Networked Control Systems (ME-427)- Exercise session 6

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1. Consider the continuous-time system $\mathcal{S}$ with dynamics

$$\dot{x} = x + u$$

along with the network control system shown in Fig. 1, where the network is ideal. We aim to control the system using a quantized control law.

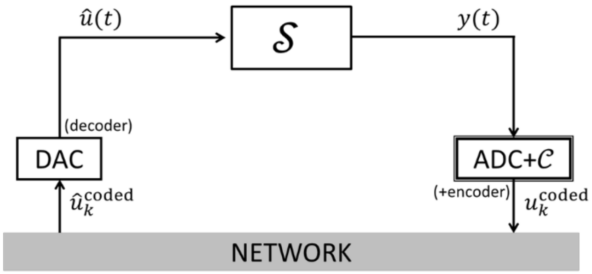


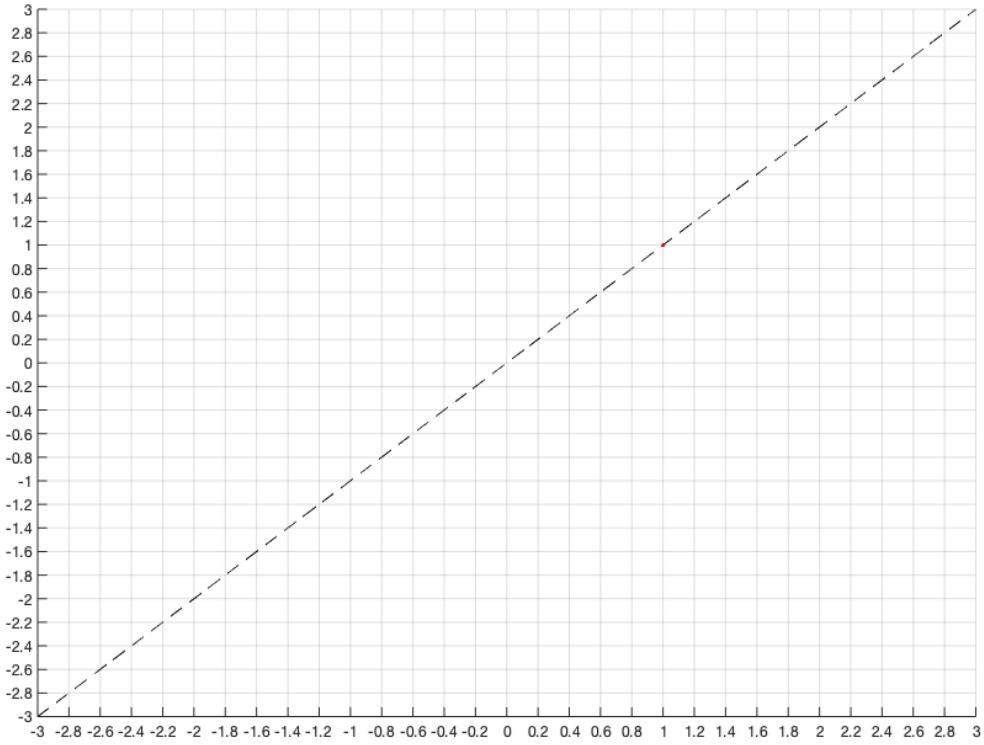
Figure 1: Networked control system

(a) According to the theory, what is the minimun rate $\frac{N_B}{T}$ required to make the system boundable?

(b) If we set the sampling time $T = \log_e(2.5) = 0.9163$ s, what is the minimum number of bits N_B required to be transmitted at each sampling instant in order to make the system boundable?

(c) Set $T = \log_e(2.5) = 0.9163$ s and assume that $N_B = 1$. Using the control law $u_k = -\text{sign}(x_k)$, show the properties of the state trajectories using the graphical method.

(d) Set $T = \log_e(2.5) = 0.9163$ s and assume that $N_B = 2$. Using the control law defined in case we operate at the rate limits (with $u_{\min} = -1, u_{\max} = 1$), study the properties of the state trajectories using the graphical method. Paper sheets for deriving relevant plots by hand are provided below. In case of boundability, define the corresponding positively invariant set.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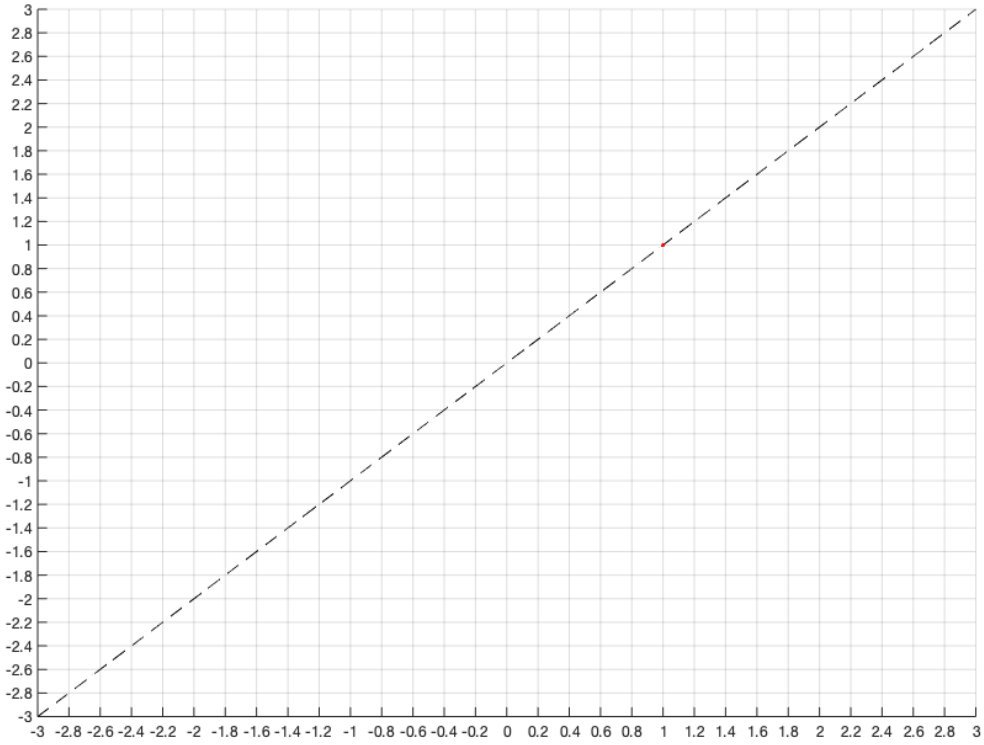
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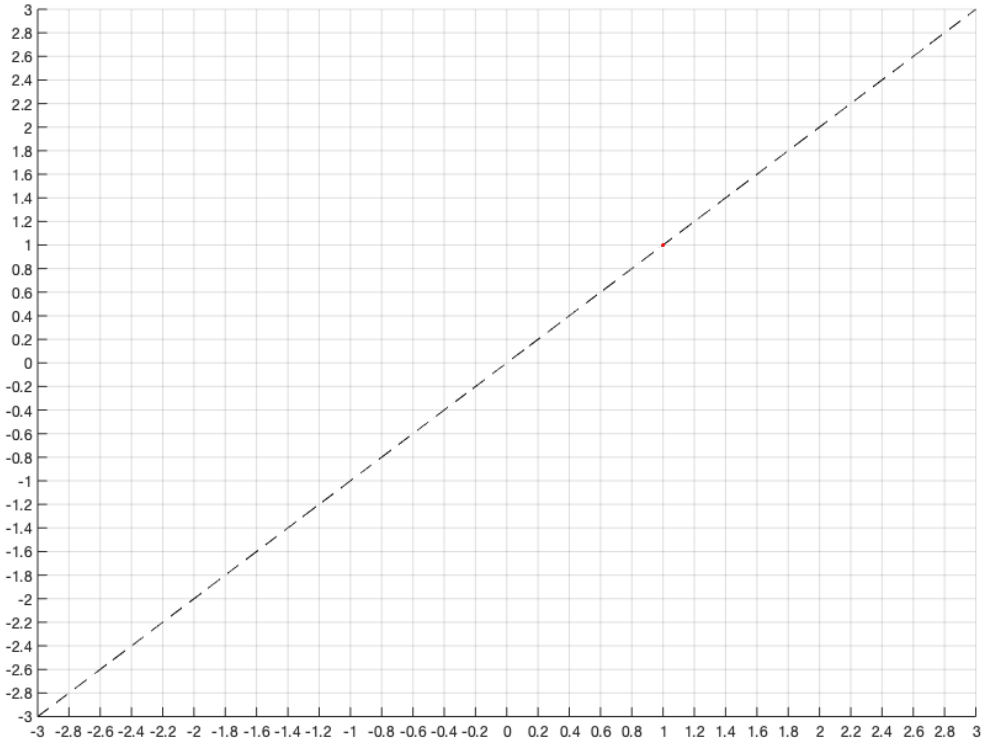
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2. **Maximal tolerable delay under high network load.** Consider the NCS in Figure 2,

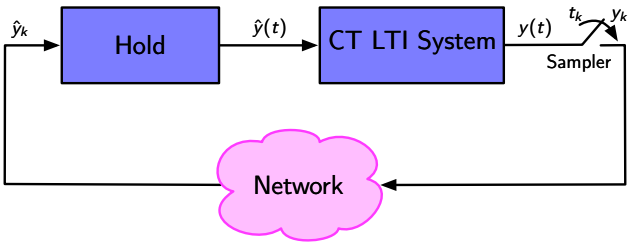

Figure 2: Networked control system

where the LTI system is the first-order model

$$\begin{cases} \dot{x} = -x - 5u \\ y = x \end{cases}.$$

Assume that, because of the network load, the nominal sampling time T and network induced delay τ are almost identical, i.e. $T \simeq \tau$.

(a) Find by hand all values of T stabilizing the NCS.

Hint: Recall the Jury's criterion: the roots of $\phi(\lambda) = \lambda^2 + \alpha\lambda + \beta$ verify $|\lambda| < 1$ if and only if

$$\begin{array}{l} \beta > -\alpha - 1 \\ \beta > \alpha - 1 \\ \beta < 1 \end{array}.$$

(b) The plot in Figure 3, seen in the lectures, represents the pairs (T, τ) verifying $T > \tau$ and yielding asymptotic stability (green points) or not (red points) of the NCS. Identify the pair $(\bar{T}, \tau)$ corresponding to maximum allowable sampling period $\bar{T}$ found in the previous point.

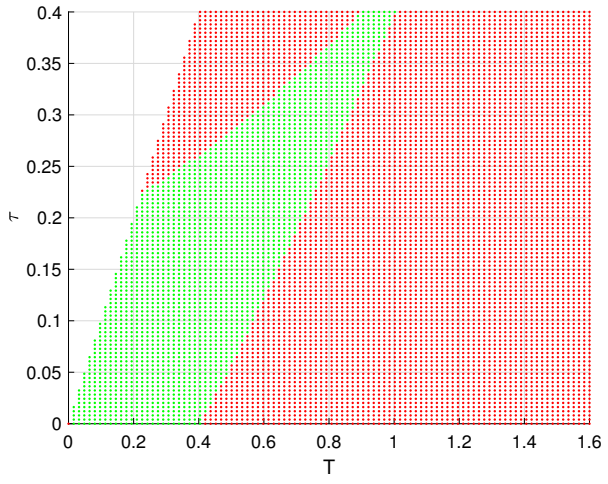

Figure 3: Region of stability for the system in Exercise 2