# Networked Control Systems (ME-427) - Exercise session 3

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1. Consider the following switched system

$$x_{k+1} = A_{\sigma(k)} x_k, \qquad \sigma(k) \in \{1, 2\}$$

$$A_1 = \begin{bmatrix} 0.4 & -0.9 \\ 0.3 & 0.5 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.7 & 0.1 \\ 0.3 & 0.6 \end{bmatrix}$$

Is there a common Lyapunov function certifying exponential stability?

**Hint:** adapt the MatLab code provided in the previous exercise session.

2. For the switched system

$$x_{k+1} = A_{\sigma(k)} x_k, \qquad \sigma(k) \in \mathcal{I} = \{0, 1, \ldots, M\} \tag{1}$$

one might think that if all matrices $A_i$, $i \in \mathcal{I}$ are Schur, then the zero solution is AS, independently of the switch signal $\sigma(k)$. This is unfortunately false, as shown by this system

$$\mathcal{I} = \{1, 2\}, \qquad A_1 = \begin{bmatrix} 0.9901 & 0.1988 \\ -0.0994 & 0.9881 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0.9424 & 0.0946 \\ -0.1892 & 0.9405 \end{bmatrix}$$

(a) Check that $A_1$ and $A_2$ are Schur.

(b) Consider

$$\sigma_k = \begin{cases} 1 & \text{if } x_k \geq 0 \text{ or } x_k \leq 0 \\ 2 & \text{otherwise} \end{cases}$$

where $x_k \geq 0$ means ($x_{k,1} \geq 0$ and $x_{k,2} \geq 0$). Write the MatLab code for simulating the system from $x(0) = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$ and plot $x(k)$ in the $x_1 - x_2$ plane.
Analyze the qualitative behavior of $x(k)$ on each orthant for understanding why stability fails.

3. Prove that

(a) If $A \in \mathbb{R}^n$ is diagonalizable, (i.e. $A = V^{-1} D V$ where $D$ is a diagonal matrix and $V$ collects the eigenvectors of $A$ as columns), then, for $t \in \mathbb{R}$, it holds $e^{At} = V^{-1} e^{Dt} V$, i.e., $A$ and $e^{At}$ can be diagonalized using the same matrix $V$.

**Hint:** use the definition of $e^{At}$.

(b) For

$$A = \begin{bmatrix} \lambda & 1 \\ 1 & \lambda \end{bmatrix} \quad \lambda \in \mathbb{R},$$

one has

$$e^{At} = \frac{1}{2} \begin{bmatrix} e^{t(1+\lambda)} + e^{t(\lambda-1)} & e^{t(1+\lambda)} - e^{t(\lambda-1)} \\ e^{t(1+\lambda)} - e^{t(\lambda-1)} & e^{t(1+\lambda)} + e^{t(\lambda-1)} \end{bmatrix}.$$

**Hint:** A is symmetric and hence diagonalizable.