Networked Control Systems (ME-427)- Exercise session 11

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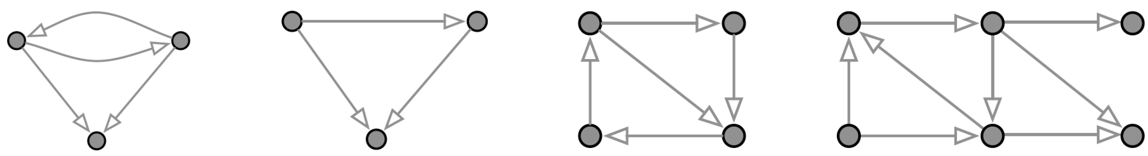
1. **Condensation digraphs.** [Textbook E3.2] Draw the condensation for each of the following digraphs.



2. **Directed spanning trees in the condensation digraph.** [Textbook E3.3] For a digraph G and its condensation digraph $C(G)$, show that the following statements are equivalent:

   (a) G contains a directed spanning tree, and

   (b) $C(G)$ contains a directed spanning tree.

   Hint: Reason on G_{rev}.

3. **Agents with self-confidence levels.** [Textbook E5.6] Consider 2 agents, labeled $+1$ and -1, described by the self-confidence levels s_{+1} and s_{-1}. Assume $s_{+1} > 0$, $s_{-1} > 0$, and $s_{+1} + s_{-1} = 1$. For $i \in \{+1, -1\}$, define
$$x_i^+ := s_i x_i + (1 - s_i) x_{-i}.$$
   Perform the following tasks:

   (a) compute the matrix A representing this algorithm and verify it is row-stochastic,

   (b) compute A^2,

   (c) compute the eigenvalues, the right eigenvectors, and the left eigenvectors of A,

   (d) compute the final value of this algorithm as a function of the initial values and of the self-confidence levels. Is it true that an agent with higher self-confidence makes a larger contribution to the final value?

4. **Two social influence networks.** [Textbook E5.2] Similarly to the DeGroot model introduced in the lectures, we consider n individuals with an initial opinion $x_i(0) \in \mathbb{R}$, $i = 1, \ldots, n$. $x_i(k)$ is the updated opinion of individual i after k communications with its neighbors. Then, the vector of opinions evolves over time according to $x(k+1) = Ax(k)$ where the coefficient $a_{ij} \in [0, 1]$ is the influence of the opinion of individual j on the update of the opinion of agent i, subject to the constraint $\sum_j a_{ij} = 1$. Consider the following two scenarios:

   (a) Everybody gives the same weight to the opinion of everybody else.

   (b) There is a distinct agent (suppose the agent with index $i = 1$) that weights equally the opinion of all the others, and the remaining agents compute the mean between their opinion and the one of first agent.

   In each case, derive the averaging matrix A, show that the opinions converge asymptotically to a final opinion vector, and characterize this final opinion vector.