AEROELASTICITY AND FLUID-STRUCTURE INTERACTION

Chapter 4:

Static Instability - Divergence

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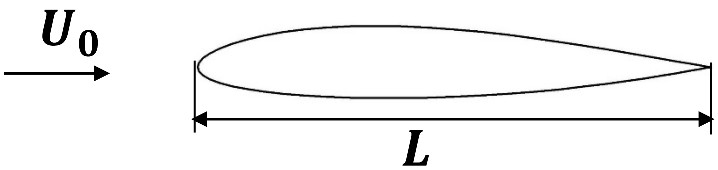
Time scales in solid and fluid domains:



$$T_{fluid} = \frac{L}{U_0}$$: *Travel time over a distance L at velocity* U_0

$$T_{solid} = \frac{L}{\sqrt{E/\rho_s}} = \frac{L}{c}$$: *Travel time over a distance L at the elestic waves celerity*

or

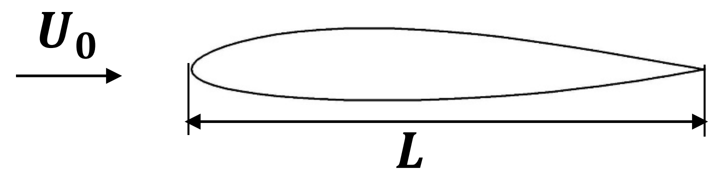
$$T_{solid} = \sqrt{m/k}$$: *Oscillation period of a given mode shape*

Hypothesis : Large reduced velocity

$$U_R = \frac{T_{solid}}{T_{fluid}} = \frac{U_0}{c} = \frac{U_0}{L}\sqrt{\frac{m}{k}} \gg 1$$

$$\Rightarrow \quad T_{solid} \gg T_{fluid}$$

$U_R \ll 1$	$U_R \gg 1$
The solid evolves in an almost still fluid	***The fluid evolves with an almost fixed solid***



→ Added stiffness & Added mass (See Chapter 2)

U_0 ~ 100 m/s; L ~ 1 m; T_{fluid} ~ 0.01 s
Torsional frequency ~ 1 Hz → T_{solid} ~ 1 s
→ U_R ~ 100 >> 1

Dimensionless numbers for large reduced velocity :

$$Re = \frac{\rho U_0 L}{\mu} \qquad Fr = \frac{U_0}{\sqrt{gL}} \qquad Cy = \frac{\rho U_0^2}{E} \qquad U_R = \frac{U_0}{c}$$

U_0 is relevant with the hypothesis of large reduced velocity

T_{solid} is no more relevant with the hypothesis of $U_R >> 1$
→ Alternate choice : T_{fluid} instead of T_{solid}

Non dimensional variables:

$$t_f = \frac{t}{T_{fluid}} = \frac{tU_0}{L} \qquad U_f = \frac{U}{U_0} \qquad p_f = \frac{p}{\rho U_0^2}$$

Dimensionless equations of fluid and solid motions

- ***On the fluid side: Navier-Stokes equations***

$$\nabla U_f = 0 \qquad \frac{dU_f}{dt_f} = -\frac{1}{Fr^2} e_z - \nabla p_f + \frac{1}{Re} \Delta U_f$$

- ***On the solid side (single mode approximation):***

$$\xi_s\,(x_s, t_s) = D q_s(t_s) \phi(x_s)$$

$$U_R{}^2 \frac{d^2 q_s}{dt_f^2} + q_s = f_s$$

Dimensionless equations of fluid and solid motions

- ***At the Fluid-Solid interface:***
  - ***Kinematic conditions:***

$$\boldsymbol{U = \frac{d\xi}{dt}} \quad \Rightarrow \quad \boldsymbol{U_f = \frac{d\xi_s}{dt_f} \approx O\left(\frac{\frac{\xi_0}{L}}{\frac{T_{solid}}{T_{fluid}}}\right) = O\left(\frac{D}{U_R}\right)}$$

  - ***Assumption of large reduced velocity:***

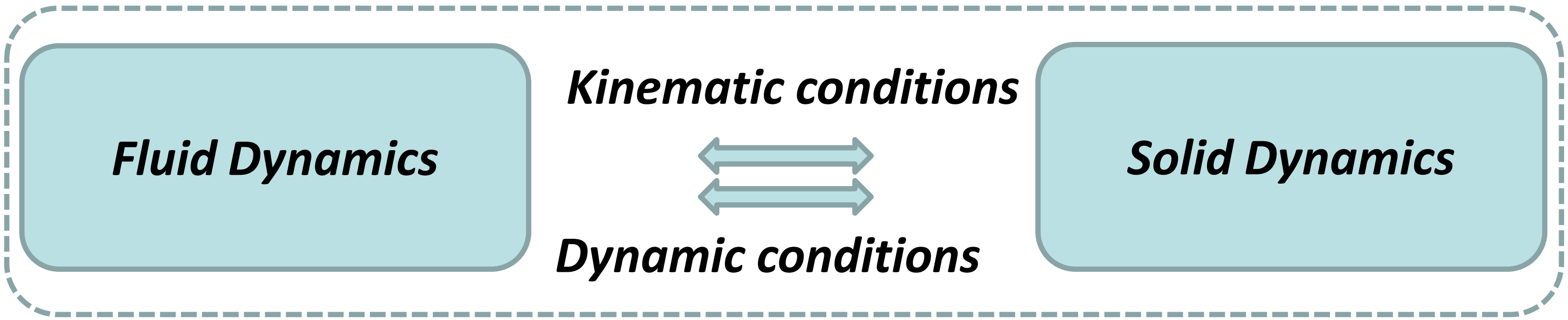
$$\boldsymbol{U_R \gg D \Rightarrow U_f \approx 0}$$ ***At the interface***

- ***Dynamic conditions along the interface (I):***

$$\boldsymbol{Cy\int_I \left\{\left[-p_f I + \frac{1}{Re}\left(\nabla U_f + \nabla^t U_f\right)\right].n\right\}.\phi dS = Df_s}$$

Quasi-Static Aeroelasticity

- ***General case:***
  - ***The fluid-solid interface has a deformation and a velocity***



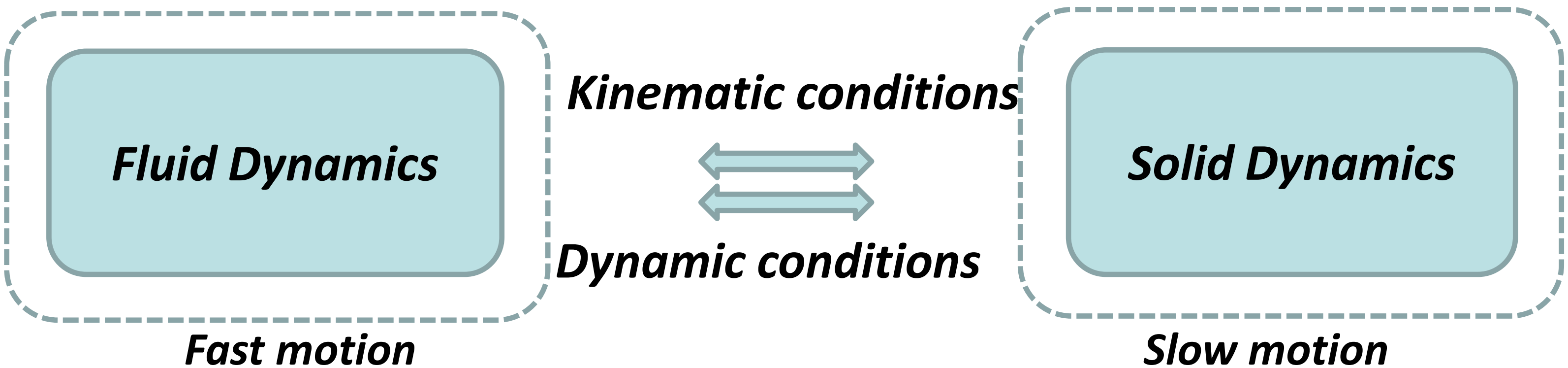
  - ***The motions in the fluid and in the solid must be solved simultaneously***

Quasi-Static Aeroelasticity

- ***Quasi-Static Aeroelasticity:***
  - ***The fluid-solid interface deforms "without" velocity***

  → ***Within the fluid time scale, the motion of the solid is so slow that we can neglect it → the solid does not move***

$$U_R \gg D$$



→ ***Due to extremely different time scales, there is no need to solve the solid and fluid motions simultaneously***
We can solve one domain at a time (iterations)

Static Instability - Divergence

- ***Fluid loading on the solid:***
  - ***Single mode approximation:***

$$\xi_s\,(x_s, t_s) = Dq_s(t_s)\phi(x_s)$$

  - ***Fixed interface → the flow depends on the position of the interface only (steady state) :***

Instantaneous position of the interface

$$p_f\,(Dq_s)\ and\quad U_f(Dq_s)$$

$$Df_s = C_y \int_I \left\{\left[-p_f I + \frac{1}{Re}\left(\nabla U_f + \nabla^t U_f\right)\right].n\right\}.\phi dS = C_y F(Re, Dq_s)$$

Static Instability - Divergence

- ***Expansion of fluid loading (small displacement Dq_s):***
  - ***Flow induced force from the motion of the interface***

$$C_y F(Re, Dq_s) = C_y F^0 + C_y \left(\frac{\partial F}{\partial Dq_s}\right) Dq_s + \cdots$$

  - ***Proportional to modal displacement q_s → Stiffness force***

→ ***The solid behaves like if it was attached to a spring with a stiffness***

$$k_f = -C_y \left(\frac{\partial F}{\partial Dq_s}\right)$$

  - ***The flow induced stiffness depends only on how the fluid reacts to the solid displacement and may be positive or negative***

Static Instability - Divergence

- ***Solid motion:***

$$U_R{}^2 \frac{d^2 q_s}{dt_f^2} + q_s = f_s$$

  - ***Using t_s instead of t_f for the reference time***

$$\Rightarrow \frac{d^2 q_s}{dt_s^2} + q_s = f_s$$

  - ***After incorporating the flow induced stiffness:***

$$\frac{d^2 q_s}{dt_s^2} + \left(1 - C_y \left(\frac{\partial F}{\partial D q_s}\right)^0\right) q_s = 0$$

Static Instability - Divergence

- ***Solid motion:***

$$\frac{d^2 q_s}{dt_s^2} + \left(1 - C_y\left(\frac{\partial F}{\partial D q_s}\right)\right) q_s = 0$$

  - ***Flow induced stiffness proportional to*** U_0^2 $\left(C_y = \rho U_0^2 / E\right)$

  - ***If*** $\left(\frac{\partial F}{\partial D q_s}\right) > 0$***, as the flow velocity is increased, the total stiffness decreases as well as the resonance frequency***

  - ***When*** $1 - C_y\left(\frac{\partial F}{\partial D q_s}\right) \leq 0$

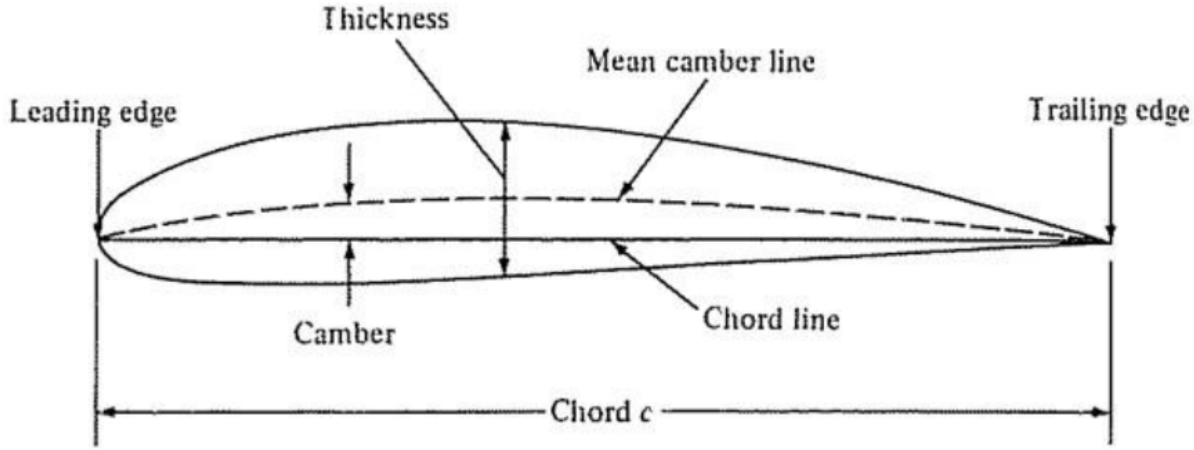
    → Static instability, also called divergence or buckling

    → Unbounded increase of oscillation amplitude

Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder*):

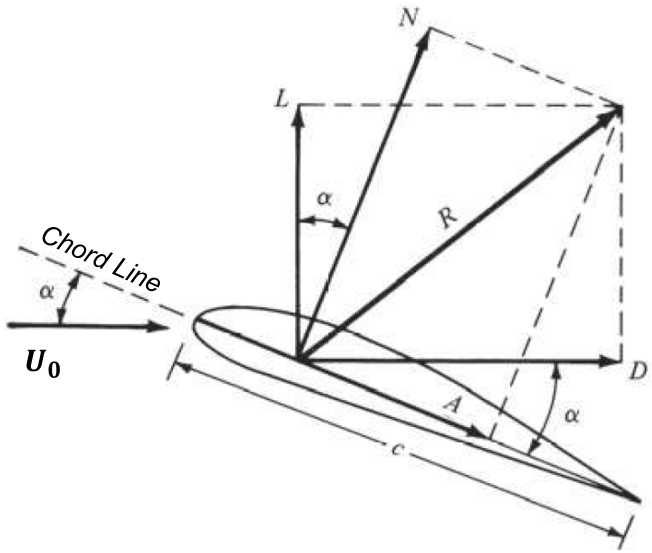
- **Nomenclature related to airfoils:**
  - **Mean camber line: located halfway between upper and lower surfaces**
  - **Chord line: Straight line connecting the leading and trailing edges**
  - **Chord length: Distance between leading and trailing edges (along the chord line)**
  - **Camber: Maximum distance between mean camber line and chord line Measured perpendicularly to the chord line.**



Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

- **Lift and drag:**
  - **Angle of attack, α: Angle between relative wind and chord line**
  - **Total aerodynamic force, R, can be decomposed into 2 components:**
    - **Lift, L: Component of aerodynamic force perpendicular to relative wind**
    - **Drag, D: Component of aerodynamic force parallel to relative wind**



Lift coefficient: $$C_L = \frac{L}{\frac{1}{2}\rho U_0^2 S} = \frac{L}{q_\infty S}$$

Drag coefficient: $$C_D = \frac{D}{\frac{1}{2}\rho U_0^2 S} = \frac{D}{q_\infty S}$$

With $$q_\infty = \frac{1}{2}\rho U_0^2$$

Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

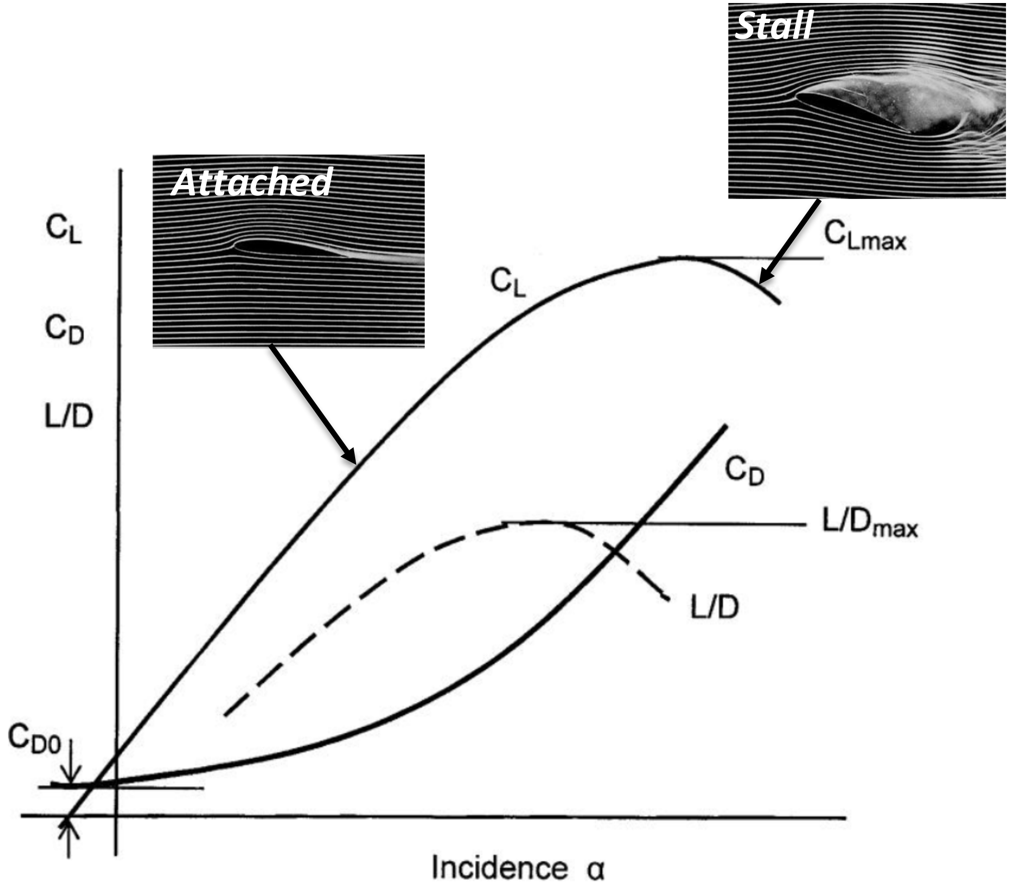
- **Lift and drag:**

Lift coefficient (per unit length):

$$c_L = \frac{L}{\frac{1}{2}\rho U_0^2 c} = \frac{L}{q_\infty c}$$

Drag coefficient (per unit length):

$$c_D = \frac{D}{\frac{1}{2}\rho U_0^2 c} = \frac{D}{q_\infty c}$$



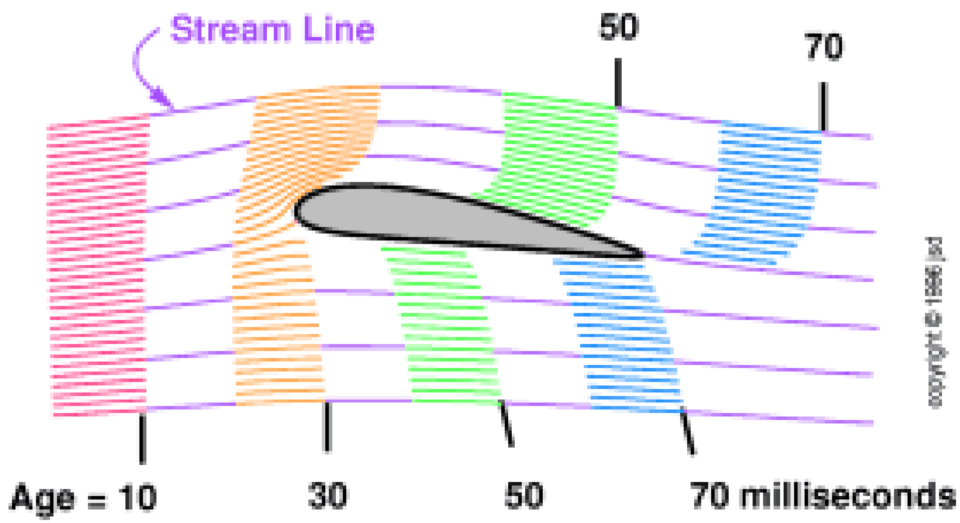
- ***Aerodynamic efficiency:*** $\frac{L}{D} = \frac{c_L}{c_D}$

  - **For a given geometry, c_L and c_D are functions of incidence angle <u>and</u> Re number**

Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

- **Mechanism of lift generation**:
  - *Illustration of the fluid displacement around a foil using transient injections of colored smokes (during 10 msec)*



  - *The flow is accelerated on the upper surface (suction side) with respect to the lower surface (pressure side) → pressure difference → lift generation*

Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

- **Thin Airfoil Theory (TAT)**:
  - Airfoil thickness < 12% of the chord length
  - Small incidence angle ($\alpha << \alpha_{stall}$)
  - Inviscid and incompressible flow

$$\rightarrow \frac{dC_L}{d\alpha} = 2\pi$$

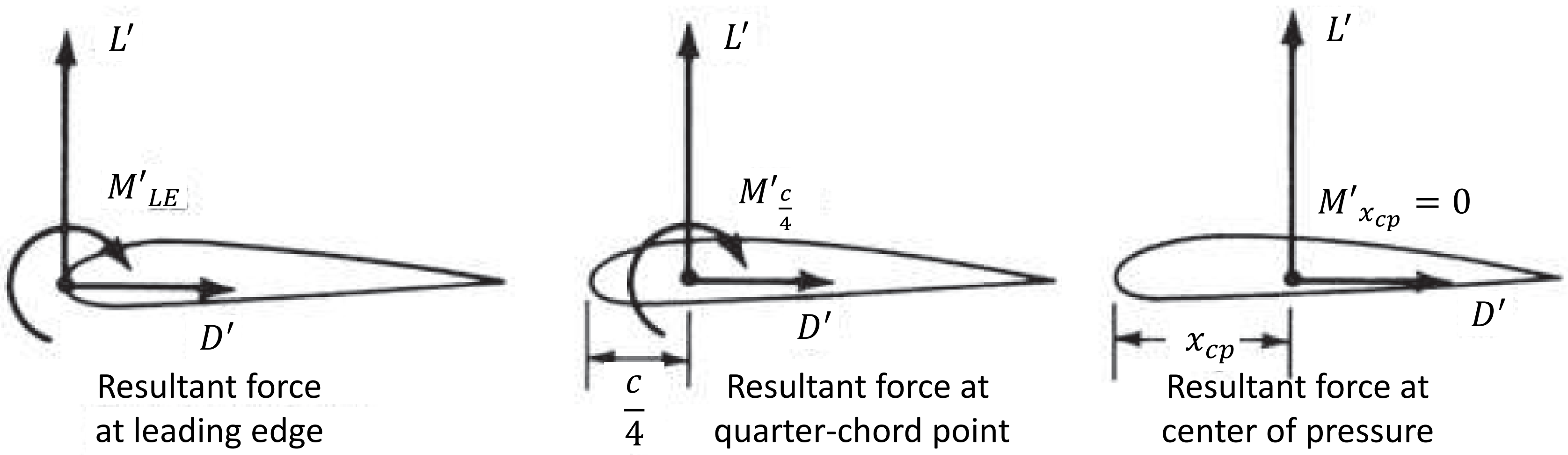
  - ***Center of pressure (CP)***: point where the moment of aerodynamic forces is zero. Its location depends on incidence angle and may be located outside of the foil. Not very practical for engineers. *The center of pressure is to pressure what the gravity center is to gravitational forces*

  - ***Aerodynamic center (AC)***: point where the moment of aerodynamic forces does not depend on incidence angle.
    TAT → *AC* is located c/4 away from the leading edge (c : chord length)

Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

- **Thin Airfoil Theory (TAT)**:



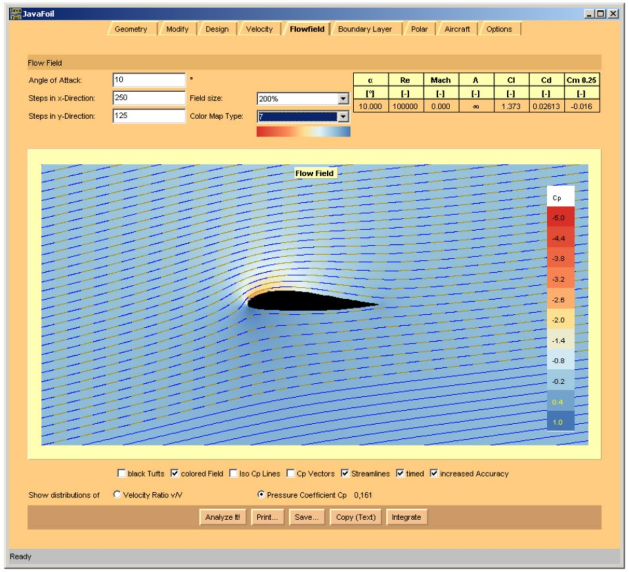
Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

- *JAVAFOIL solver: is a relatively simple program, which uses several traditional methods for the analysis of a subsonic flow around a profiled body and provides a quick and fair estimation of the velocity and pressure distributions along with the lift and drag forces.*

- *Training:*
  - *Install JAVAFOIL in your computer and use it for a foil of simple geometry (NACA)*
  - *Analyze the flow for different operating parameters:*
    - *Plot velocity and pressure fields for different incidence angles*
    - *Plot $C_L(\alpha)$, $C_D(\alpha)$ and $C_L(C_D)$*
    - *Verify that $\frac{dC_L}{d\alpha} = 2\pi$*
    - *Verify that the moment of aerodynamic forces is independent on incidence angle, when computed at the aerodynamic center.*
    - *Examin the influence of upstream velocity*

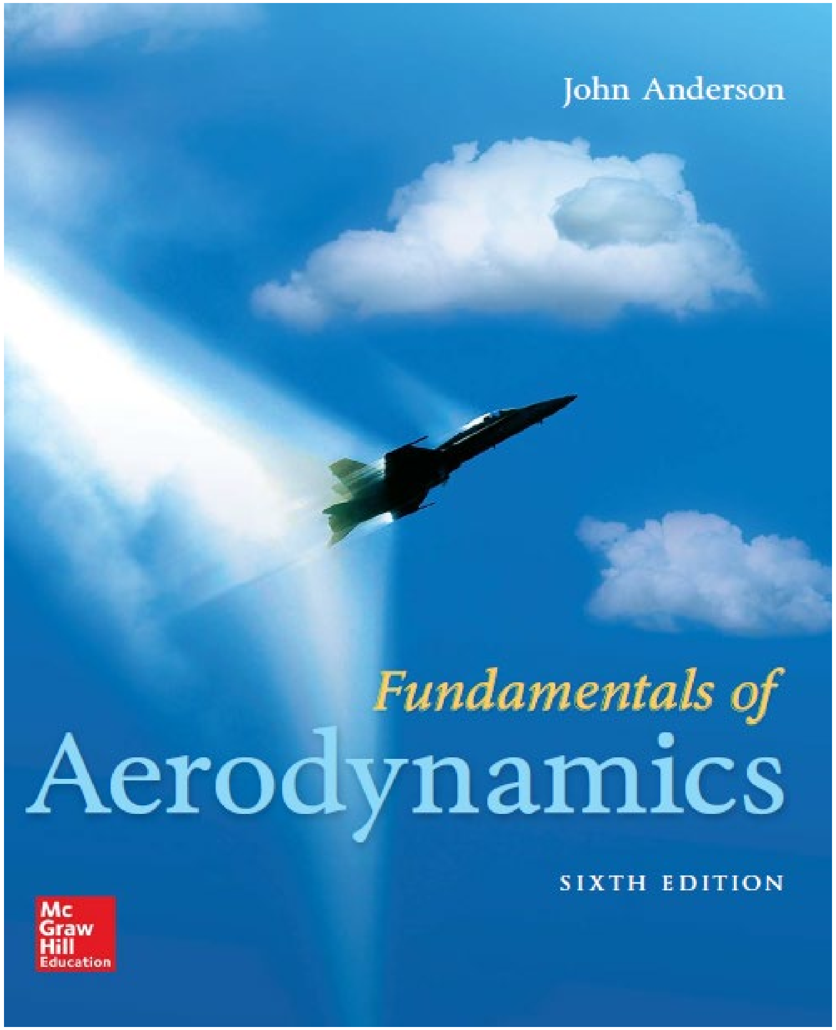
    - *Reference:*
      - *JAVAFOIL User's Guide, M. Hepperle, 2017*
      - https://www.mh-aerotools.de/airfoils/jf_applet.htm



Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

- ***Good reference for aerodynamics:***



Application: Torsional divergence of an airfoil

- **The case study:**
  - We consider a two-dimensional airfoil restrained by a rotation spring (stiffness k_α), placed in air stream of velocity U_0. The angle of attack, α, is the sum of the rigid foil angle, α_r, and the structural twist angle, θ. The aerodynamic center (resp. gravity center) is located at a distance e (resp. d) of the elastic center, counted positive towards the leading edge.

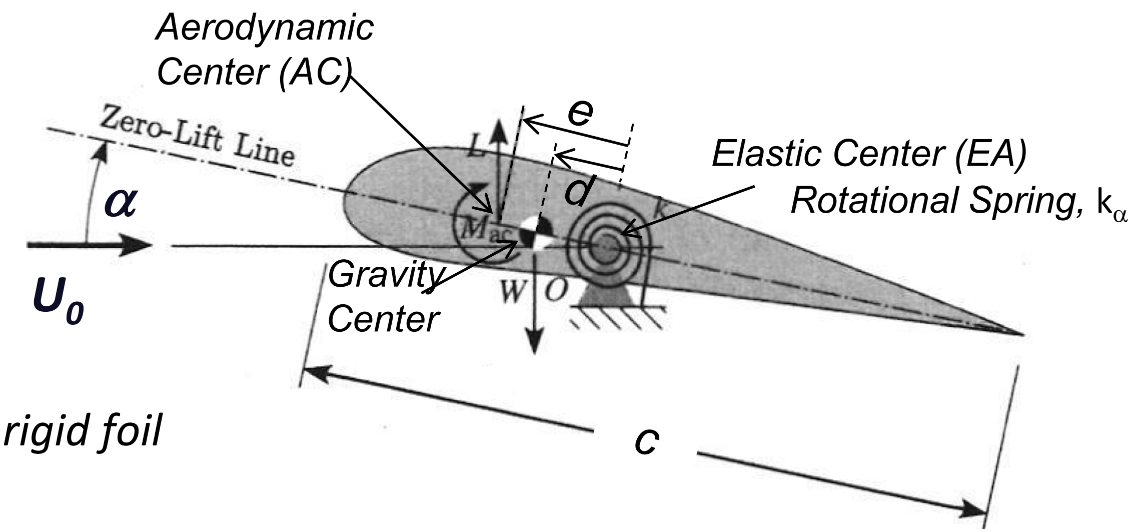
$\boldsymbol{L}$: *Lift force*

$\boldsymbol{W}$: *Weight*

$\boldsymbol{\alpha = \alpha_r + \theta}$

$\boldsymbol{\alpha_r}$ *: incidence angle of a rigid foil*

$\boldsymbol{\theta}$ *: elastic twist angle*



Application: Torsional divergence of an airfoil

- **Assumption: Incompressible flow**
  - Compressibility may be taken into account using Prandtl-Glauert corrections for Mach Nb up to ~0.8

- **The lift always acts upstream of the elastic center (e>0)**

  → an increase of the lift force leads to an increase of α, which in its turn, increases the lift !

  → Unstable

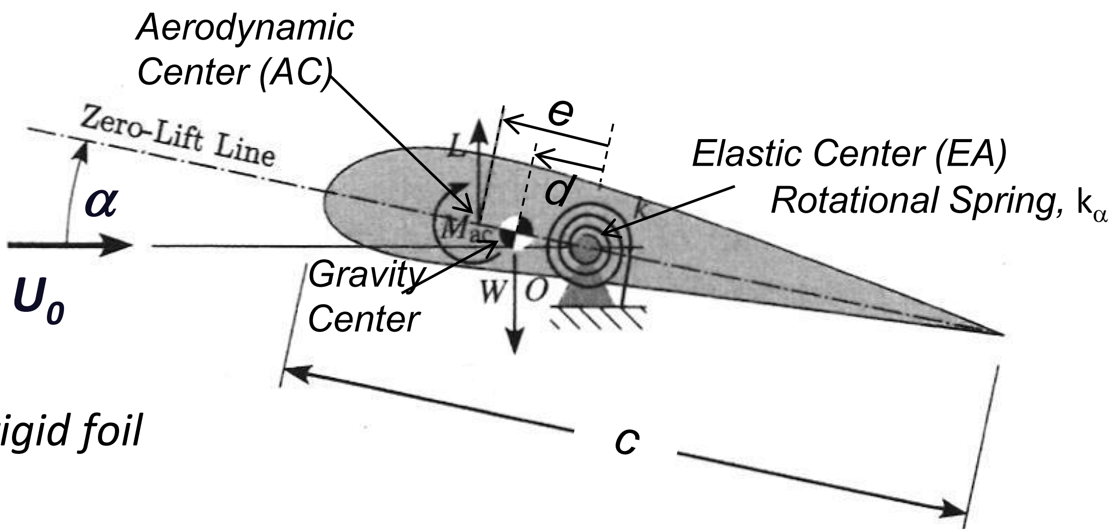
$\boldsymbol{L}$: Lift force

$\boldsymbol{W}$: Weight

$\boldsymbol{\alpha} = \boldsymbol{\alpha_r} + \boldsymbol{\theta}$

$\boldsymbol{\alpha_r}$: incidence angle of a rigid foil

$\boldsymbol{\theta}$: elastic twist angle



Application: Torsional divergence of an airfoil

- We assume the airfoil is pivoted about its elastic center (EC) with an angle of attack sufficiently small, so that **cosα ~ 1 and sinα ~ α**
Equilibrium equation (expressed at the elastic center):

$$\sum \boldsymbol{moments} = \mathbf{0} \quad \rightarrow \quad \boldsymbol{M_{ac}} + \boldsymbol{Le} - \boldsymbol{Wd} - \boldsymbol{k_\alpha \theta} = \mathbf{0}$$

Lift for an elastic foil: $\boldsymbol{L = q_\infty S C_L = q_\infty S C_{L,\alpha} \alpha = q_\infty S C_{L,\alpha}(\alpha_r + \theta)}$

with: $\boldsymbol{q_\infty = \frac{1}{2}\rho U_0^2}$, $\boldsymbol{C_{L,\alpha} = \frac{dC_L}{d\alpha}}$ and $\boldsymbol{S}$ is the airfoil area

Moment of aerodynamic forces: $\boldsymbol{M_{ac} = q_\infty S c C_{M_{ac}}}$

Thin airfoil assumption: $\boldsymbol{C_{L,\alpha}}$ & $\boldsymbol{C_{M_{ac}}}$ constant ($\boldsymbol{C_{M_{ac}}}$ =0 for symmetric foil)

Application: Torsional divergence of an airfoil

- The equilibrium equation becomes:

$$q_\infty S c C_{M_{ac}} + q_\infty S e C_{L,\alpha}(\alpha_r + \theta) - W d = k_\alpha \theta$$

$$\rightarrow \theta = \frac{q_\infty S c C_{M_{ac}} + q_\infty S e C_{L,\alpha}\alpha_r - W d}{k_\alpha - q_\infty S e C_{L,\alpha}}$$

- Divergence occurs when the denominator vanishes:

$$\rightarrow q_\infty = q_d = \frac{k_\alpha}{S e C_{L,\alpha}}$$

  - *Thin airfoil theory (2D airfoil) :*

$$C_{L,\alpha} = 2\pi \implies q_d = \frac{k_\alpha}{2\pi S e}$$

Application: Torsional divergence of an airfoil

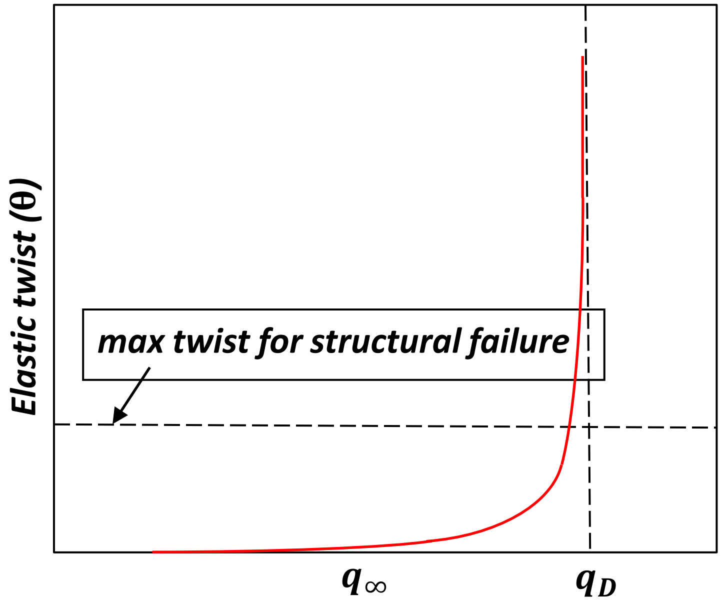
- Divergence speed $\boldsymbol{U_D}$:

$$q_D = \frac{1}{2}\rho U_D^2 = \frac{k_\alpha}{SeC_{L,\alpha}} \qquad U_D = \sqrt{\frac{2k_\alpha}{\rho SeC_{L,\alpha}}}$$

- Assumption:
  - Symmetric foil : $\boldsymbol{M_{ac} = 0}$
  - Gravity center = elastic center $(d = 0)$

Then the twist angle may be written as:

$$\theta = \frac{\alpha_r}{\frac{q_D}{q_\infty} - 1} \qquad \lim_{q_\infty \to q_D} \theta = +\infty$$



Elastic twist vs. Dynamic pressure

Application: Torsional divergence of an airfoil

Interpretation of divergence:

- **Twist angle:**

$$\boldsymbol{\theta = \frac{q_\infty ScC_{M_{ac}} + q_\infty SeC_{L,\alpha}\alpha_r - Wd}{k_\alpha - q_\infty SeC_{L,\alpha}}}$$

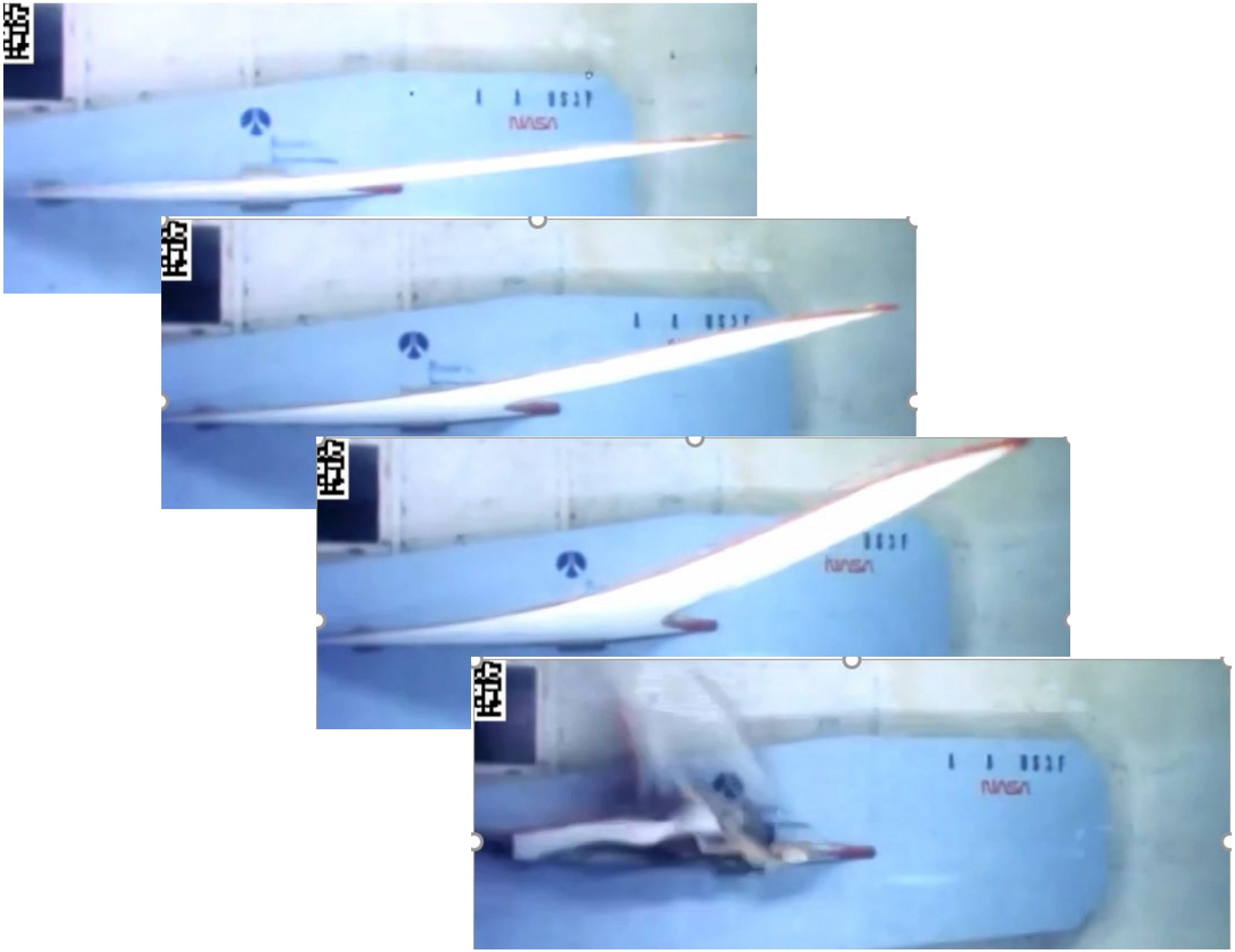
$$\boldsymbol{\Longrightarrow\ \theta = \frac{q_\infty ScC_{M_{ac}} - Wd}{k_\alpha - q_\infty SeC_{L,\alpha}} + \frac{q_\infty SeC_{L,\alpha}}{k_\alpha - q_\infty SeC_{L,\alpha}}\alpha_r}$$

- **Assumption:**
  - **Symmetric foil : $M_{AC} = 0$ and Gravity center = elastic center $(d = 0)$ Then the twist angle may be written as:**

$$\boldsymbol{\theta = \frac{q_\infty SeC_{L,\alpha}}{k_\alpha - q_\infty SeC_{L,\alpha}}\alpha_r}$$

  - **The twist angle varies linearly with α_r, but the slope tends to infinity as q_∞ tends to q_D**

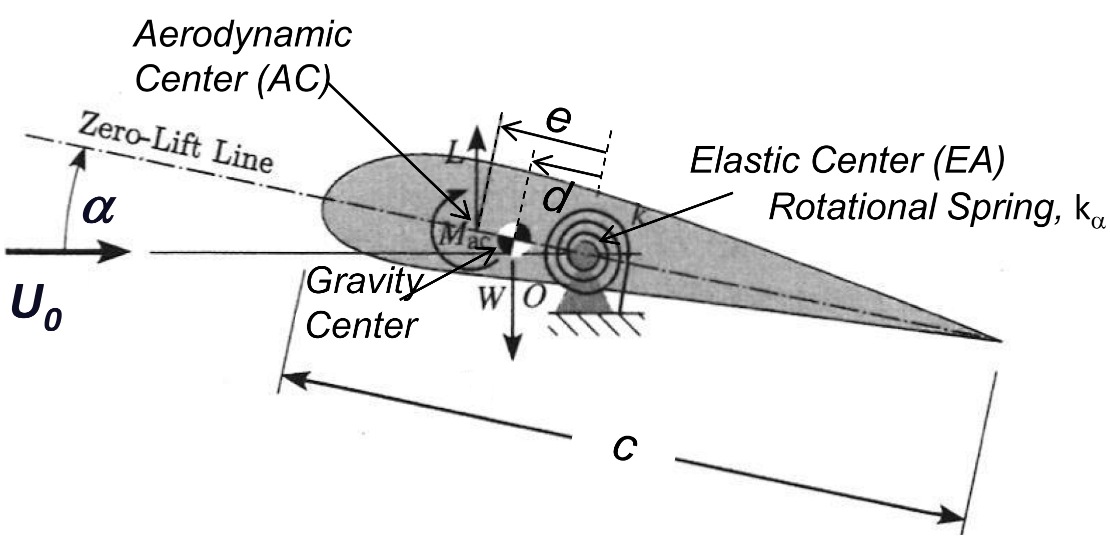
Illustration of a static instability (divergence) of an airfoil



Application: Torsional divergence of an airfoil

- Divergence might be avoided by locating the aerodynamic center between the elastic center and the trailing edge $(\mathrm{e} < 0)$
  - In this case, an increase of the lift decreases α, which in its turn decreases the lift (Stable)
  - This solution is hardly feasible in practice

$$\boldsymbol{q_d} = \frac{\boldsymbol{k_\alpha}}{\boldsymbol{SeC_{L,\alpha}}}$$



Application: Torsional divergence of an airfoil

- Divergence might be avoided by locating the aerodynamic center between the elastic center and the trailing edge ($e < 0$)
    - In this case, an increase of the lift decreases α, which in its turn decreases the lift (Stable)
    - This solution is hardly feasible in practice

$$\boldsymbol{q_d = \frac{k_\alpha}{SeC_{L,\alpha}}}$$

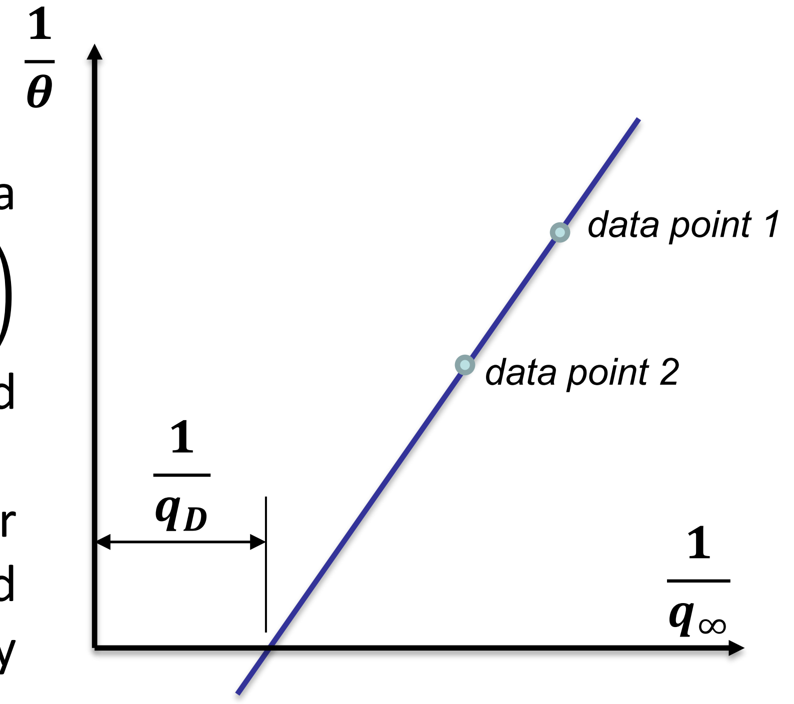
- Another alternative is to increase the wing twist stiffness, although this solution typically penalizes the total weight of the wing

- Divergence is disastrous and design rules in aeronautics require a divergence speed 1.15 times higher than the maximum diving speed

Application: Torsional divergence of an airfoil

- Experimental evaluation of divergence speed ? (non destructive method)
  - Inverting the expression of the twist angle leads to a linear relationship between $\frac{1}{\theta}$ and $\frac{1}{q_\infty}$, whose slope is $\frac{q_D}{\alpha_r}$:

$$\theta = \frac{\alpha_r}{\frac{q_D}{q_\infty} - 1} \Longrightarrow \frac{1}{\theta} = \frac{q_D}{\alpha_r}\left(\frac{1}{q_\infty} - \frac{1}{q_D}\right)$$



- Non destructive tests may be carried out in a wind tunnel to plot the line $\frac{1}{\theta} = f\left(\frac{1}{q_\infty}\right)$ (known as divergence Southwell Plot) and estimate the divergence speed.
- 2 experimental data points (at speeds lower than U_D) are enough to draw the line and have an estimate q_D in a non destructive way

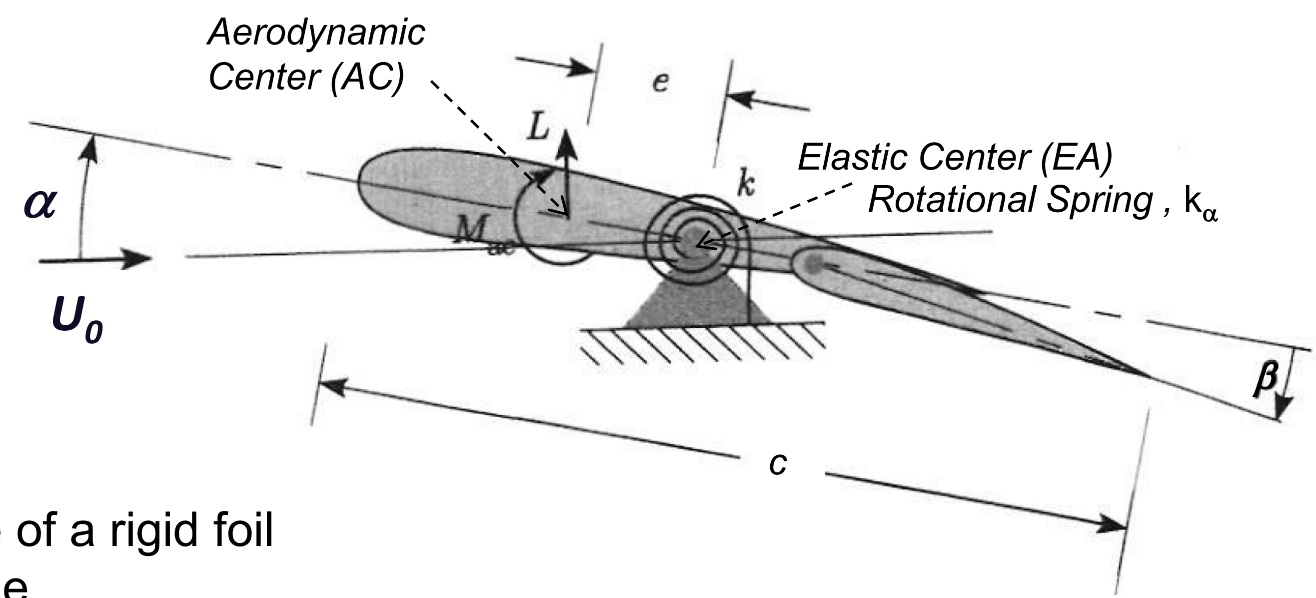
Application: Torsional divergence of an airfoil
Aileron reversal

- The so-called “Aileron Reversal” problem occurs when the response of an aileron is opposite to the one expected, because of structural deformation of the wing:
  - For example, wing torsional flexibility, can cause ailerons to lose their effectiveness as the dynamic pressure increases beyond “reversal dynamic pressure”

→ Dangerous consequences:
  - The pilot cannot control the aircraft in the usual way.
  - Impossible to execute evasive maneuvers.

Application: Torsional divergence of an airfoil
Aileron reversal

- We consider the same 2D wing, with a flap attached to its trailing edge at an angle β, which may be arbitrarily set by the flight-control system.
- For the sake of simplicity, <u>the weight is ignored</u>.



L: Lift force
$\alpha=\alpha_r+\theta$
α_r : incidence angle of a rigid foil
θ : elastic twist angle

Application: Torsional divergence of an airfoil
Aileron reversal

- Moment equilibrium about the pivot: $M_{ac} + Le = k_\alpha \theta$

With: $L = q_\infty S C_L$ $\qquad$ $M_{ac} = q_\infty S c C_{M_{ac}}$

The flap changes the effective camber of the wing. Owing to linear theory (small α and β), the resulting changes in lift and pitching moment may be approximated as follows:

$$C_L = C_{L,\alpha}\alpha + C_{L,\beta}\beta \qquad and \qquad C_{M_{ac}} = C_{M_0} + C_{M,\beta}\beta$$

$$\alpha = \alpha_r + \theta$$

(Foil without the flap)

Application: Torsional divergence of an airfoil
Aileron reversal

- After combining the previous relations, with the assumption of a symmetric wing ($\boldsymbol{C_{M_0} = 0}$), the twist angle $\boldsymbol{\theta}$ may be written as follows:

$$\boldsymbol{\theta = \frac{q_\infty S\left[eC_{L,\alpha}\alpha_r + (eC_{L,\beta} + cC_{M,\beta})\beta\right]}{k_\alpha - eq_\infty SC_{L,\alpha}}}$$

  - The twist angle θ, due to the flexibility of the wing, is a function of β

- Note: The divergence dynamic pressure $\left(\boldsymbol{q_D = \frac{k_\alpha}{eSC_{L,\alpha}}}\right)$ is unaffected by the flap since it does not depend on β.

Application: Torsional divergence of an airfoil
Aileron reversal

- The lift equation $\left(C_L = C_{L,\alpha}(\alpha_r + \theta) + C_{L,\beta}\beta\right)$ leads to the following relation of the lift force, using the previous expression of θ:

$$L = \frac{q_\infty S\left[C_{L,\alpha}\alpha_r + \left(C_{L,\beta} + \dfrac{c q_\infty S C_{L,\alpha} C_{M,\beta}}{k_\alpha}\right)\beta\right]}{1 - \dfrac{e q_\infty S C_{L,\alpha}}{k_\alpha}}$$

- From the coefficient of β, the lift is a function of β in 2 counteracting ways:
  - The 1st term is purely aerodynamic: an increase of β leads to an increase of lift because of a change in the effective camber
  - 2nd term: aeroelastic effects
    Since $C_{M,\beta} < 0$, an increase of β induces a nose down pitching moment, which tends to decrease θ and consequently the lift

Application: Torsional divergence of an airfoil
Aileron reversal

- At low speed, the purely aerodynamic increase in lift overpowers the aeroelastic tendency to decrease the lift → the lift increases with β (expected behavior)
- As the speed increases, the aeroelastic term rapidly increases and there is a risk that it cancels the aerodynamic effect (reversal):

$$\frac{\partial L}{\partial \beta} = \frac{q_\infty S\left(C_{L,\beta} + \dfrac{c q_\infty S C_{L,\alpha} C_{M,\beta}}{k_\alpha}\right)}{1 - \dfrac{e q_\infty S C_{L,\alpha}}{k_\alpha}} = 0$$

→ Reversal dynamic pressure, q_R, and reversal speed, U_R, read:

$$q_r = -\frac{k_\alpha C_{L,\beta}}{c S C_{L,\alpha} C_{M,\beta}} \qquad U_r = \sqrt{-\frac{2 k_\alpha C_{L,\beta}}{\rho c S C_{L,\alpha} C_{M,\beta}}}$$

- Note: q_r and U_r do not depend on the location of the aeroelastic center (e)

Application: Torsional divergence of an airfoil
Aileron reversal

- The lift may be written as follows:

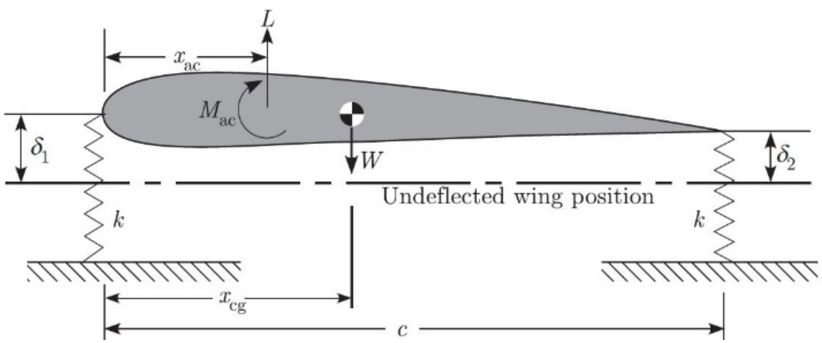
$$\boldsymbol{L} = \frac{\boldsymbol{q_\infty S\left[C_{L,\alpha}\alpha_r + C_{L,\beta}\left(1 - \frac{q_\infty}{q_R}\right)\beta\right]}}{\boldsymbol{1 - \frac{q_\infty}{q_D}}}$$

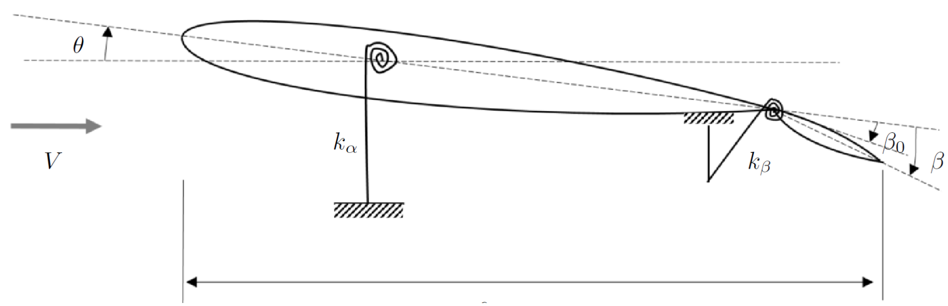
- Aileron's lift efficiency η:
  - Definition: Ratio of lift changes for aeroelastic and rigid wings

$$\boldsymbol{\eta = \frac{\left(\frac{\Delta L}{\Delta \beta}\right)_{elastic}}{\left(\frac{\Delta L}{\Delta \beta}\right)_{rigid}} = \frac{1 - \frac{q_\infty}{q_R}}{1 - \frac{q_\infty}{q_D}}}$$

Application: Torsional divergence of an airfoil

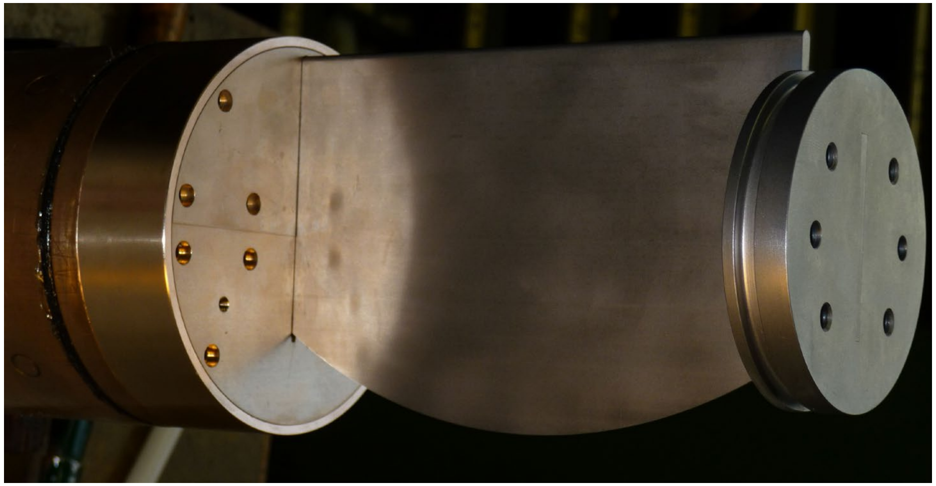
- ***Other configurations of airfoil divergence (Exercise session)***

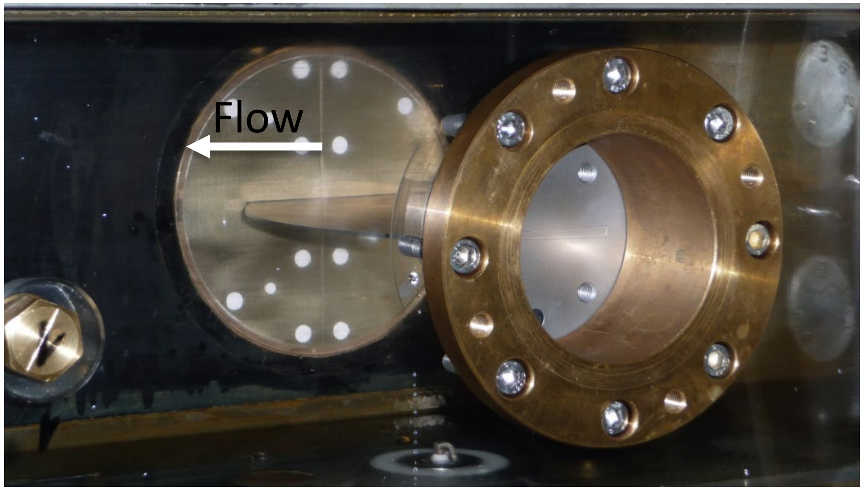




Torsional divergence in real life

- Static instability (divergence) is always associated with aeroelasticity:
  - Flexible structures in air flow (e.g. wings of airplanes, …)
  - Divergence is almost never considered in hydrodynamic applications:
    - Marine propellers, hydraulic turbines and pumps
    - The question is recently raised because the blades of hydraulic machines are getting thinner and thinner.





Torsional divergence in real life

- Thin leading edge hydrofoil (SP 80 foil):
  - Foil with sharp leading edge (supecavitating foil)
  - Tested in EPFL Cavitation Tunnel with air injection
  - As the flow speed is increased, divergence occurs and the foil is severely bent