

Networked Control Systems (ME-427) - Exercise session 2

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1. Solving LMIs with Yalmip (guided exercise)

Consider the discrete-time LTI system

$$x^+ = Ax, \quad A = \begin{bmatrix} 0.9053 & 0.0928 \\ 0.0098 & 0.9512 \end{bmatrix}$$

Find $P = P^T \in \mathbb{R}^{2 \times 2}$ such that

$$\begin{cases} A^T P A - P \leq -Q \\ P > 0 \end{cases} \quad (1)$$

where $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Note that, for square matrices M_1 and M_2 of the same size, $M_1 \leq M_2$ means that $M_1 - M_2 \leq 0$.

Workflow for solving LMIs (1)

- Define LMI unknowns `P = sdpvar(n,n)`. Square matrices are symmetric by default.
- Define LMI constraints `Li`, $i = 1, \dots, N$.
- Combine constraints by concatenation `L = [L1; L2]`.
- Solve for unknowns

```
infosol = optimize(L)
```

- Extract and check solutions

```
Psol = double(P)
```

- `infosol.info` should be 'Successfully solved';
- double-check the solution fulfills the constraints.

Other useful commands

- For LMI
 - Equality operator: `==`
 - Inequality operators: `<=`, `>=`
`P>=0` means P positive semi-definite. Warning: Yalmip does not support strict inequalities.
- Solver options
 - `ops = sdpsetting('solver', 'mosek')` *%set the solver*
 - `ops = sdpsetting(ops, 'verbose', 0)` *%suppresses the output*

Exercise (1) is solved by the following MatLab code

```
% Define the system
A = [0.9053 0.0928; 0.0098 0.9512];

% set yalmip options
ops = sdpsettings('solver','mosek');
ops = sdpsettings(ops,'verbose',0);

% Define unknowns, parameters and constraints

P = sdpvar(2,2); % Unknown 2x2 symmetric matrix
Q = 1 * eye(2,2);
CONS1=[A'*P*A-P<=-Q]; % Constraint 1
CONS2=[P>=0]; % Constraint 2
CONS=[CONS1 ,CONS2]; % Combine all constraints

% Solving for P
infosol = optimize(CONS,[],ops);

Psol = double(P) % Converts to standard matrix format

% Check if the solution is OK
infosol.info

% Double-check the solution verifies the constraints

% Psol must be >0. Check that all eigenvalues of Psol are > 0
eigP=eig(Psol)

% the matrix -(A'*Psol*A-Psol+Q) must be >0. Check that all its eigenvalues are > 0
eigcons0=eig(-(A'*Psol*A-Psol+Q))
```

2. LMI analysis

Answer to the following question about exercise (1). Use MatLab for computing eigenvalues.

- (a) Analyzing only the matrix A , do you expect feasibility of (1), without solving it?
Is (1) feasible for other matrices $Q \in \mathbb{R}^{2 \times 2}$?

Hint: Use the Lyapunov equation $A^T P A - P = -Q$ and Lyapunov theory for showing feasibility.

- (b) For solving (1) with Yalmip, we have replaced $P > 0$ with $P \geq 0$. Prove that, in spite of this change, any solution to $A^T P A - P \leq -I$ verifies $P > 0$.

Hint: Show first that $M = -(I + A^T P A)$ is negative definite. Then, show that $-P \leq M$ implies $P > 0$.

3. Solving a Lyapunov equation

Consider the discrete-time LTI system

$$x^+ = Ax, \quad A = \frac{1}{10} \begin{bmatrix} 4 & 0 \\ 3 & 5 \end{bmatrix}$$

Solve by hand the Lyapunov equation

$$A^T P A - P = -Q \tag{2}$$

where $P = P^T \in \mathbb{R}^{2 \times 2}$ and $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Hint: Since P is symmetric, set $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}$.