Networked Control Systems (ME-427)- Exercise session 9

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1. **Bounded evolution for averaging systems** Consider the fictitious train map given in Figure 1.

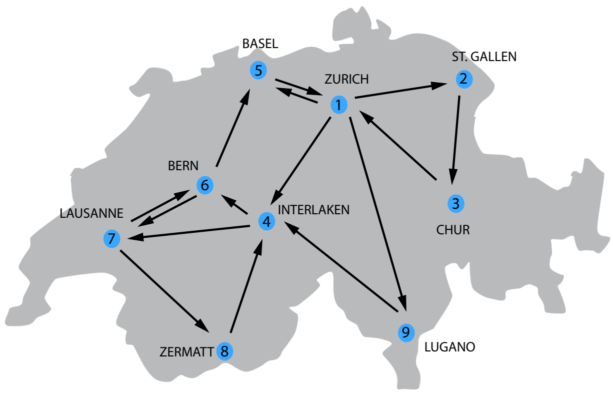

Figure 1: Fictitious map of trains in Switzerland.

   (a) Write the unweighted adjacency matrix of this graph. By using the theory developed in class and Matlab answer the following questions.
      i. Is the graph strongly connected? What does this mean for a passenger?
      ii. Is the graph acyclic?
      iii. What is the number of links of the shortest path connecting St. Gallen to Zermatt?
      iv. Is it possible to go from Bern to Chur using 4 links? And 5?
      v. How many different routes, with strictly less then 9 links, start from Zurich and end in Lausanne? (You may even pass by Zurich and Lausanne more then once during your trip)

2. **Simple properties of stochastic matrices** [Textbook E2.1] Let $A_1, A_2, \dots, A_k$ be $n \times n$ matrices and $A_1 A_2 \dots A_k$ be their product. Show that
   (a) if $A_1, A_2, \dots, A_k$ are non-negative, then their product is non-negative,
   (b) if $A_1, A_2, \dots, A_k$ are row-stochastic, then their product is row-stochastic, and
   (c) if $A_1, A_2, \dots, A_k$ are doubly-stochastic, then their product is doubly stochastic.

3. **Powers of primitive matrices** [Textbook E2.5] Let $A \in \mathbb{R}^{n \times n}$ be non-negative. Show that $A^k \succ 0$, for some $k \in \mathbb{N}$, implies $A^m \succ 0$ for all $m \geq k$.

   Hint: If column j of A is zero, then column j of $A^k, k > 0$ is zero. This fact has a simple proof and can be useful for solving the exercise.

4. **On some non-negative matrices** [Textbook E2.9] How many 2×2 matrices exist that are simultaneously doubly stochastic, irreducible and not primitive? Justify your claim.

   Hint: Parametrize the entry in position $(1, 1)$ as $x \in [0, 1]$.

5. **An example reducible or irreducible matrix.** [Textbook E4.7] Consider the binary matrix:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Prove that A is irreducible or prove that A is reducible by providing a permutation matrix P that transforms A into an upper block-triangular matrix, i.e., $P^T AP = \begin{bmatrix} \star & \star \\ \mathbf{0} & \star \end{bmatrix}$.