AEROELASTICITY AND FLUID-STRUCTURE INTERACTION

Complement

Sloshing Dynamics

Equivalent Mechanical Model

Sloshing Dynamics – Equivalent Mechanical Model

In certain situations, sloshing can influence the behavior of the structure surrounding the moving liquid
→ there is a need to incorporate its effects into structural models.

1.

2.

1. *Coffee mug*
2. *Cruise ship swimming pool.*
3. *Rocket liquid O_2 fuel tank*

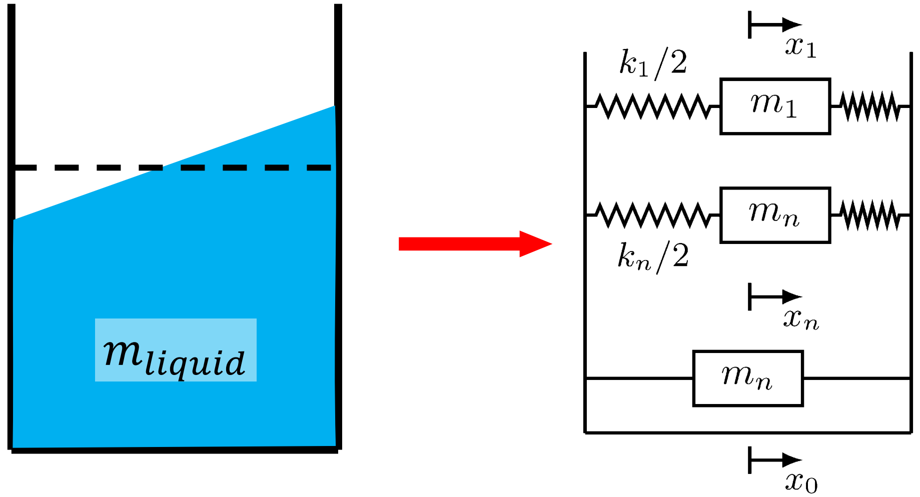
→ *Find the first asymmetric natural frequency for case 1 and 2.*
→ *How to model the fluid-structure interactions?*

3.

Sloshing Dynamics – Equivalent Mechanical Model

To incorporate the effects of sloshing in a dynamical system, it is often convenient to conceptually replace the liquid by an equivalent linear mechanical system:

- *We will use a linear spring-mass model Each of the n spring-mass corresponds to one of the infinite sloshing modes.*



Main benefits:

- *Easier to include in a system*
- *Reduced computation cost*
- *Fluid damping can be easily incorporated in the model by adding linear dashpots*

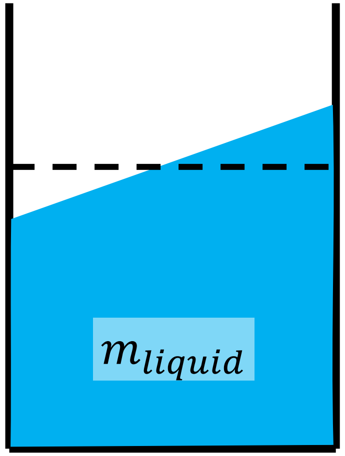
- *How to define the parameters m_n and k_n?*

Sloshing Dynamics – Equivalent Mechanical Model

Defining the model parameters for horizontal motion

- **Static properties:**
  - The sum of all the masses must be the same as the liquid mass

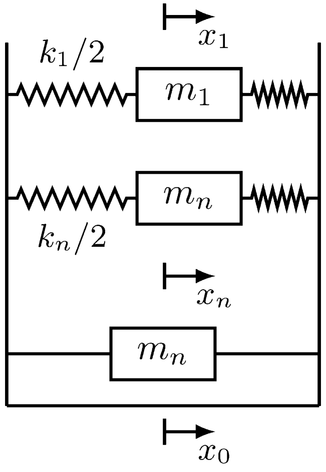
$$m_{liqu} = m_0 + \sum_{n=1}^{\infty} m_n$$



- **Dynamic properties:**
  - The natural frequencies must duplicate the ones of the liquid

$$\frac{k_n}{m_n} = \omega_n^2$$

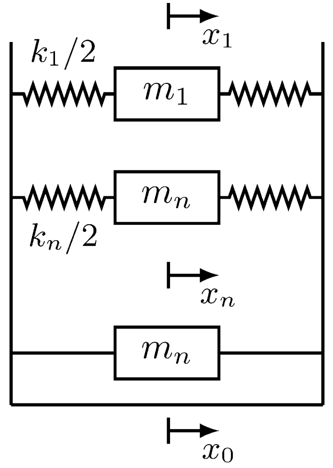
where ω_n is the natural frequency of the n^th sloshing mode.
(known from potential theory)



Defining the model parameters for horizontal motion

- **Dynamic properties:**
  - *The force components exerted on the tank under certain excitation must be equivalent to the one produced by the actual system (= the ones derived from the potential flow theory)*

$$-F = m_0\ddot{x}_0 + \sum_{n=1}^{\infty} m_n\left(\ddot{x}_n + \ddot{x}_0\right)$$



  - *In the case of pure translational excitation of the tank:* $x_0(t) = X_0\sin(\Omega t)$

    From the steady state solution of the undamped equation

    $m_n(\ddot{x}_n + \ddot{x}_0) + k\,\ddot{x}_n = 0$ *we obtain* $x_n = \frac{\Omega^2}{\omega_n^2-\Omega^2}X_0\sin(\Omega t)$

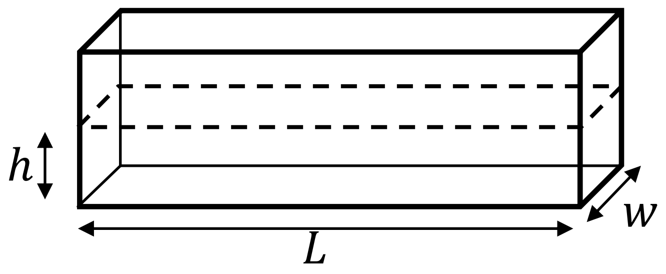
$$F = X_0\Omega^2\sin(\Omega t)\left[m_0 + \sum_{n=1}^{\infty} m_n\left(\frac{\Omega^2}{\omega_n^2-\Omega^2}+1\right)\right]$$

$$= m_{liqu}X_0\Omega^2\sin(\Omega t)\left[1 + \sum_{n=1}^{\infty}\frac{m_n}{m_{liqu}}\left(\frac{\Omega^2}{\omega_n^2-\Omega^2}\right)\right]$$

Defining the model parameters for horizontal motion

- ***The model parameters depend on the liquid and on the tank shape (not demonstrated here)***
  Rectangular tank with motion along the a-direction

- $m_{liqu} = \rho L b h$
- $\frac{m_n}{m_{liqu}} = 8\left(\frac{L}{h}\right)\frac{\tanh((2n-1)\pi\frac{h}{L})}{(2n-1)^3\pi^3}$
- $\frac{k_n}{m_{liqu}} = 8\left(\frac{g}{h}\right)\frac{\tanh^2((2n-1)\pi\frac{h}{L})}{(2n-1)^2\pi^2}$

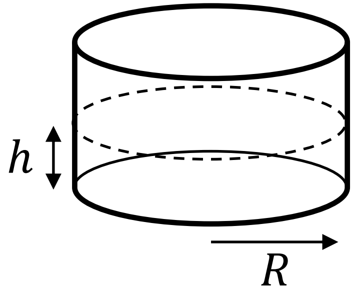


- *One may observe that the masses rapidly decrease for all modes exceeding the first one.*

Defining the model parameters for horizontal motion

- ***The model parameters depend on the liquid and on the tank shape (not demonstrated here)***
  Cylindrical tank with motion along the R-direction (linear sloshing)

  - $m_{liqu} = \rho \pi R^2 h$
  - $\frac{m_n}{m_{liqu}} = \left(\frac{2R}{\varepsilon_{1n} h (\varepsilon_{1n}^2 - 1)}\right) \tanh\left(\frac{\varepsilon_{1n} h}{R}\right)$
  - $\frac{k_n}{m_{liqu}} = \left(\frac{2g}{h (\varepsilon_{1n}^2 - 1)}\right) \tanh^2\left(\frac{\varepsilon_{1n} h}{R}\right)$



  - ε_{1n} *corresponds to the roots of the derivatives of the Bessel function of the first kind* *(*$\varepsilon_{11} = 1.841, \varepsilon_{12} = 5.331, \varepsilon_{13} = 8.536, \ldots$ *)*
  - *Since* ε_{1n} *increases with n, the size of the masses also decreases for all modes exceeding the first.*

- *From potential theory we can model the free-response motion.*

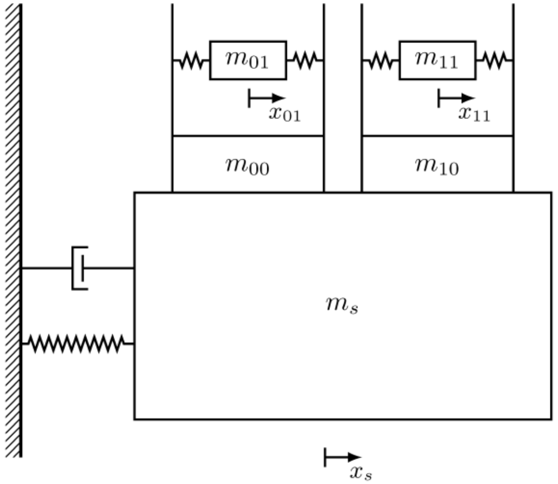
$$\ddot{x} + \omega_1^2 x = 0$$

$$\omega_1 \approx \frac{2\pi g}{L}, 1.841\,\frac{g}{R} \text{(h>>2L, respectively Rect. Cyl.)}$$

→ *We can now model dynamic interaction with any force as a function of time*

$$\ddot{x} + \frac{k_1}{m_1} x = F(t)$$

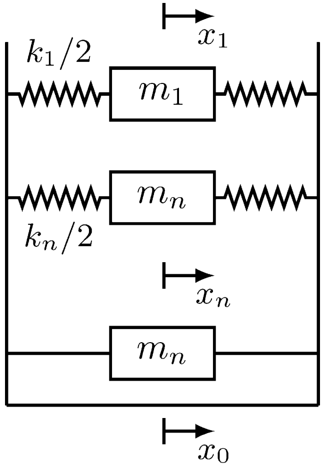
→ *Or model coupled dynamical systems*

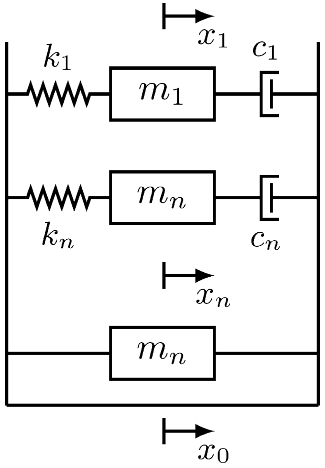


Sloshing Dynamics – Equivalent Mechanical Model

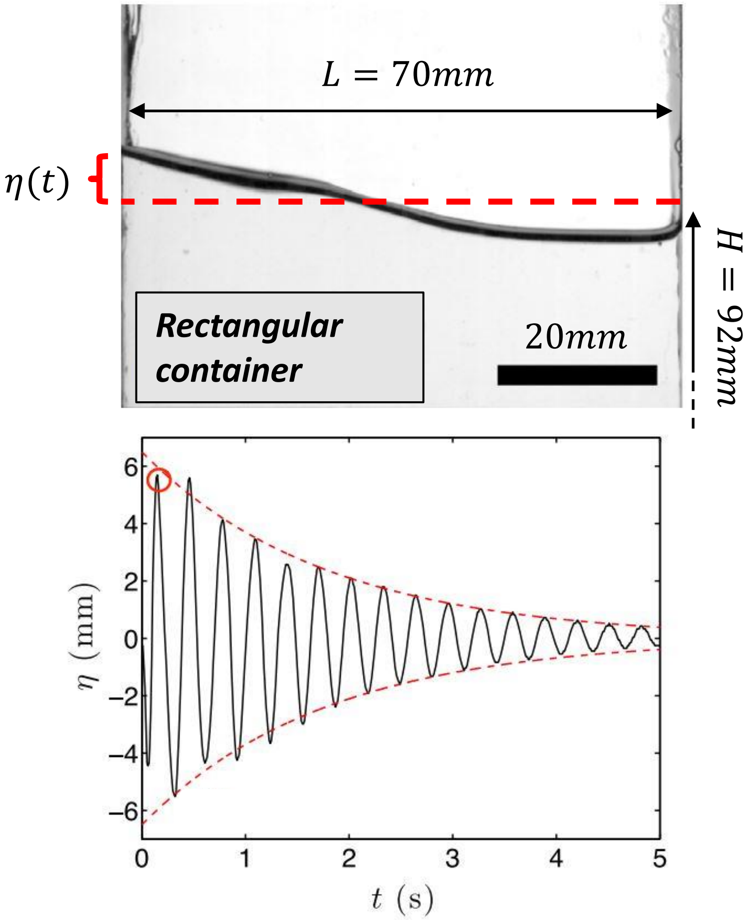
- *Isn't there something missing to model our experimental observations?*

  → *Viscous damping !*





Modeling the liquid damping – linear dashpots



Damping of liquid sloshing by foams: from everyday observations to liquid transport, Capello et al., 2015

- ***Experiments show that the free-surface elevation, after an impulse motion, follows a damped harmonic motion:***

$$\eta(t) = \eta_0 e^{(-t/\tau)} cos(2\pi f t)$$

With $f_1 = \frac{\omega_1}{2\pi} = \frac{1}{2\pi}\sqrt{g k_1 \tanh(k_1 H)} = 3.34\ Hz$

→ $f_{exp} = 3.22\ \pm 0.11\ Hz$

- ***In the linear framework, the effect of damping can thus be modeled by a set of linear dashpots.***

Sloshing Dynamics – Equivalent Mechanical Model

Modeling the liquid damping – linear dashpots

- *In real liquids, energy dissipation occurs at the tank walls and free surface due to the viscous boundary layer and within the liquid because of viscous stresses.*
- *For small tanks, the boundary layer dissipation dominates, while for large tanks, the dissipation in the liquid interior may be the larger contribution.*
- *Most results for the damping ratio have been obtained experimentally (first sloshing mode):*
  - *In cylindrical tanks (Stephens et al. 1962):*

$$\zeta_1 = 0.83 \sqrt{\frac{\nu}{g^{1/2} R^{3/2}}} \left[\tanh\left(\frac{\varepsilon_{11} h}{R}\right) \left(1 + 2 \frac{1 - \frac{h}{R}}{\cosh\left(\frac{\varepsilon_{11} h}{R}\right)} \right) \right]$$

  - *In rectangular tanks (Sun, 1991):*

$$\zeta_1 = \frac{1}{2h} \sqrt{\frac{2\nu}{\omega_1}} \left(1 + \frac{h}{w} \right)$$

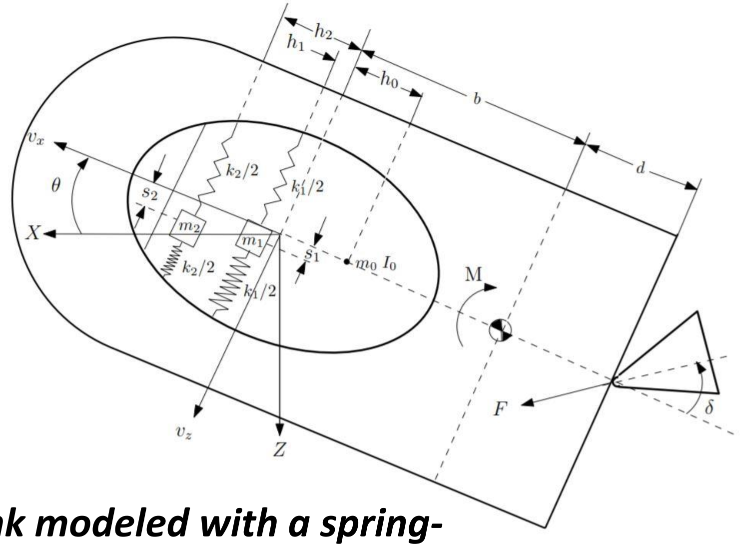
ν *is the liquid kinematic viscosity*

Sloshing Dynamics – Equivalent Mechanical Model

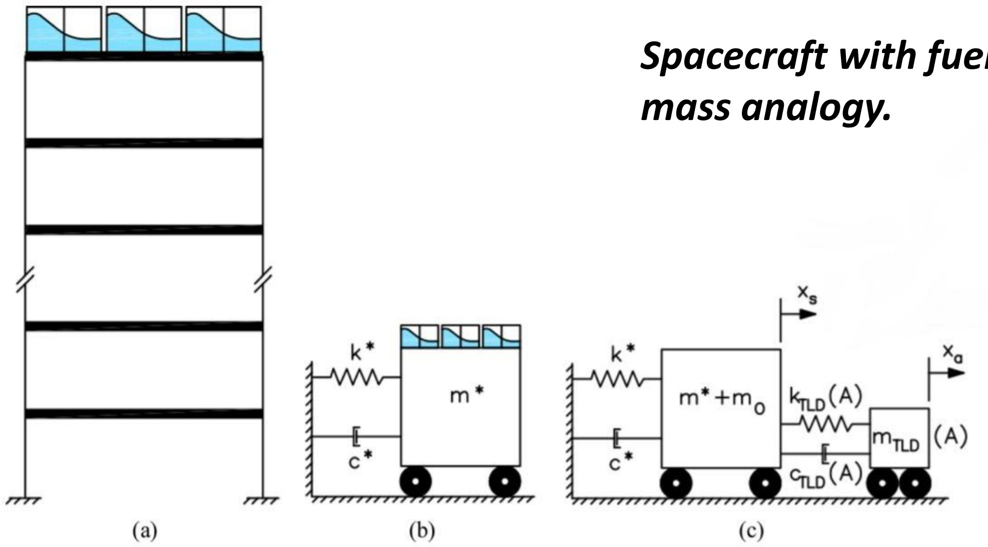
Examples of application:

- Control of spacecraft with liquid fuel tanks
- Vibration absorbtion (Tuned Liquid Dampers or TLD)
- Modeling of fuel tanks in aircraft wings , in oil tankers or in trucks
- …

Dynamics and Control of Higher-Order Nonholonomic Systems, Jaime Rubio Hervas, PhD Thesis, 2013



Spacecraft with fuel tank modeled with a spring-mass analogy.

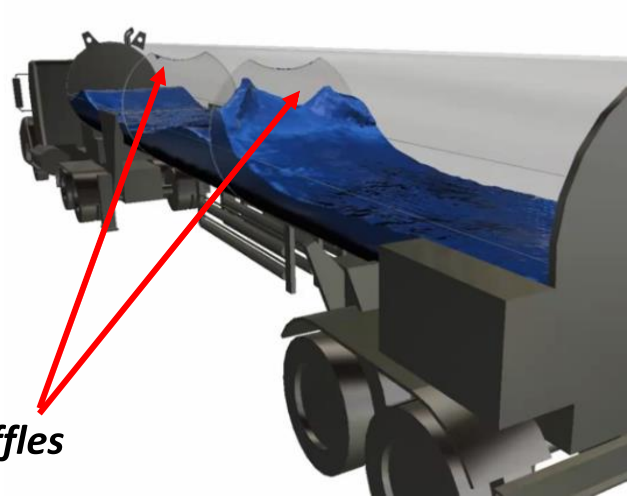


Development and Validation of Finite Element Structure-Tuned Liquid Damper System Models, Soliman et al., 2015

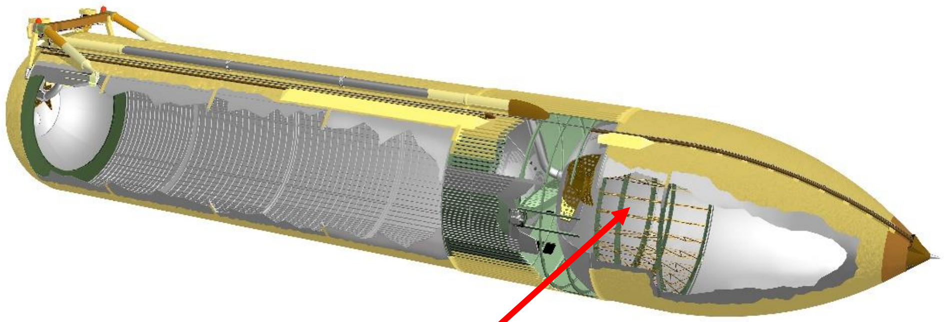
The evolution of (a) a structure-TLD system into, (b) a generalized structural system with TLDs and then into, (c) a system with equivalent Tuned Mass Damper (TMD) representation

Sloshing mitigation techniques

- ***Liquid containers: Sub-division with baffles or bulkheads are widely used***
  - *Reduces wave amplitude and wall pressure, increases the liquid damping*
  - *Ongoing research to define the optimal shape, number and locations of the baffles*



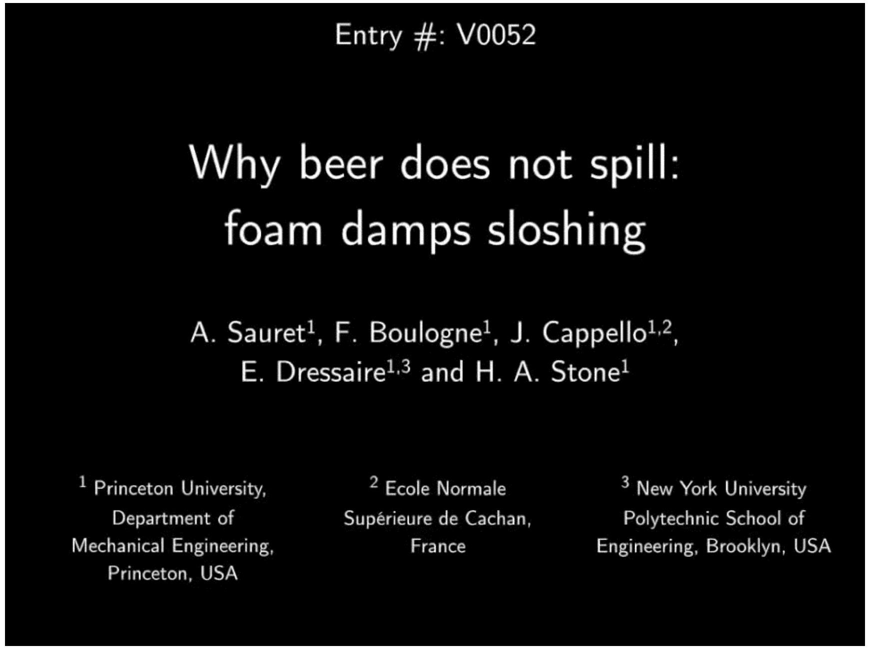
baffles



Space shuttle liquid oxygen tank anti-slosh baffles

Sloshing mitigation techniques – the effect of a foam layer

- ***The addition of foam of the free surface damps the sloshing of the liquid.***



- *Foam generates an additionnal friction force that adds damping to the fluid oscillations.*

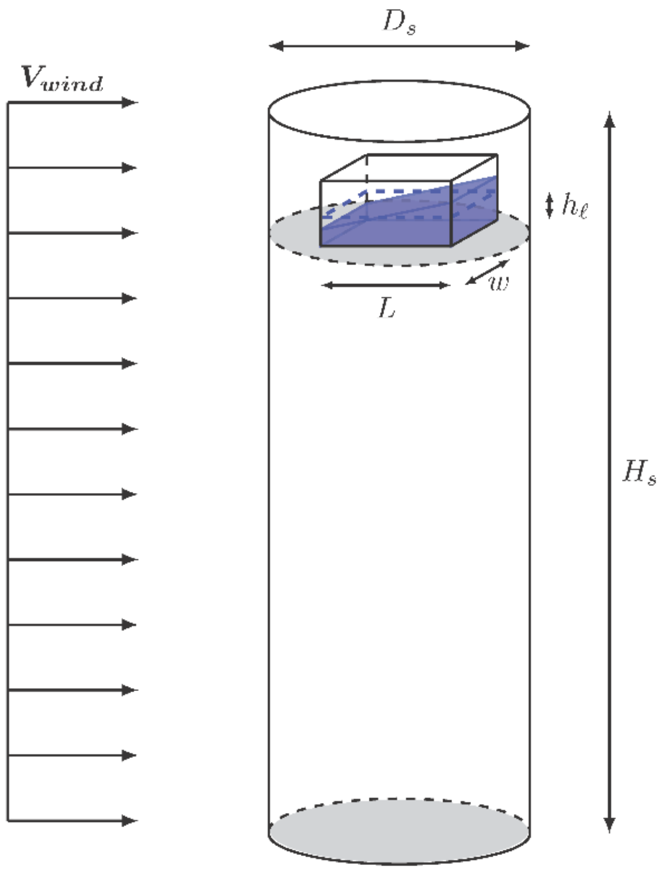
- *This additionnal force scales as:*

$$f_{foam} \sim K\gamma Ca^{2/3}$$

where γ is the foam surface tension, K represents geometrical properties of the foam layer and $Ca = \frac{\mu \dot{x}}{\gamma}$ is the capillary number → the foam friction force in non-linear.

Exercise 1 – Tuned Liquid damper.

Consider a tall building represented by a cylinder with an equivalent diameter, $D_s = 50\ m$ and a total height $H_s = 300\ m$ as depicted on figure 1. The structure is subjected to strong winds, vortices are periodically shed from its surface thus forcing the structure to oscillate. To dampen those oscillations, a tuned liquid damper (TLD), in the form of rectangular tank partially filled with water, is installed within the building. The tank measures $L = 10\ m$ by $w\ =\ 6\ m$.



Sloshing Dynamics – Equivalent Mechanical Model

a) *What is the frequency of the vortex shedding if we expect winds ranging up to 50 m/s. Could the associated excitation match the building eigen-frequency measured at $f_s = 0.15$ Hz.*

Taking a Strouhal number of $St = 0.2$ we obtain frequencies up to $f = \frac{St\ U}{D} = 0.2$ Hz. The vortex shedding frequency matches the building eigen-frequency when the wind speed is $U = \frac{f_S D}{St} = 37.5$ m/s.

Sloshing Dynamics – Equivalent Mechanical Model

b) *As said earlier, a rectangular container will be used in the building as a tuned liquid damper. At what height the container should be filled to obtain the same natural frequency as the building.*

Using the formula of the frequency of the first sloshing mode, we find

$$\omega_1^2 = \frac{g\pi}{L} tanh\left(\frac{\pi h}{L}\right)$$

$$h_\ell = \frac{L}{\pi} tanh^{-1}\left[(2\pi f_s)^2 \frac{L}{\pi g}\right] = 0.944m$$

Sloshing Dynamics – Equivalent Mechanical Model

c) *As we are only interested in the first asymmetric mode, we will replace the liquid inside the container with two masses: m_0 a rigid mass fixed to the container and m_1 moving mass restrained with a spring of rigidity k_1 and a dashpot with a damping c_1. Compute the value of all the said parameters.*

$$m_{liqu} = \rho L b h = 56.640 \times 10^3 \ kg$$

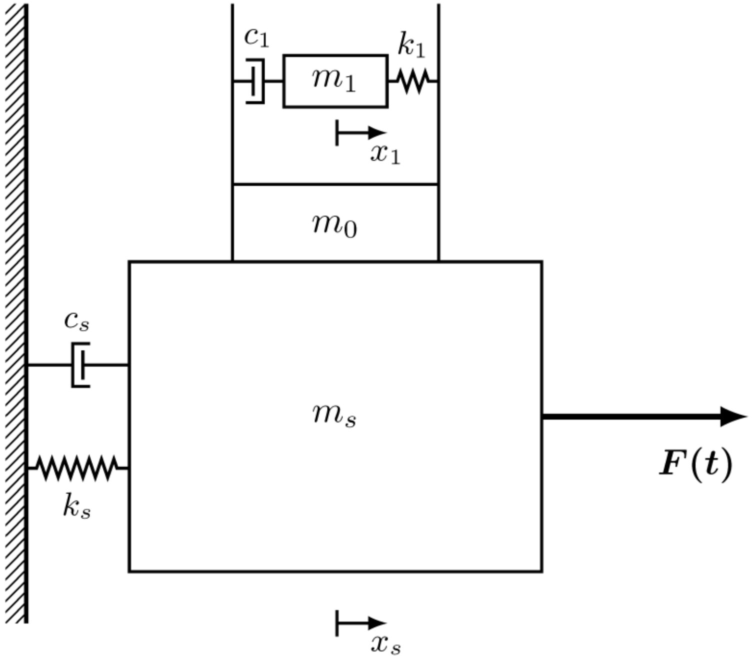
$$m_1 = 8\, m_{liqu} \left(\frac{L}{h}\right) \frac{tanh(\pi \frac{h}{L})}{1^3 \pi^3} = 44.610 \times 10^3 \ kg$$

$$m_0 = m_{liqu} - m_1 = 12.029 \times 10^3 \ kg$$

$$k_1 = 8\, m_{liqu} \left(\frac{g}{h}\right) \frac{tanh^2((2n-1)\pi \frac{h}{L})}{(2n-1)^2 \pi^2} = 39.618 \times 10^3 \ N$$

$$\zeta_1 = \frac{1}{2h} \sqrt{\frac{2\nu}{\omega_1}} \left(1 + \frac{h}{b}\right) = 3.563 \times 10^{-4}$$

$$c_1 = 2\zeta_1 \sqrt{(m_1 k_1)} = 29.955 \ Ns/m$$

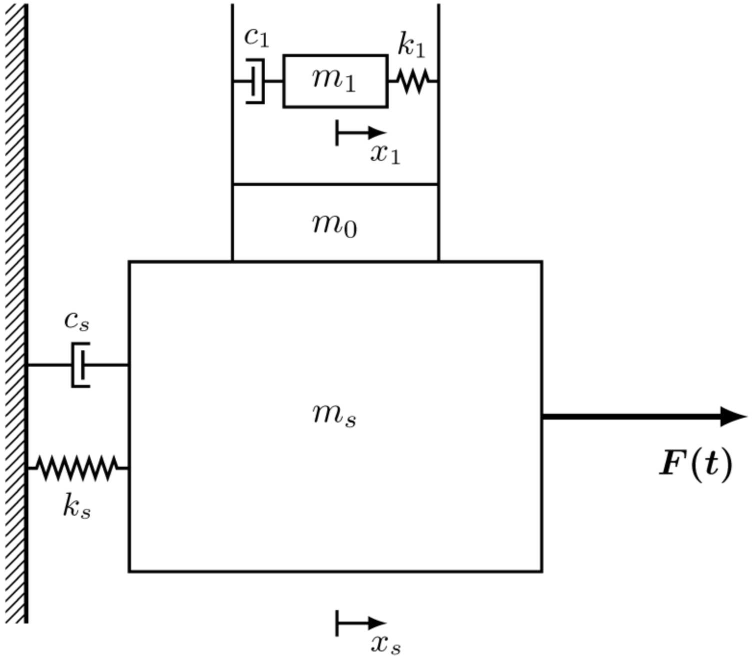


Sloshing Dynamics – Equivalent Mechanical Model

d) *Assuming the mechanical parameters of the building are* $m_s = 250 \times 6\,\mathrm{kg}$, *and* $c_s = 500\ Ns/m$, *and the aerodynamic parameter is* $C_{L,osci} = 0.684$. *We will model our system using the mechanical system shown. Find the equation of motion.*

$$(m_s + m_0)\ddot{x}_s + c_s\dot{x}_s + k_s x_s + c_l(\dot{x}_s - \dot{x}_l) + k_l(x_s - x_l) = F(t)$$
$$m_l\ddot{x}_l + c_l(\dot{x}_l - \dot{x}_s) + k_l(x_l - x_s) = 0$$

With $F(t) = \frac{1}{2}\rho V_{wind}^2 D_s H_s C_{L,osci} sin(2\pi f_{st} t)$



Sloshing Dynamics – Equivalent Mechanical Model

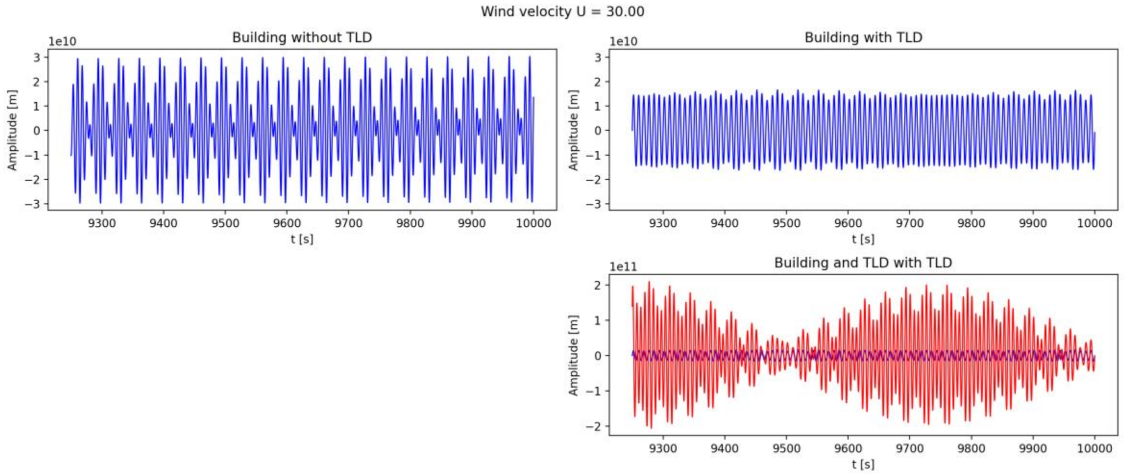
e) *Numerically compute the response of the system to wind blowing at 30 m/s, 37.25 and 37.5 m/s. Also compute the response of the system if the TLD was not here.*

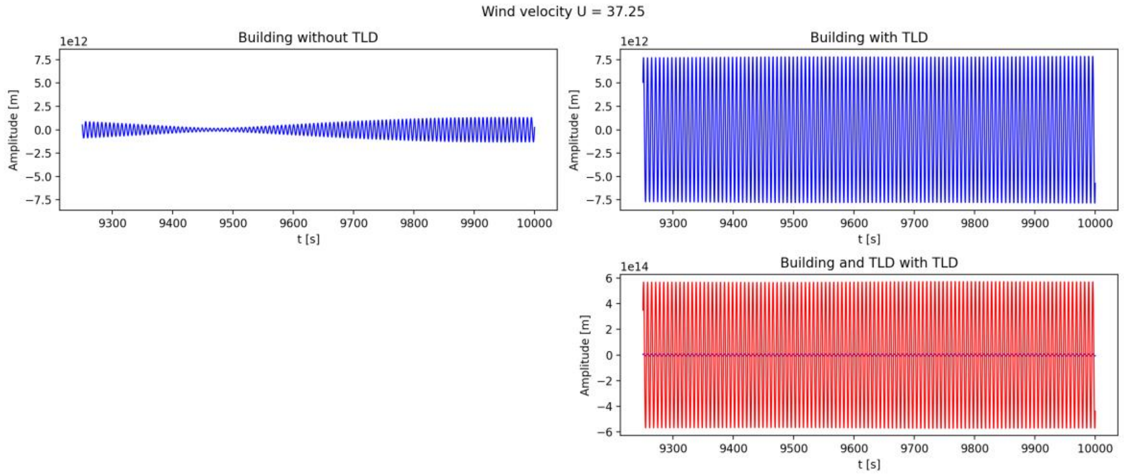
To numerically compute the response, we will need to transform our second order system to a system of 4 first order equations.

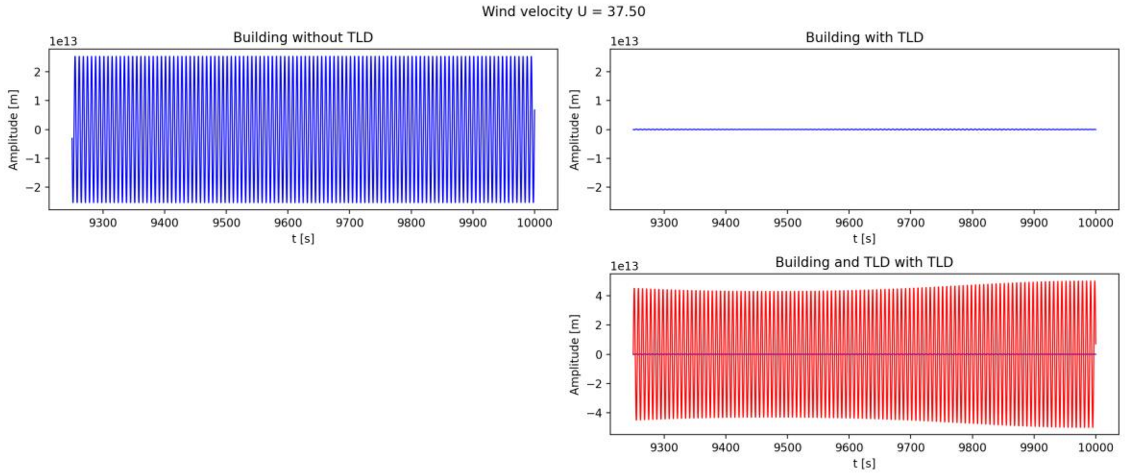
We find the following system:

$$\frac{d}{dt}\begin{pmatrix} x_s \\ \dot{x}_s \\ x_\ell \\ \dot{x}_\ell \end{pmatrix} = \begin{pmatrix} \dot{x}_s \\ -\frac{c_s}{(m_s+m_0)}\dot{x}_s - \frac{k_s}{(m_s+m_0)}x_s - \frac{c_\ell}{(m_s+m_0)}(\dot{x}_s-\dot{x}_\ell) - \frac{k_\ell}{(m_s+m_0)}(x_s-x_\ell) + F \\ \dot{x}_\ell \\ -\frac{c_\ell}{m_\ell}(\dot{x}_\ell-\dot{x}_s) - \frac{k_\ell}{m_\ell}(x_\ell-x_s) \end{pmatrix}$$

Sloshing Dynamics – Equivalent Mechanical Model





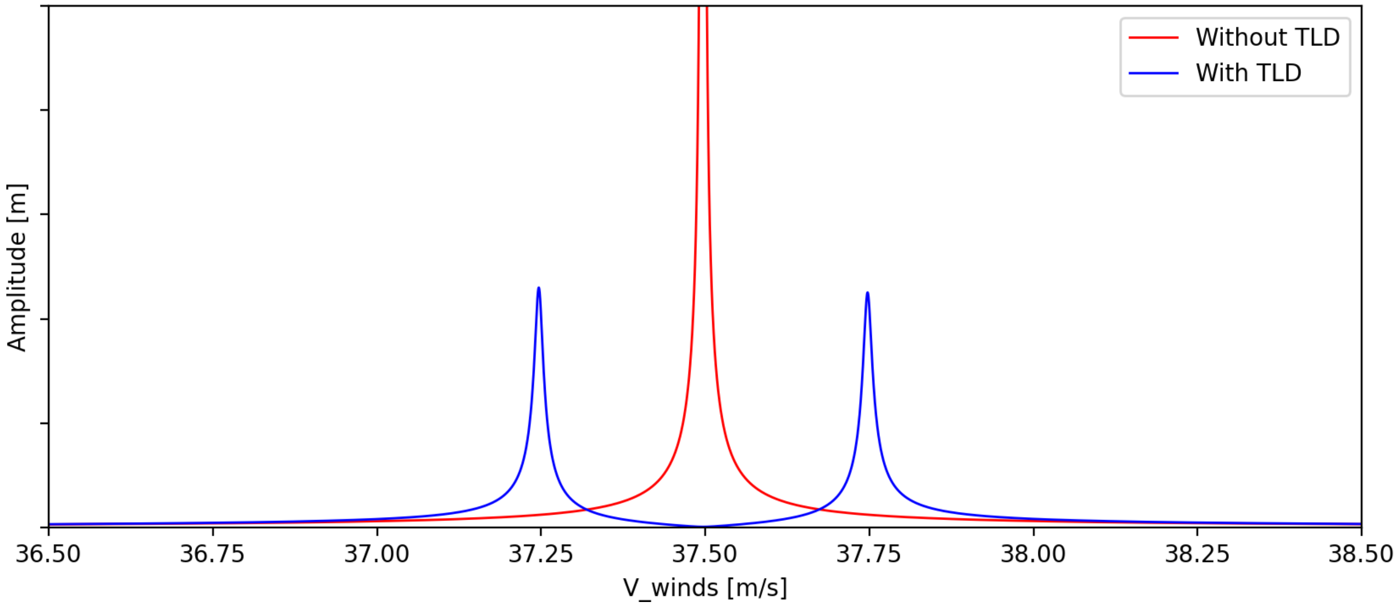


Sloshing Dynamics – Equivalent Mechanical Model

$$x_s(t) = X_s e^{i\omega t}, \qquad x_l(t) = X_l e^{i\omega t}, \qquad x_l(t) = X_l e^{i\omega t}$$

$$-\omega^2 M \begin{bmatrix} X_s \\ X_l \end{bmatrix} + i\omega C \begin{bmatrix} X_s \\ X_l \end{bmatrix} + K \begin{bmatrix} X_s \\ X_l \end{bmatrix} = \begin{bmatrix} A \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X_s \\ X_l \end{bmatrix} = \mathbf{B}^{-1} \begin{bmatrix} A \\ 0 \end{bmatrix}, \qquad \mathbf{B} = -\omega^2 M + i\omega C + K$$



Sloshing Dynamics – Equivalent Mechanical Model

f) *What advantages and disadvantages can you think of for a tuned liquid damper compared to a classic one.*

Advantages:

- *Easy to incorporate in existing buildings or structures. Easy to incorporate on various building shapes (chimneys, …)*
- *The sloshing frequency can easily be adjusted by increasing or decreasing the height of water for a given tank.*
- *There are no moving parts.*

Disadvantages:

- *Often poor damping capabilities when compared to TMDs (apparatus such as baffles, nets or contaminants are often added to increase the damping properties of the TLDs).*
- *Heavy when compared to TMDs.*