Networked Control Systems (ME-427)- Exercise session 8

Prof. G. Ferrari Trecate

1. **Bounded evolution for averaging systems** [Textbook E1.1]. Given a matrix $A \in \mathbb{R}^{n\times n}$ with non-negative entries and unit row sums, consider the *averaging model*

$$x(k+1) = Ax(k). \tag{1}$$

Show that, for all initial conditions $x(0)$ and times k,

$$\min_i x_i(0) \leq \min_i x_i(k) \leq \min_i x_i(k+1) \leq \max_i x_i(k+1) \leq \max_i x_i(k) \leq \max_i x_i(0),$$

where i takes values in $\{1,\dots,n\}$. Note that these inequalities imply that all evolutions of the averaging model are bounded.

Hint: Start with the maximum, show that $\max_i (Ax)_i \leq (\star) \cdot \max_h x_h$ for a suitable $(\star)$.

2. **Simulating the average dynamics** [Textbook E1.3]. Simulate in Matlab the linear averaging algorithm

$$x_i^+ := \text{average}(x_i, \{x_j, \text{for all neighbor nodes } j\}). \tag{2}$$

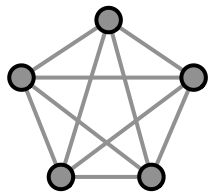
Set $n = 5$, select the initial state equal to $(1, -1, 1, -1, 1)$, and use the following undirected unweighted graphs (depicted in figure):

   (a) the complete graph,

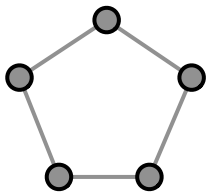
   (b) the ring graph, and

   (c) the star graph with node 1 as center.

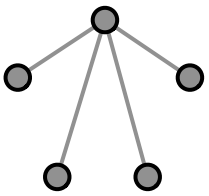
   Which value do all nodes converge to? Is it equal to the average of the initial values?



(a) Complete graph



(b) Ring graph



(c) Star graph

3. **Directed spanning trees.** Draw if possible a directed spanning tree of the digraph in Figure 2.

4. **Acyclic digraphs** [Textbook E3.1] Let G be an acyclic digraph with nodes $\{1,\dots,n\}$. Show that G contains at least one sink, i.e., a vertex without out-neighbors and at least one source, i.e., a vertex without in-neighbors.

   Hint: By contradiction assume that G has no sinks and show it must have a cycle.

5. **Properties of trees.** Consider an undirected graph G with $n > 1$ nodes(recall that self-loops are not allowed in undirected graphs). Show that if G is a tree, then every pair of distinct nodes of G is connected by a unique path.

   Hint: By contradiction ...

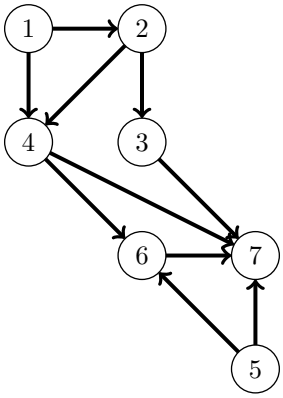


Figure 2: Graph for problem 3

6. **Properties of directed spanning trees.** Assume $G = (V, E)$ is a directed spanning tree. Show that each node has at most one in-neighbor.

7. Check if the following statements are true or false.

	T	F
(a) The diagraph $G = (V, E),\ V = \{1\},\ E = \phi$ is a directed tree.	☐	☐
(b) A directed tree has no self-loops	☐	☐
(c) A digraph contains at most a directed spanning tree	☐	☐