Networked Control Systems (ME-427)- Exercise session 5

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1. **Stabilizing state-feedback design.** Consider the NCS in Figure 1,

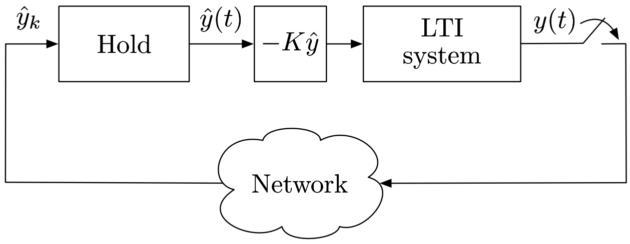


Figure 1: Networked control system

where the LTI system is the first-order model

$$\begin{cases} \dot{x} = -x - 5u \\ y = x \end{cases}.$$

Let $T = 0.5$ and $\tau = 0.2$ be the nominal sampling time and network induced delay, respectively. Find by hand all values of K stabilizing the NCS.

Hint: Recall the Jury's criterion: the roots of $\phi(\lambda) = \lambda^2 + \alpha\lambda + \beta$ verify $|\lambda| < 1$ if and only if

$$\begin{aligned} &\beta > -\alpha - 1 \\ &\beta > \alpha - 1 \\ &\beta < 1 \end{aligned}.$$

2. **Remote control with delay compensation.** Consider the cart-stick balancer system in Figure 2, with states x_1: stick angle $\theta/10$, x_2: stick angular velocity $\dot{\theta}$, x_3: cart velocity v. The input u is the voltage to the motor driving the wheels. The measured ouput $y = \theta$ is the stick angle. The control goal is to mantain the stick vertical by moving the cart through u. Consider the remote controller with delay compensation defined in Figure 3 with uniform sampling period $T = 0.1$ s.

Setting $x^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$ one has the linearized model

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 31.33 & 0 & 0.016 \\ -31.33 & 0 & -0.216 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ -0.649 \\ 8.649 \end{bmatrix}}_{B} u$$

Recall from the lectures that, setting $\delta_k = T + \tau_{sc,k+1} - \tau_{sc,k}$, the closed-loop NCS model is given by the discrete-time system

$$\begin{aligned} &x(t_{k+1} + \tau_{sc,k+1}) = \tilde{A}_k x(t_k + \tau_{sc,k}) \\ &\tilde{A}_k = e^{A\delta_k} - \Gamma(\delta_k)BK, \quad \Gamma(\delta_k) = \int_0^{\delta_k} e^{As} ds \end{aligned} \tag{1}$$

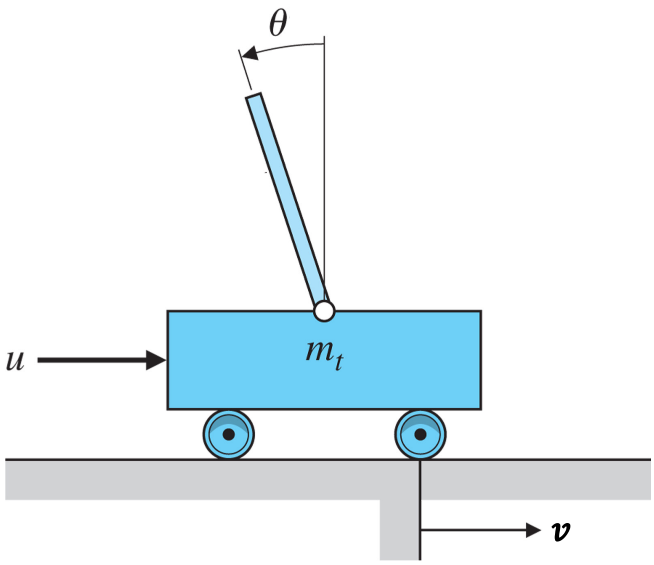


Figure 2: Cart-stick balancer.

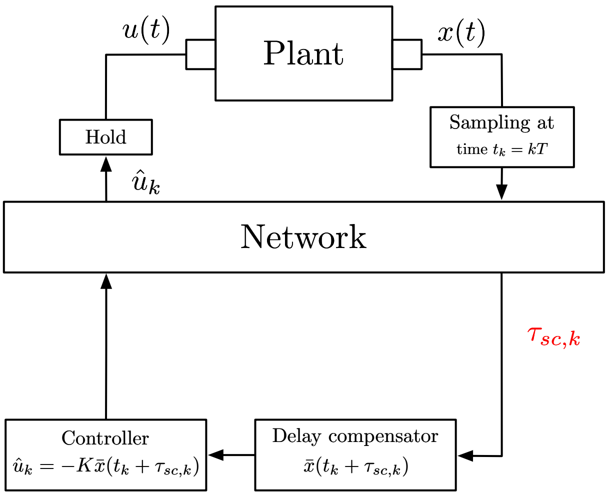


Figure 3: Remote controller with delay compensation.

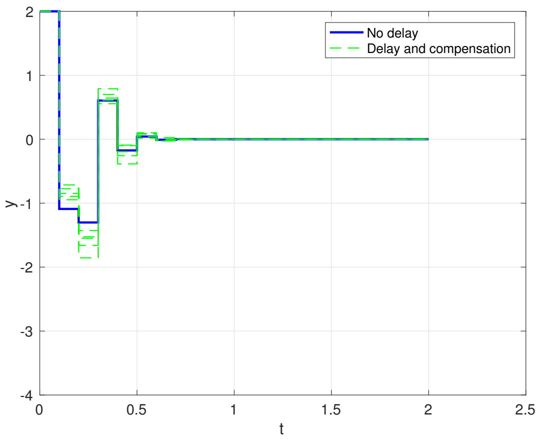
A simulator of the NCS is provided by the file `NCS_car_stick_balancer.m` available on moodle. The `.m` file is configured for running the *experiment* defined by

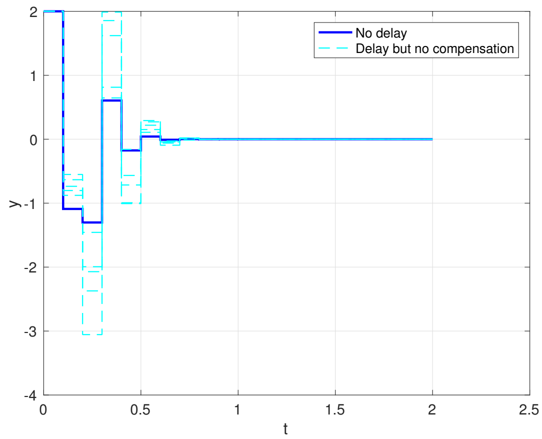
$$t_0 = 0$$
$$x(0) = \begin{bmatrix} 0.2 & 0.3 & -0.5 \end{bmatrix}^T$$
$$K = \begin{bmatrix} -556.1829 & -208.3171 & -12.9905 \end{bmatrix}$$

and it produces two plots, similar to the ones below

Both figures display the angular position of the stick (in degrees) and

- the blue line is the ideal NCS with $\tau_{sc,k} = 0$, $k = 0, 1, \ldots$
- other lines are obtained extracting the values of $\tau_{sc,k}$ five times from the uniform distribution on $[\tau_{min}, \tau_{max}]$ (these parameters can be specified at the beginning of the file) and
  - using a delay-compensation controller, as seen in the lectures (green lines);
  - using an uncompensated controller (cyan lines).





(a) **Familiarize with the simulator.** Perform the following experiments.

i. The default gain K has been produced by nominal design (see the lecture slides for the precise meaning of "nominal design") for placing the closed-loop eigenvalues in -0.42, -0.49, and -0.56. Check stability in simulation by looking at the plots for

- $\tau_{min} = \tau_{max} = 0$
- $\tau_{min} = \tau_{max} = \tau < T$. In this case performances are different if using delay compensation or not. Why ?

ii. Assume that performance of the NCS is acceptable if

$$|\theta(t_k)| < 15, \ \forall k = 0, 1, \dots \tag{2}$$

By increasing $\tau = \tau_{min} = \tau_{max}$, find $\bar{\tau}$ such that (2) is verified by the delay-compensated controller, but not by the uncompensated controller.

iii. Run simulations with $\tau_{sc,k}$ generated randomly in $[0, \bar{\tau}]$. The system behavior gets worse (for instance, oscillations are less dampened). Can you guess why ?

iv. Can the NCS become unstable by increasing $\bar{\tau}$ in the previous point ? Run simulations for answering.

(b) **Control design.** Network delays that can be tolerated for stability and performance depend on the eigenvalues of the nominal NCS. To see this,

- design a nominal gain K for placing the closed-loop eigenvalues in -0.12, -0.14, and -0.16 (so that NCS transients are shorter than before).

  Hint: Fill in the missing code in `NCS_car_stick_balancer.m` for computing K.
- run simulations with $\tau_{sc,k}$ extracted randomly in $[0, \bar{\tau}]$ and increase $\bar{\tau}$ until unstable behaviors start appearing. How does $\bar{\tau}$ compare with the result of point (2(a)iv) above?