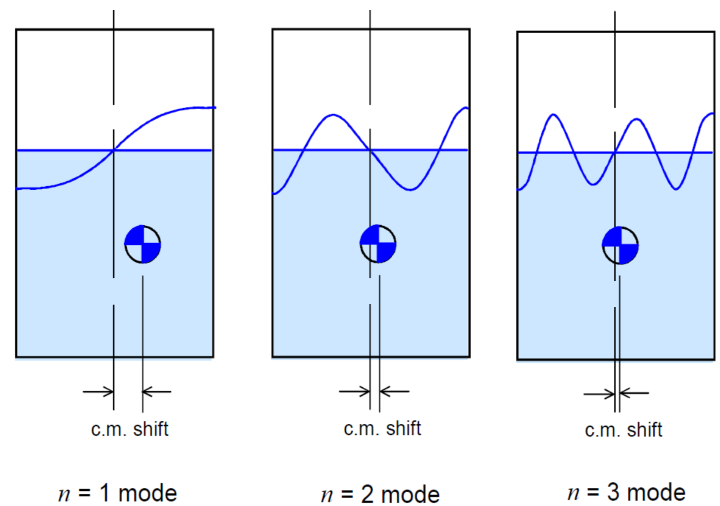
- ***Linear sloshing dynamics in a 2D container:***

1st symmetric modes (m=1, 2, 3)



Produces no drift, no lateral forces, no torque

1st asymmetric modes (n=1, 2, 3)



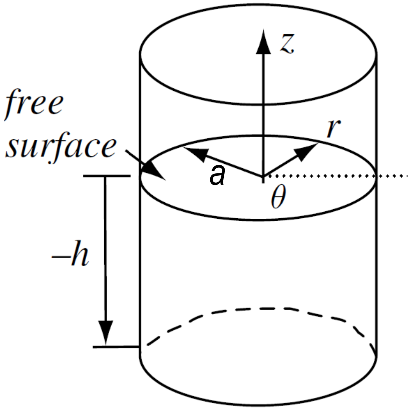
Produces drift, lateral forces and torque
Maximum drift due to 1st mode (n=1)

Sloshing Dynamics

- *Linear sloshing dynamics in a cylindrical container - Assumptions :*
  - *Cylindrical rigid container*
  - *Incompressible, inviscid and irrotational flow*
- *Equations of motion: Derivation of the sloshing velocity potential is similar to the rectangular container. The main difference is that the sines & cosines are replaced by Bessel functions $J_1(r)$ of the 1st kind (relevant solutions of Bernoulli Eq. in cylindrical coordinates).*
- *The eigen solutions and eigenvalues are:*

$$\Phi_{mn}(r,z) = J_1\left(\frac{\lambda_{mn} r}{a}\right)\cos(m\theta)\frac{\cosh[\lambda_{mn}(z/a + h/2a)]}{\cosh[\lambda_n h/a]}$$

$$\omega_{mn}^2 = \frac{g\lambda_{mn}}{a}\tanh\left(\frac{\lambda_{mn} h}{a}\right)$$

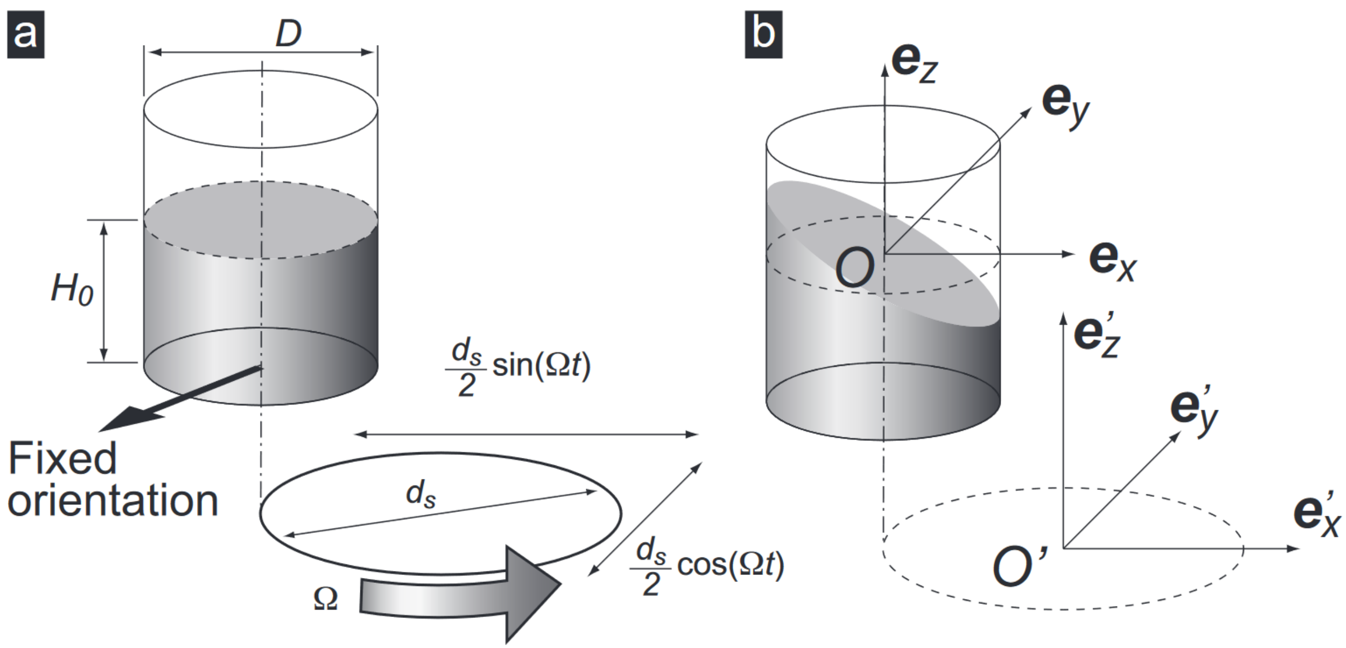


where r and θ are the radial and angular coordinates, a is the tank radius, and λ_{mn} is a root of the eigenvalue equation $dJ_1(\lambda r/a)/dr = 0$ for $r = a$.

Sloshing Dynamics

- *Orbital sloshing dynamics in a cylindrical container - Assumptions :*
  - *Upright rigid cylinder*
    - *Diameter: D, liquid height at rest: H_0, Eccentricity: ds, Rotational speed: Ω, Interface position: ξ*
  - *Incompressible, inviscid and irrotational liquid flow*
  - *Small oscillation amplitude of the interface*
- *Non-dimensional numbers:*

$$\begin{cases} \tilde{d}_s = \dfrac{d_s}{D} \\ \tilde{H}_0 = \dfrac{H_0}{D} \\ Fr = \sqrt{\dfrac{d_s \Omega^2}{g}} \\ \tilde{\xi} = \dfrac{\xi}{D} \end{cases}$$



Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container :***
  - ***Motion of the container (anywhere in the solid):***

Displacement

$$\boldsymbol{X_0(r,\theta,t)} = \begin{cases} \dfrac{d_s}{2} cos(\Omega t-\theta) e_r \\ \dfrac{d_s}{2} sin(\Omega t-\theta) e_\theta \end{cases}$$

Velocity

$$\boldsymbol{\dot{X}_0(r,\theta,t)} = \begin{cases} -\dfrac{d_s\Omega}{2} sin(\Omega t-\theta) e_r \\ \dfrac{d_s\Omega}{2} cos(\Omega t-\theta) e_\theta \end{cases}$$

  - ***Fluid motion:***

$$\nabla^2\varphi = 0 \qquad \left.\frac{\partial\varphi}{\partial r}\right|_{r=D/2} = 0 \qquad \left.\frac{\partial\varphi}{\partial z}\right|_{z=-H_0} = 0 \qquad \left.\frac{\partial\varphi}{\partial z}\right|_{z=\xi} = \frac{\partial\xi}{\partial t}$$

$$\left[\frac{\partial\varphi}{\partial t} - \frac{d_s\Omega^2 r}{2} cos(\Omega t-\theta) + gz\right]_{z=\xi} = 0$$

For more information, refer to M. Reclari, Hydrodynamics of orbital shaken bioreactors, EPFL Thesis 2014

Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container :***
  - ***The solution of Laplace's equation is obtained by a variables separation and by assuming exponential, harmonic and Bessel's functions, respectively for axial, tangential and radial directions.***

  - ***The solution reads:***

$$\tilde{\Phi}(r,\theta,z,t) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\left[\alpha_{mn}(t)\cos m\theta + \beta_{mn}(t)\sin m\theta\right] J_m(\lambda_{mn} r)\frac{\cosh[\lambda_{mn}(z+H_0)]}{\cosh\lambda_{mn}H_0}$$

Where: α_{mn} and β_{mn} are time dependent functions, J_m are the Bessel's function of the first kind of order m, $\lambda_{mn}= \varepsilon_{mn} / (D/2)$ are the roots of $\partial J_m(\lambda_{mn} r)/\partial r=0$ at $r=D/2$

For more information, refer to M. Reclari, Hydrodynamics of orbital shaken bioreactors, EPFL Thesis 2014

Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container :***
  - ***The natural frequencies of the free surface oscillations are found for non forcing case (Ω=0) and assuming***

$$\alpha_{mn}=a_{mn}\cos\omega_{mn}t \text{ and } \beta_{mn}=b_{mn}\sin\omega_{mn}t$$

$$\omega_{mn}^2 = g\lambda_{mn}\tanh(\lambda_{mn}H_0) = \frac{2g\varepsilon_{mn}}{D}\tanh\left(\frac{2\varepsilon_{mn}H_0}{D}\right)$$

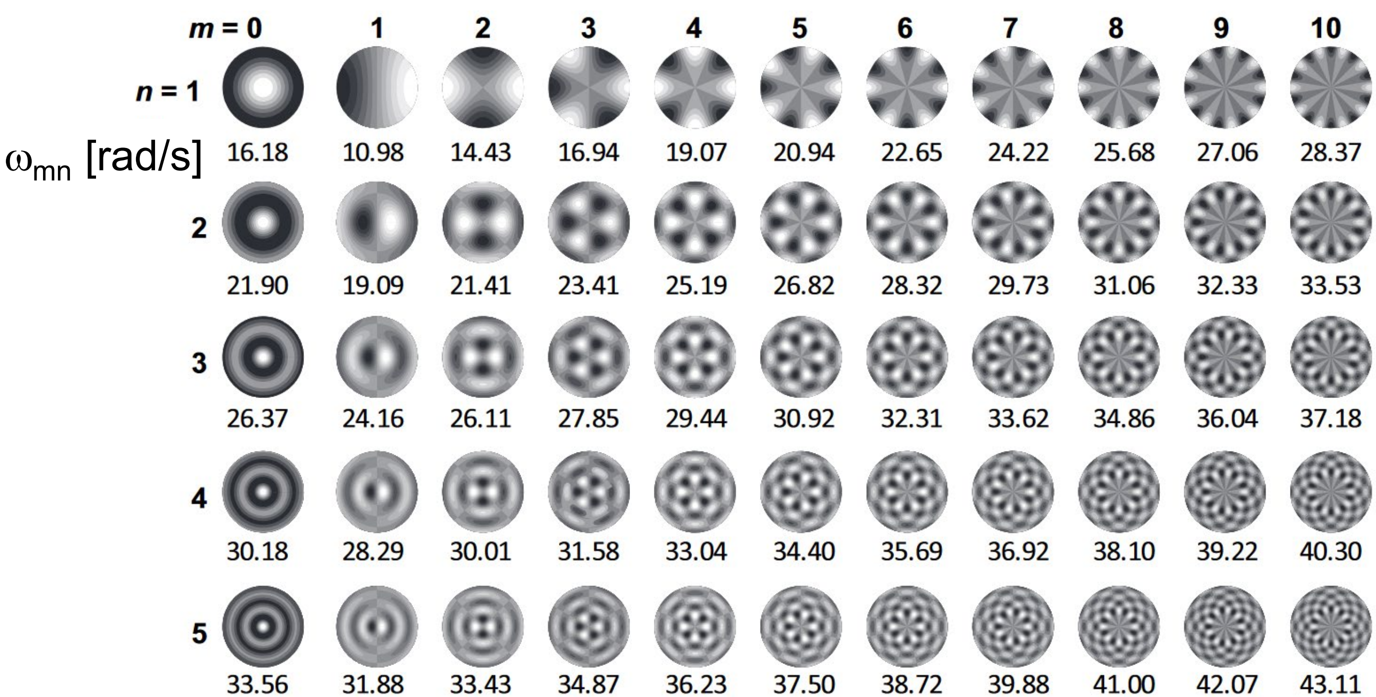
ε_{mn} Roots of the derivative of the Bessel's function of the first kind

	n=1	n=2	n=3	n=4	n=5
m=0	3.8317059702	7.0155866698	10.173468135	13.323691936	16.470630051
m=1	1.8411837813	5.3314427735	8.5363163663	11.706004903	14.863588634
m=2	3.0542369282	6.7061331941	9.9694678230	13.170370856	16.347522318
m=3	4.2011889412	8.0152365983	11.345924310	14.585848286	17.788747866
m=4	5.3175531260	9.2823962852	12.681908442	15.964107038	19.196028800
m=5	6.4156163757	10.519860873	13.987188630	17.312842488	20.575514521
m=6	7.5012661446	11.734935953	15.268181461	18.637443009	21.931715018
m=7	8.5778364897	12.932386237	16.529365884	19.941853367	23.268052926
m=8	9.6474216519	14.115518907	17.774012367	21.229062623	24.587197486
m=9	10.711433970	15.286737667	19.004593538	22.501398727	25.891277277
m=10	11.770876674	16.447852748	20.223031413	23.760715860	27.182021527

For more information, refer to M. Reclari, Hydrodynamics of orbital shaken bioreactors, EPFL Thesis 2014

- ***Orbital sloshing dynamics in a cylindrical container :***
  - ***Mode shapes of the free surface oscillations***

$$\omega_{mn}^2 = g\lambda_{mn}\tanh(\lambda_{mn}H_0) = \frac{2g\varepsilon_{mn}}{D}\tanh\left(\frac{2\varepsilon_{mn}H_0}{D}\right)$$



ω_{mn} are given in [rad/s] for D=0.287 m and H_0=0.15 m

Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container :***
  - ***The free surface elevation ξ, under forcing:***
    - ***Rotation speed Ω and eccentricity d_s***
    - ***Modes (m=1, n) are the most likely to be excited***

$$\xi(r,\theta,t) = \frac{d_s\Omega^2}{2g}\cos(\Omega t-\theta)\cdot\left\{r+\sum_{n=1}^{\infty}\left[\frac{D}{(\varepsilon_{1n}^2-1)}\frac{\Omega^2}{(\omega_{1n}^2-\Omega^2)}\frac{J_1(2\varepsilon_{1n}r/D)}{J_1(\varepsilon_{1n})}\right]\right\}$$

- ***Non dimensional form:***

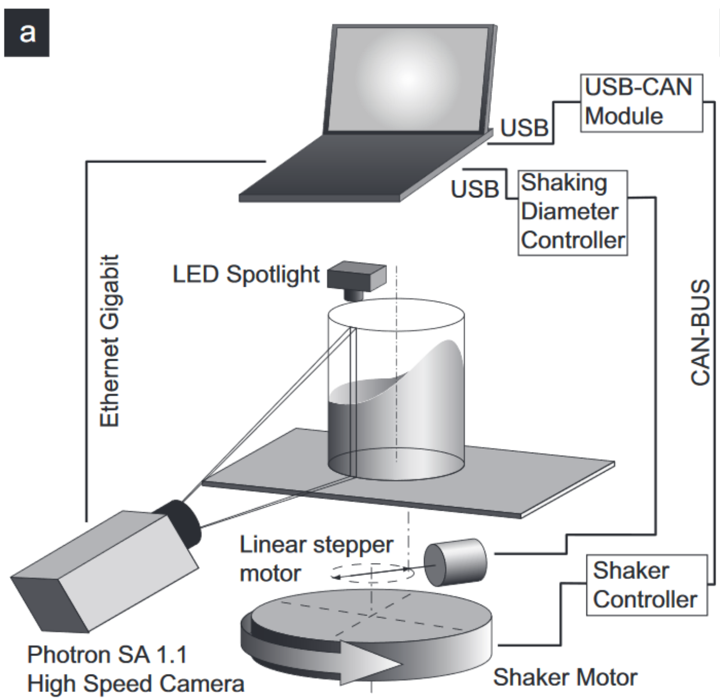
$$\tilde{\xi}(\tilde{r},\theta,t) = \frac{Fr^2}{2}\cos(\theta-\Omega t)\left\{\tilde{r}+\sum_{n=1}^{\infty}\left[\frac{1}{(\varepsilon_{1n}^2-1)}\frac{Fr^2}{(Fr_{1n}^2-Fr^2)}\frac{J_1(2\varepsilon_{1n}\tilde{r})}{J_1(\varepsilon_{1n})}\right]\right\}$$

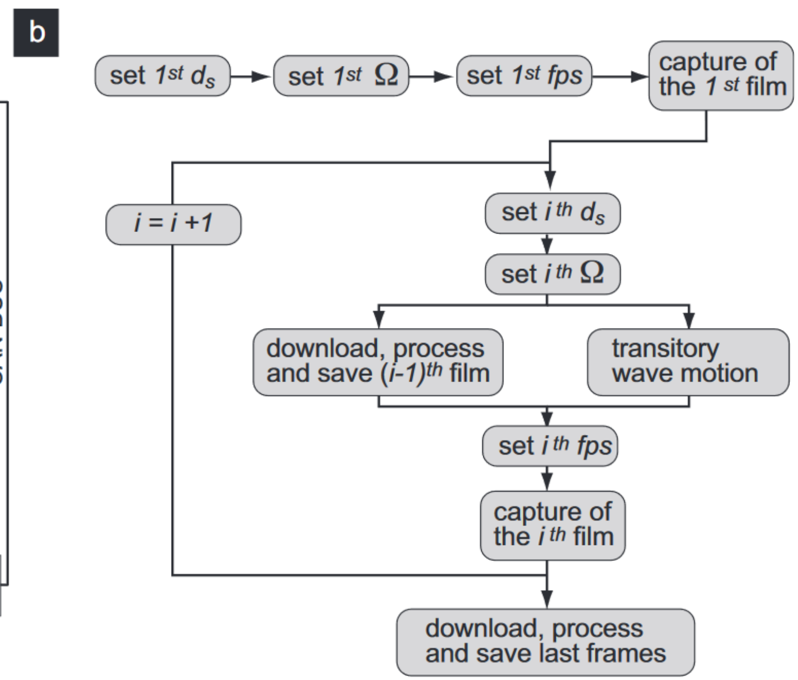
$$Fr_{1n}^2 = 2\varepsilon_{1n}\tilde{d}_s\tanh(2\varepsilon_{1n}\tilde{H}_0).$$

  - ***The free surface height increases radially as a combination of a linear function of r and of a Bessel's function***
  - ***The free surface height evolves tangentially as a sine function***

Sloshing Dynamics

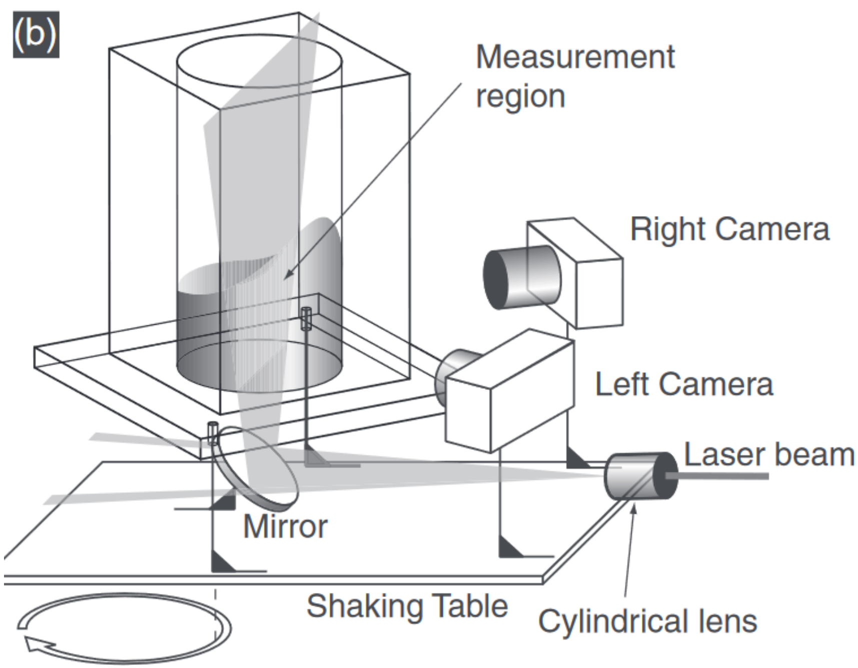
- ***Orbital sloshing dynamics in a cylindrical container :***
  - ***Experimental investigations (M. Reclari, PhD 2015):***
    - ***A Shaking table and cylindrical containers with different radii***
    - ***High speed visualization → Shape & amplitude of the interface***
      - ***Automated procedure → Wide parameters space (>6000)***

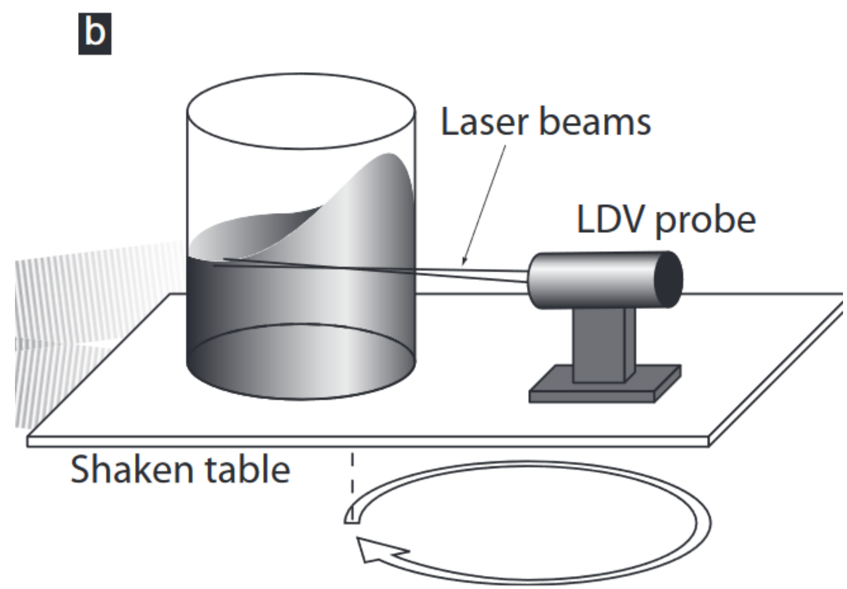




Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container :***
  - ***Experimental investigations:***
    - ***Stereo PIV setup → Measurement of the 3D velocity field***
    - ***LDV setup → time resolved measurement of local velocities***



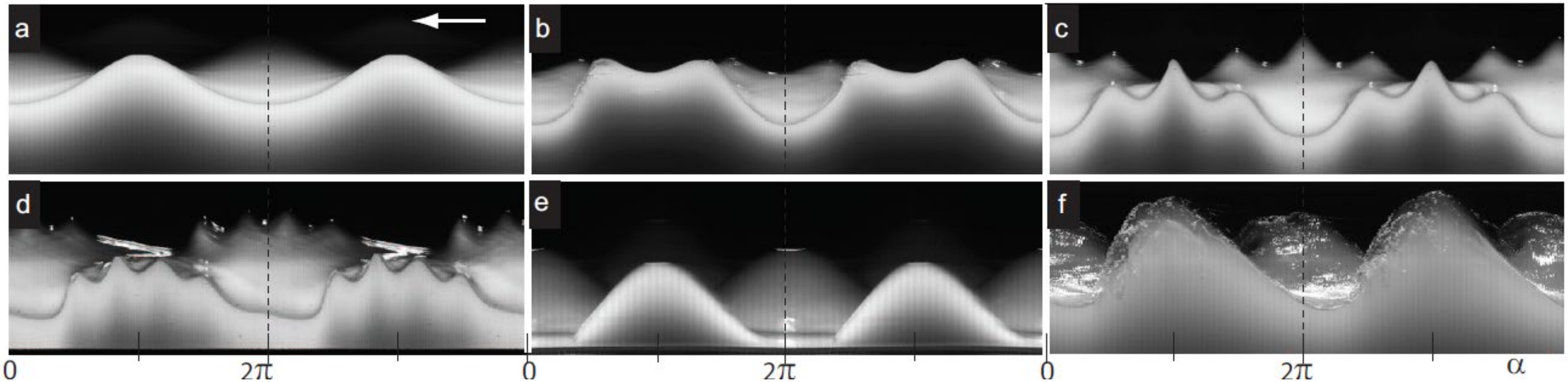


Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container :***
  - ***Experimental observation: Wave patterns***

Wave patterns reconstructed from high speed movies, depicted for two revolutions of the vessel. All waves are travelling from right to left.

- *a: single crested wave (the most usually observed).*
- *b: double crested wave.*
- *c: triple crested wave.*
- *d: quadruple crested wave.*
- *e: wave drying a portion of the vessel bottom.*
- *f: breaking single crested wave.*

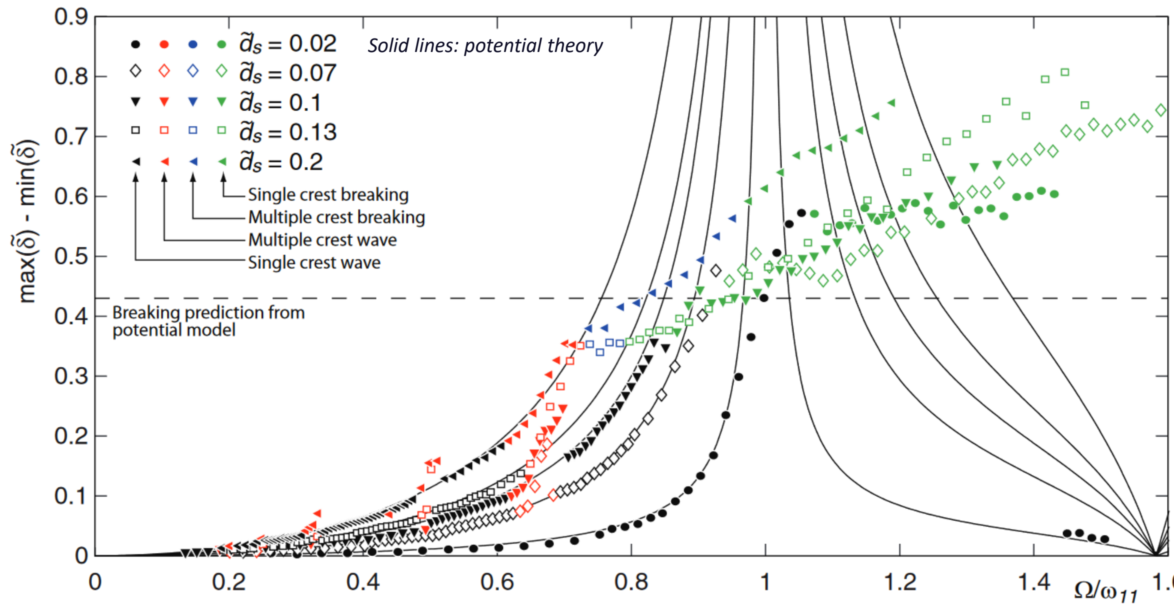


Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container :***
  - ***Experimental results - Wave amplitude:*** $\tilde{A}_\xi$

$$\tilde{\boldsymbol{\delta}} = \tilde{\boldsymbol{\xi}}\left(\boldsymbol{r} = \frac{\boldsymbol{D}}{\mathbf{2}}, \boldsymbol{\theta}, \boldsymbol{z}, \boldsymbol{t}\right)$$ *Free surface elevation at the wall*

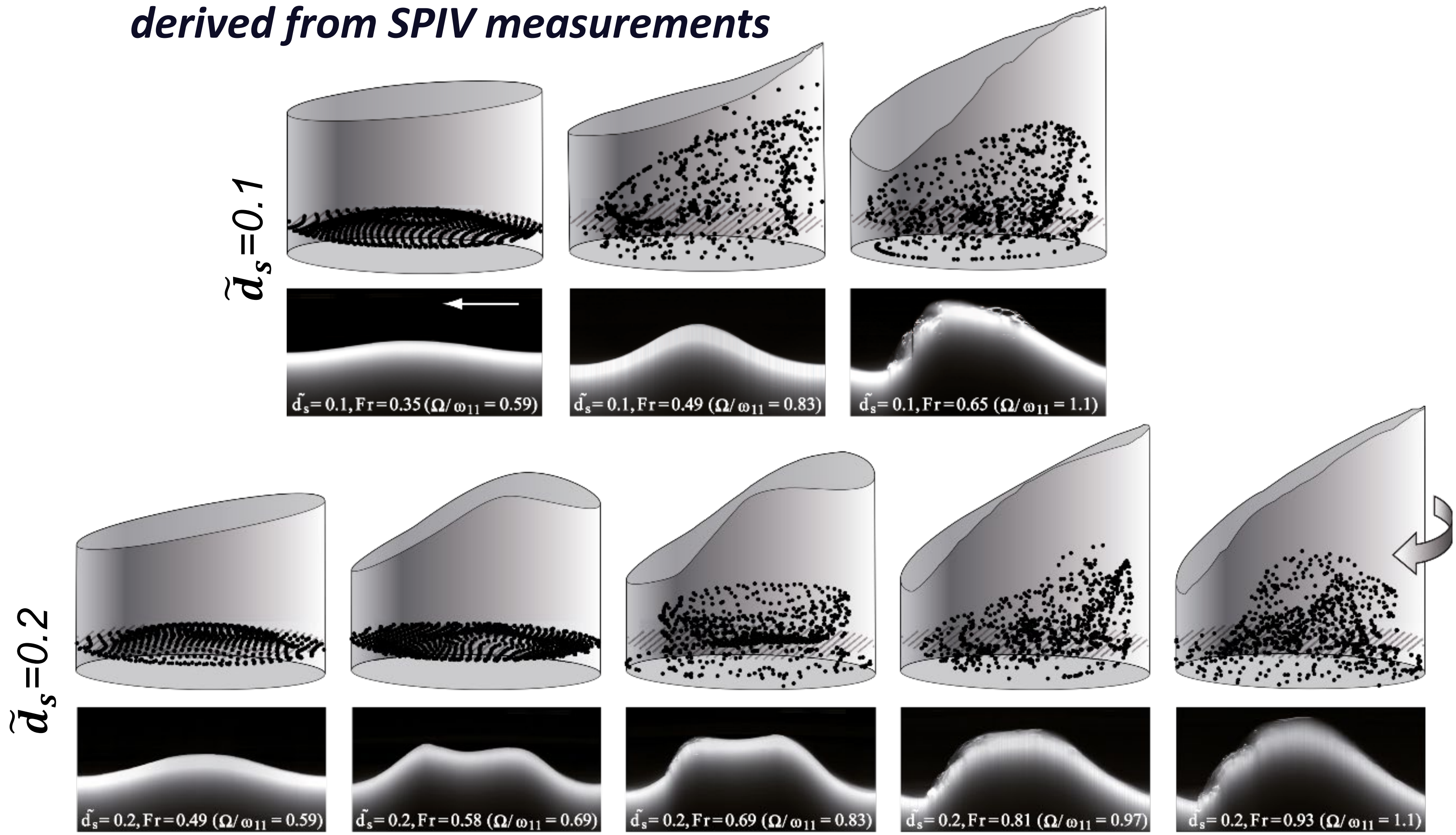
$$\tilde{A}_\xi = max(\tilde{\delta}) - min(\tilde{\delta}) = \frac{d_s\Omega^2}{g} \cdot \left\{ \frac{1}{2} + \sum_{n=1}^{\infty} \left[\frac{1}{(\varepsilon_{1n}^2 - 1)} \frac{\Omega^2}{(\omega_{1n}^2 - \Omega^2)} \right] \right\}$$



- ***Fair agreement with potential theory for small eccentricities and outside resonance***
- ***Multiple crested waves, likely excited by subharmonics of the forcing frequency***
- ***Multiple crested waves tend to break for large eccentricities***

Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container :***
  - ***Mixing performances - Effect of shaking velocity and eccentricity:***
  - ***Motion of particles released in a plane close to the container bottom derived from SPIV measurements***

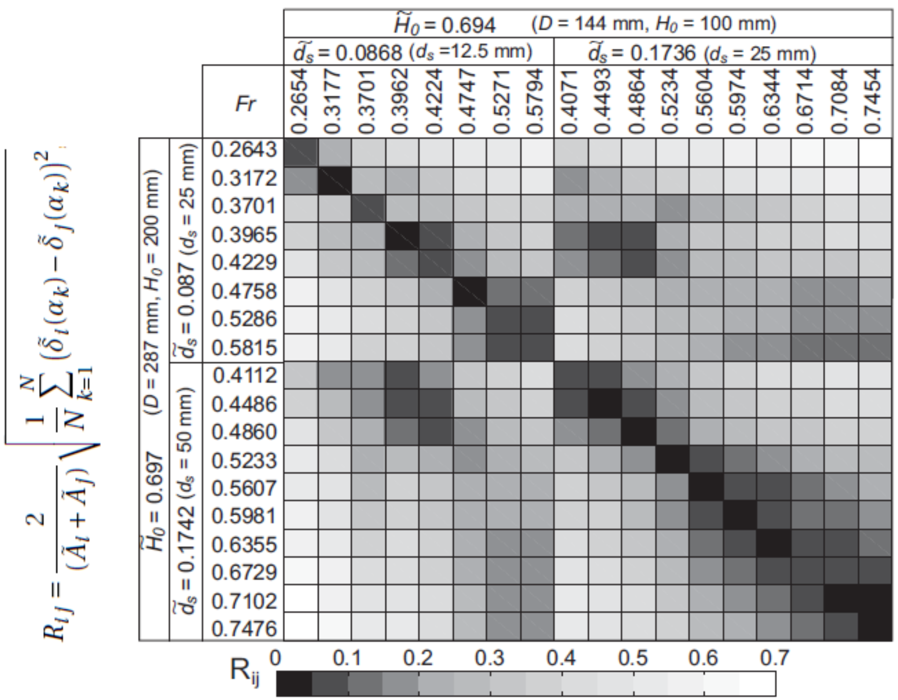


Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container :***
  - ***Scale-up (dimensionless numbers):***

$$\tilde{d}_s \equiv \frac{d_s}{D} \qquad \tilde{H}_0 \equiv \frac{H_0}{D} \qquad Fr^2 \equiv \frac{(\Omega^2 d_s)}{g} \qquad Re \equiv \frac{\rho \Omega d_s^2}{\nu}.$$

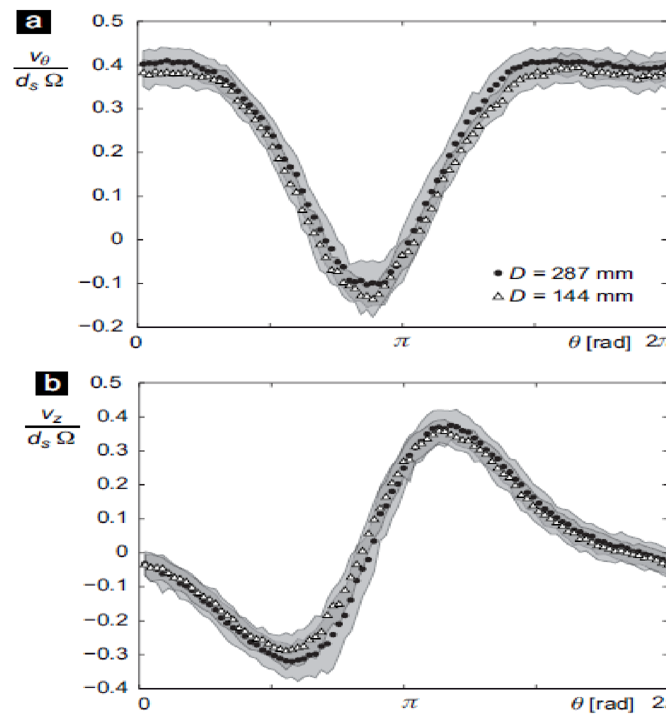
Comparison of water level for different operating conditions



Comparison of LDV velocity profiles for two container scales

$\tilde{d}_s = 0.17, Fr = 0.597$

$\tilde{H}_0 = 0.695, \Omega/\omega_{11} = 0.75$

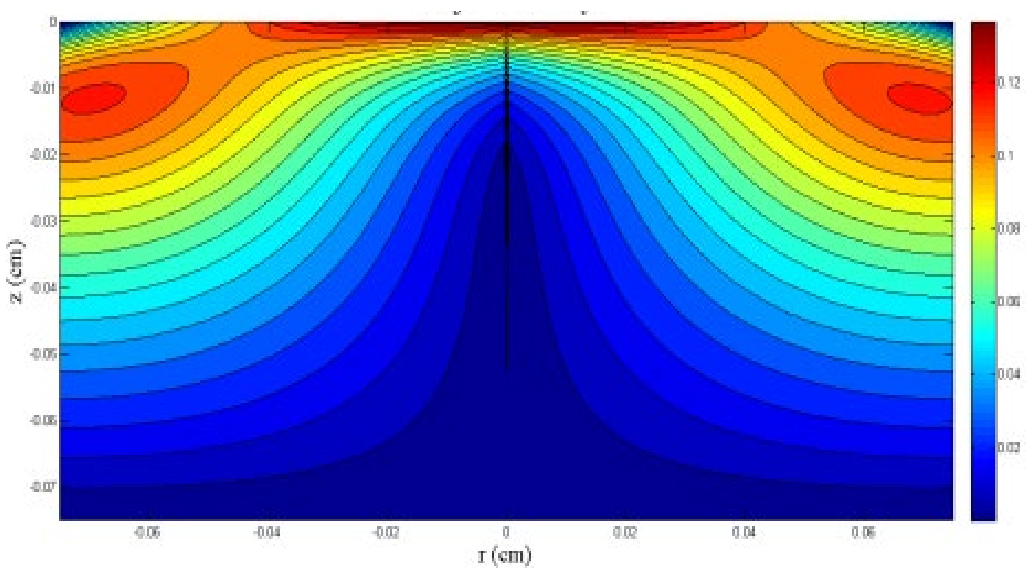


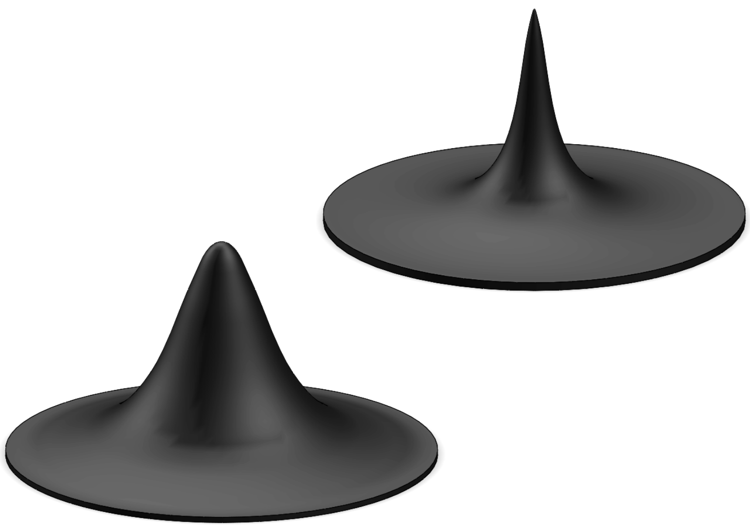
Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container***
- ***Effect of container shape ?***
    - ***Semester projects, M. Mosavi 2015, S. Eghbali 2016***

Contour of major velocity component
(Potential theory)
Existence of a low velocity area (low mixing)

Bump profiles for the container bottom
2 velocity thresholds
(3D printing)





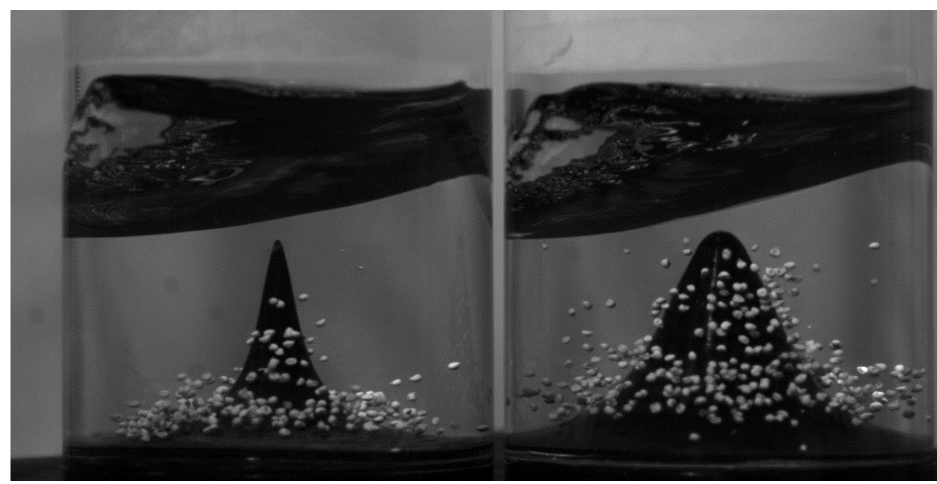
Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container***
- ***Effect of container shape ?***
  - ***Semester projects, M. Mosavi 2015, S. Eghbali 2016***

Visualisation of mixing improvements with the bump bottom using solid particles in suspension

Ω=135 rpm, d_S=44 mm, H_0=84 mm

Ω=135 rpm, d_S=44 mm, H_0=84 mm

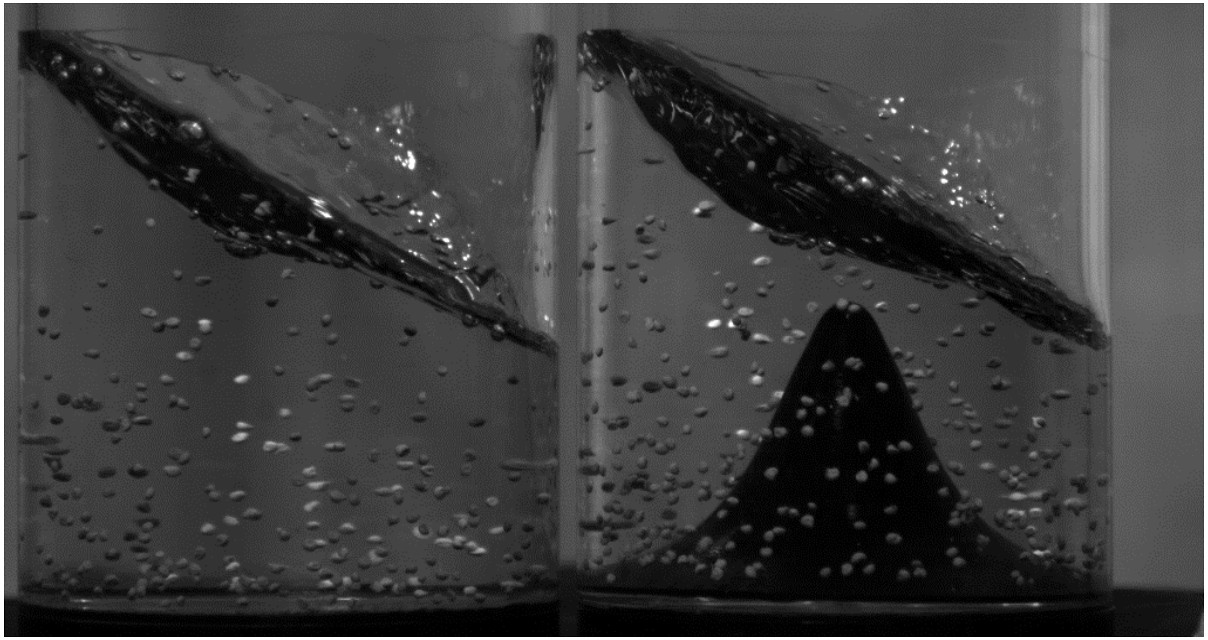


Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container***
- ***Effect of container shape ?***
    - ***Semester projects, M. Mosavi 2015, S. Eghbali 2016***

Visualisation of mixing improvements with the bump bottom using solid particles in suspension
No significant improvement in the case of breaking waves

Ω=175 rpm, d_S=44 mm, H_0=84 mm



Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container***
- ***Sloshing in Francis turbines and pump turbines in synchronous condenser mode***
    - ***Condenser mode is used to supply reactive power to the grid to cope with the fluctuations due to the intermittent renewable energies***

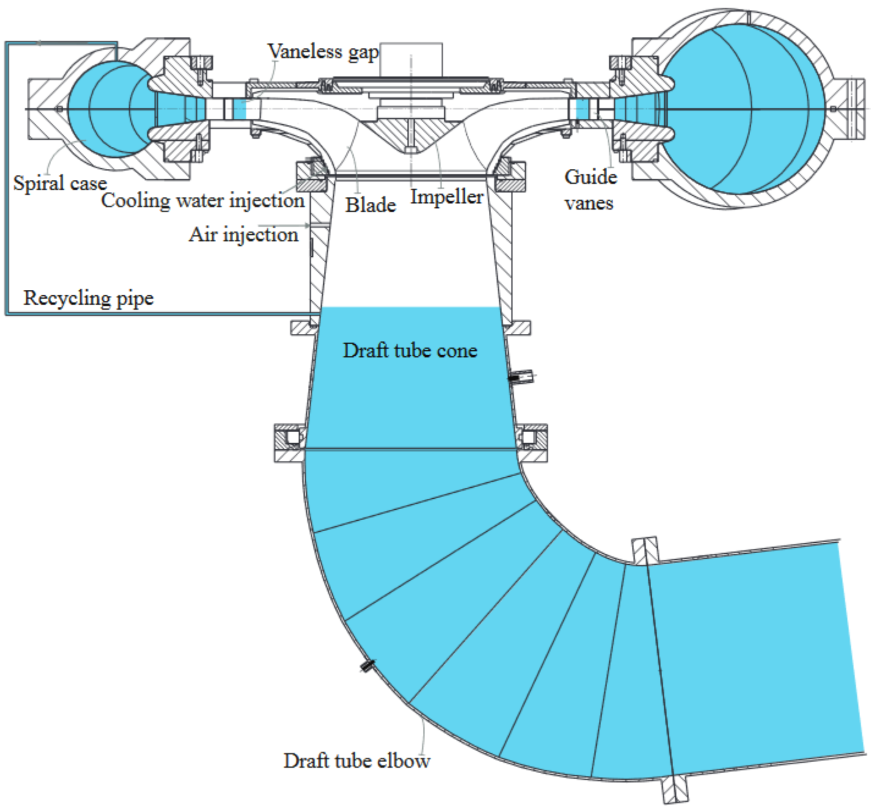


Figure 3 – Francis-type pump-turbine operating in synchronous condenser mode.

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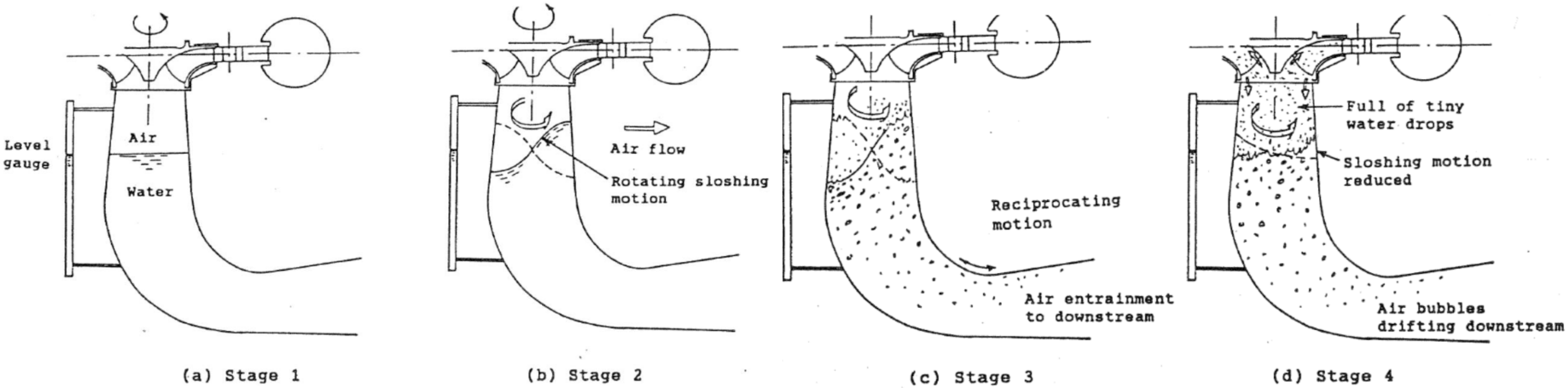
Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container***
- ***Sloshing in Francis turbines and pump turbines in synchronous condenser mode***
  - ***Condenser mode is used to supply reactive power to the grid to cope with the fluctuations due to the intermittent renewable energies***
  - ***The water column in the diffuser is excited by the runner induced air flow and develops a sloshing at the 1st resonance frequency***
  - ***Problem: Enhanced gas diffusion → increase of water level***
    → Air supply is needed to maintain the water column



Sloshing Dynamics

- *Orbital sloshing dynamics in a cylindrical container*
- *Sloshing in Francis turbines and pump turbines in synchronous condenser mode*
  - *Condenser mode is used to supply reactive power to the grid to cope with the fluctuations due to the intermittent renewable energies*
  - *The water column in the diffuser is excited by the runner induced air flow and develops a sloshing at the 1st resonance frequency*
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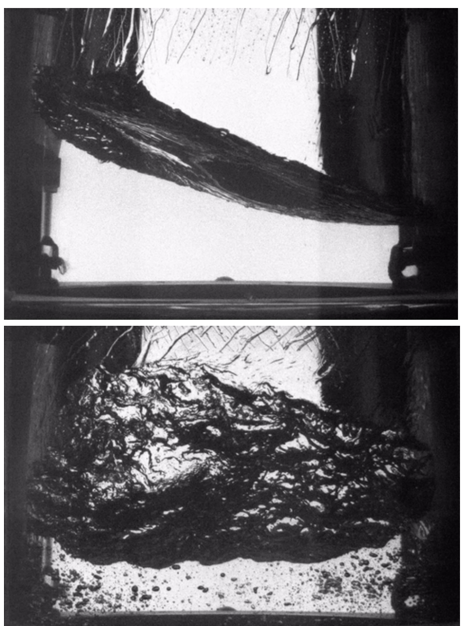

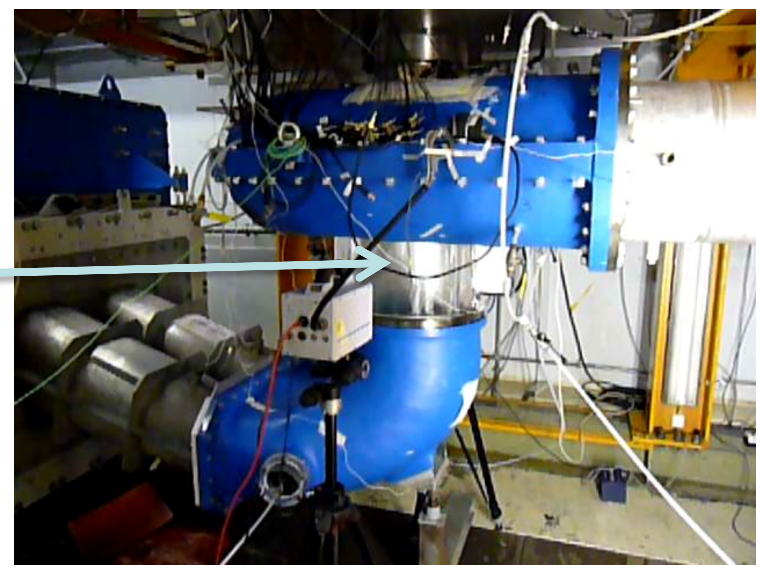

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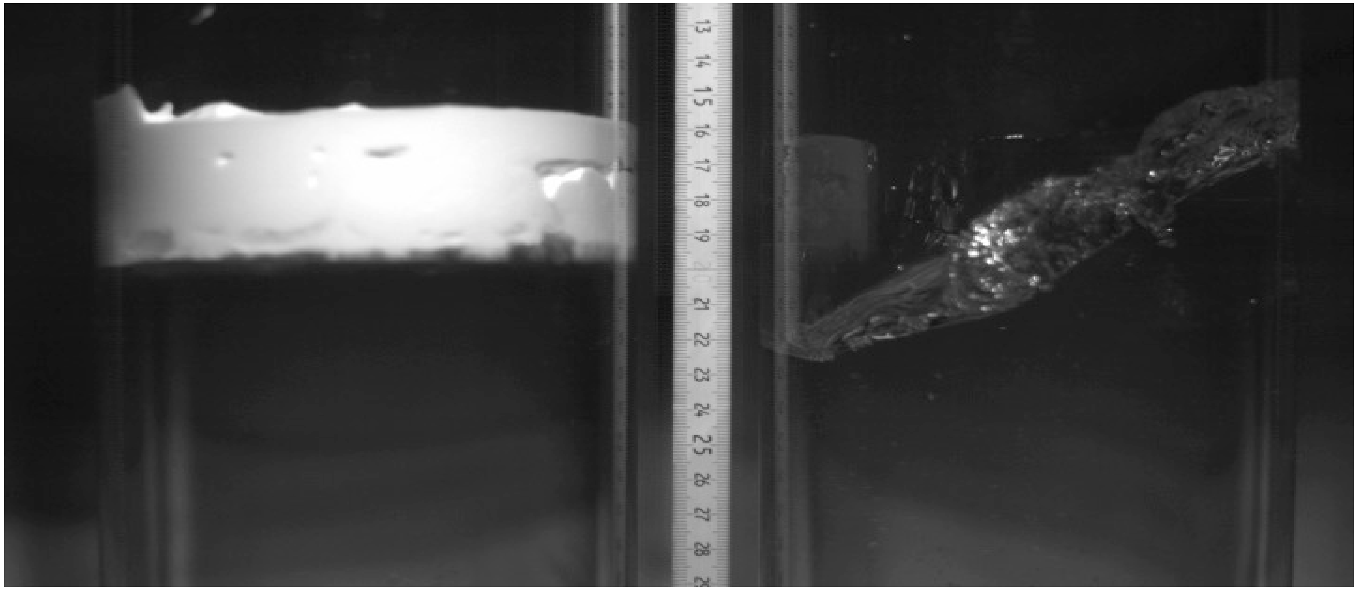
Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container***
- ***Damping of sloshing using foam - Semester project, C. Daguet, 2016***
  - ***Promising results for liquid transportation applications***
  - ***Open issues: Improve the foam stability over time***

$d_s = 8$ [mm], $H_0 = 20$ [cm], $\Omega = 195$ [rpm], $h_0 = 3$ [cm]



Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container***

- ***Additional open questions:***
    - ***Characterization of gas exchange vs operating conditions***
    - ***Effects of viscosity***
    - ***Orbital sloshing of non Newtonian fluids***
    - ***Precise measurement of the free surface shape***
    - ***Unsteady motion of the container***
    - ***Sloshing of bubbly fluids and foams***
    - ***Damping***
    - ***….***