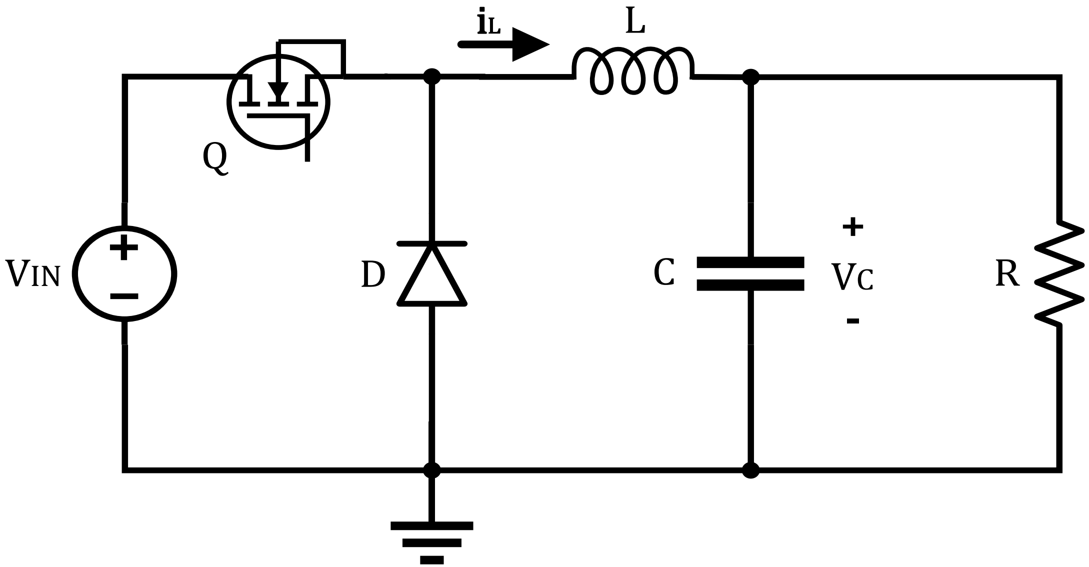
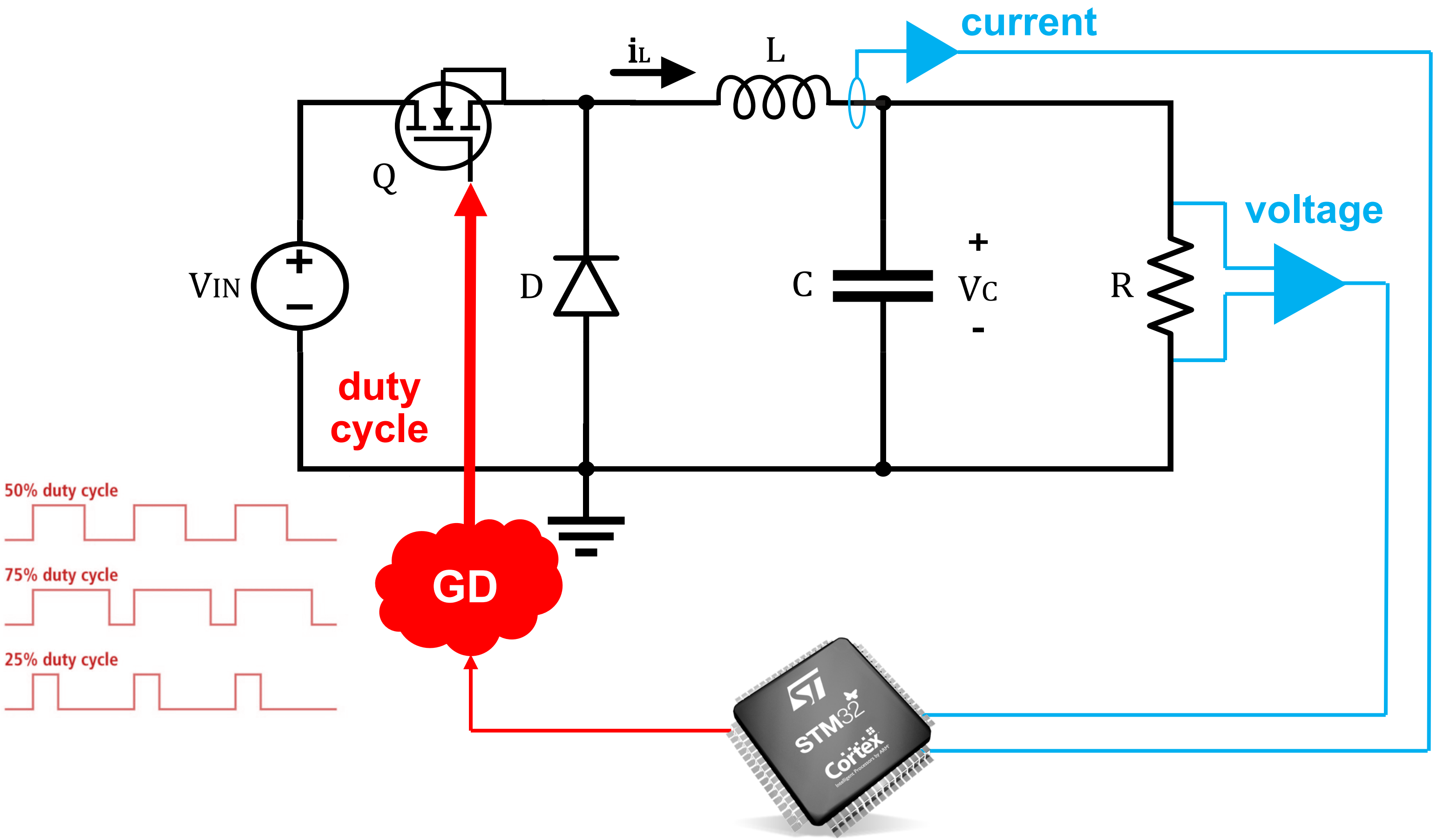
Embedded Explicit MPC of a Dc-Dc Converter

Emilio T. Maddalena, Colin N. Jones – Fall 2021

System overview

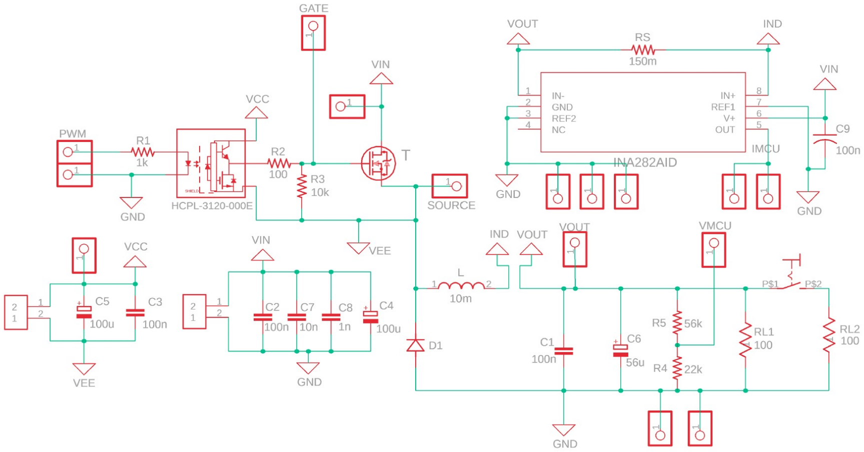


System overview



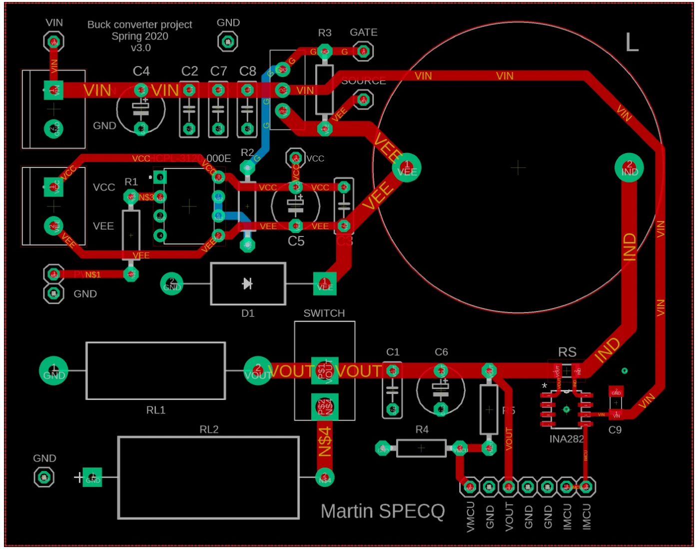
System overview

Schematic



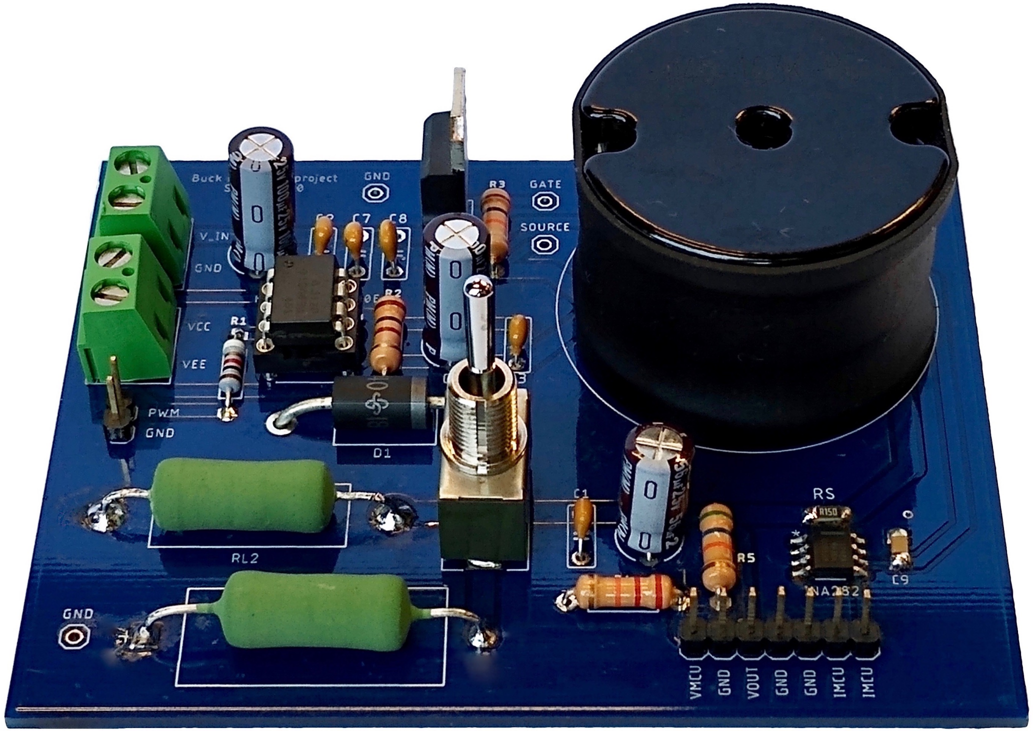
AUTODESK® EAGLE

Board layout



Ack: Martin W. F. Specq

System overview



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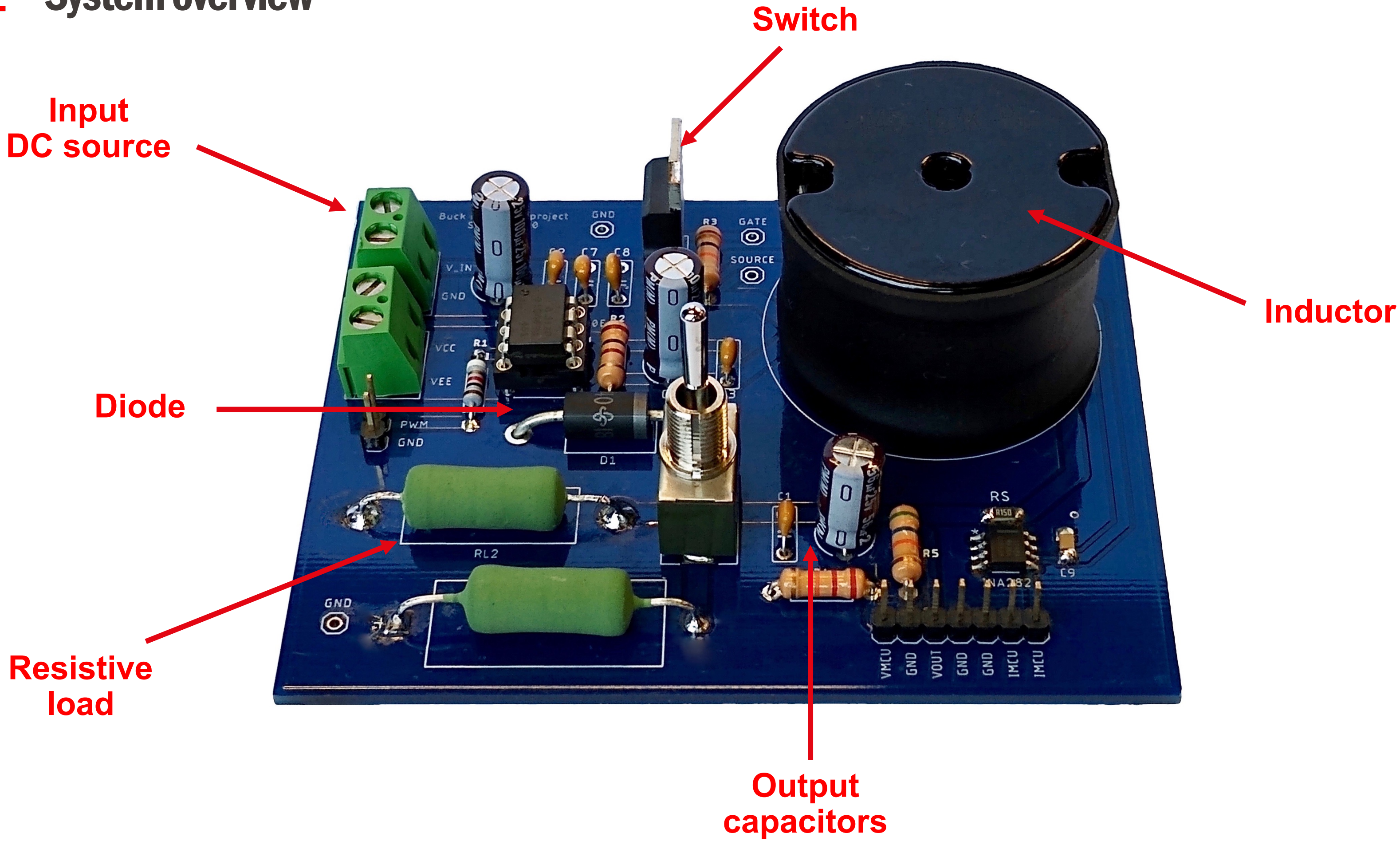
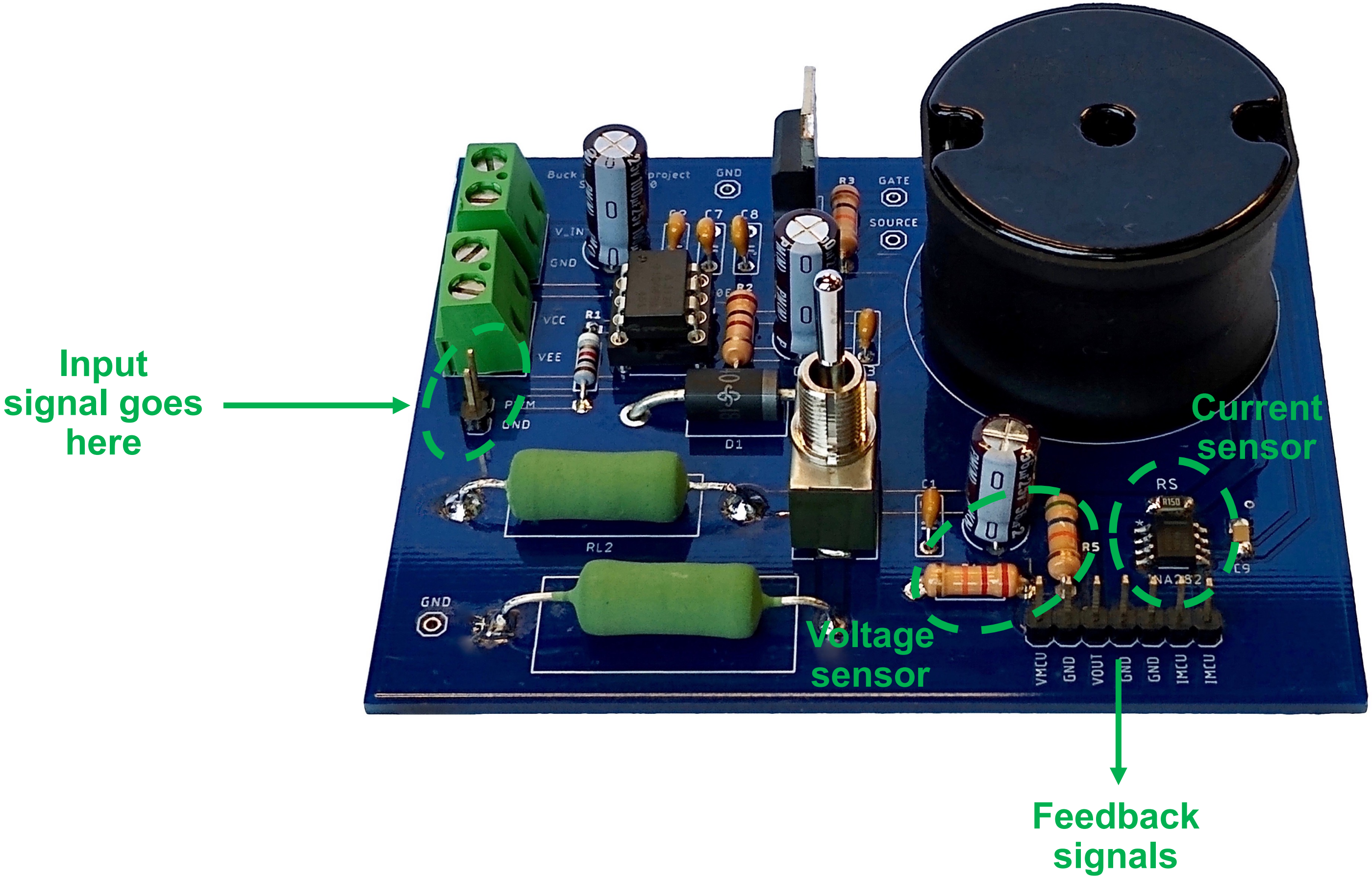
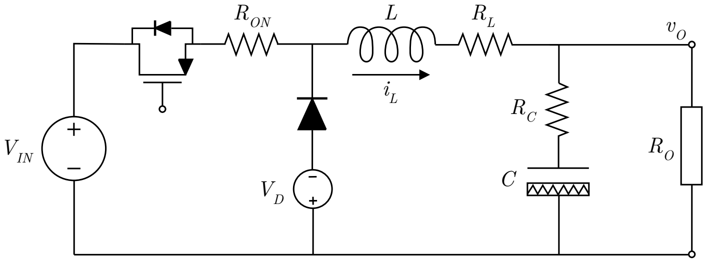


System overview



Obtaining a model

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i_L \\ v_O \end{bmatrix} \qquad u = \delta$$



Average dynamics over a switching period:

$$\dot{x}_1 = -\frac{R_L}{L} x_1 - \frac{1}{L} x_2 + \frac{V_{IN} + V_D}{L} u - \frac{R_{ON}}{L} x_1 u - \frac{V_D}{L}$$

$$\dot{x}_2 = -\frac{R_C R_O R_L C + R_O L}{(R_C + R_O) L C} x_1 - \frac{R_C R_O C + L}{(R_C + R_O) L C} x_2 + \frac{R_C R_O (V_{IN} + V_D)}{(R_C + R_O) L} u - \frac{R_C R_O R_{ON}}{(R_C + R_O) L} x_1 u - \frac{R_C R_O V_D}{(R_C + R_O) L}$$

Linearization around a desired equilibrium point:

$$x_{1eq} = \frac{x_{2eq}}{R_O} \qquad u_{eq} = \frac{R_O V_D + (R_L + R_O) x_{2eq}}{R_O (V_{IN} + V_D) - R_{ON} x_{2eq}}$$

$$\dot{x} = A_{ct} x + B_{ct} u \qquad A_{ct} = \begin{bmatrix} -\frac{R_L + R_{ON} u_{eq}}{L} & -\frac{1}{L} \\ -\frac{R_C R_O (R_L C - R_{ON} C u_{eq} + R_O L)}{(R_C + R_O) L C} & -\frac{R_C R_O C + L}{(R_C + R_O) L C} \end{bmatrix} \qquad B_{ct} = \begin{bmatrix} \frac{V_{IN} + V_D - R_{ON} x_{1eq}}{L} \\ \frac{R_C R_O (V_{IN} + V_D - R_{ON} x_{1eq})}{(R_C + R_O) L} \end{bmatrix}$$

- + ZOH discretization

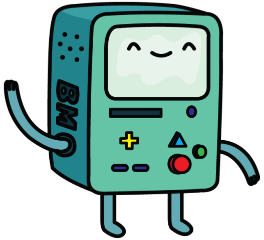
Controller design

Standard MPC formulation with guarantees

$$\min_{X,U} \sum_{t=0}^{N-1} \left(\|x_t - x_{\text{eq}}\|_Q^2 + \|u_t - u_{\text{eq}}\|_R^2\right) + \|x_N - x_{\text{eq}}\|_P^2$$

$$\begin{aligned}
\text{s.t.}\quad & \forall t = 0, \ldots, N-1 \\
& x_{t+1} = Ax_t + Bu_t \\
& \begin{bmatrix} i_L^{\min} \\ v_O^{\min} \end{bmatrix} \leq x_t \leq \begin{bmatrix} i_L^{\max} \\ v_O^{\max} \end{bmatrix} \\
& u^{\min} \leq u_t \leq u^{\max} \\
& x_N \in \mathcal{X}_N \\
& x_0 = x(0)
\end{aligned}$$

Control frequency of <u>10 kHz</u>!



STM32L476:

- 80 MHz clock
- 128 kB of RAM
- 1 MB of flash

Parameters and constraints

$$v_{eq} = 5\ V,\ i_{eq} = 50\text{ mA},\ u_{eq} = 0.338$$

$$N = 10,\ Q = \text{diag}(90, 1),\ R = 1$$

$$x^{\min} = \begin{bmatrix} i_L^{\min} & v_O^{\min} \end{bmatrix}^\top = \begin{bmatrix} 0\,\text{mA} & 0\,\text{V} \end{bmatrix}^\top$$

$$x^{\max} = \begin{bmatrix} i_L^{\max} & v_O^{\max} \end{bmatrix}^\top = \begin{bmatrix} 200\,\text{mA} & 7\,\text{V} \end{bmatrix}^\top$$

$$u^{\min} = 0, \quad u^{\max} = 1$$

Controller design

Standard MPC formulation with guarantees

$$
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& \begin{bmatrix} i_L^{\min} \\ v_O^{\min} \end{bmatrix} \le x_t \le \begin{bmatrix} i_L^{\max} \\ v_O^{\max} \end{bmatrix} \\
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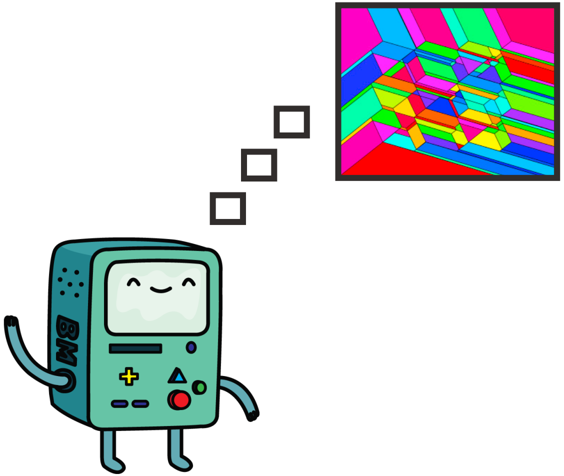

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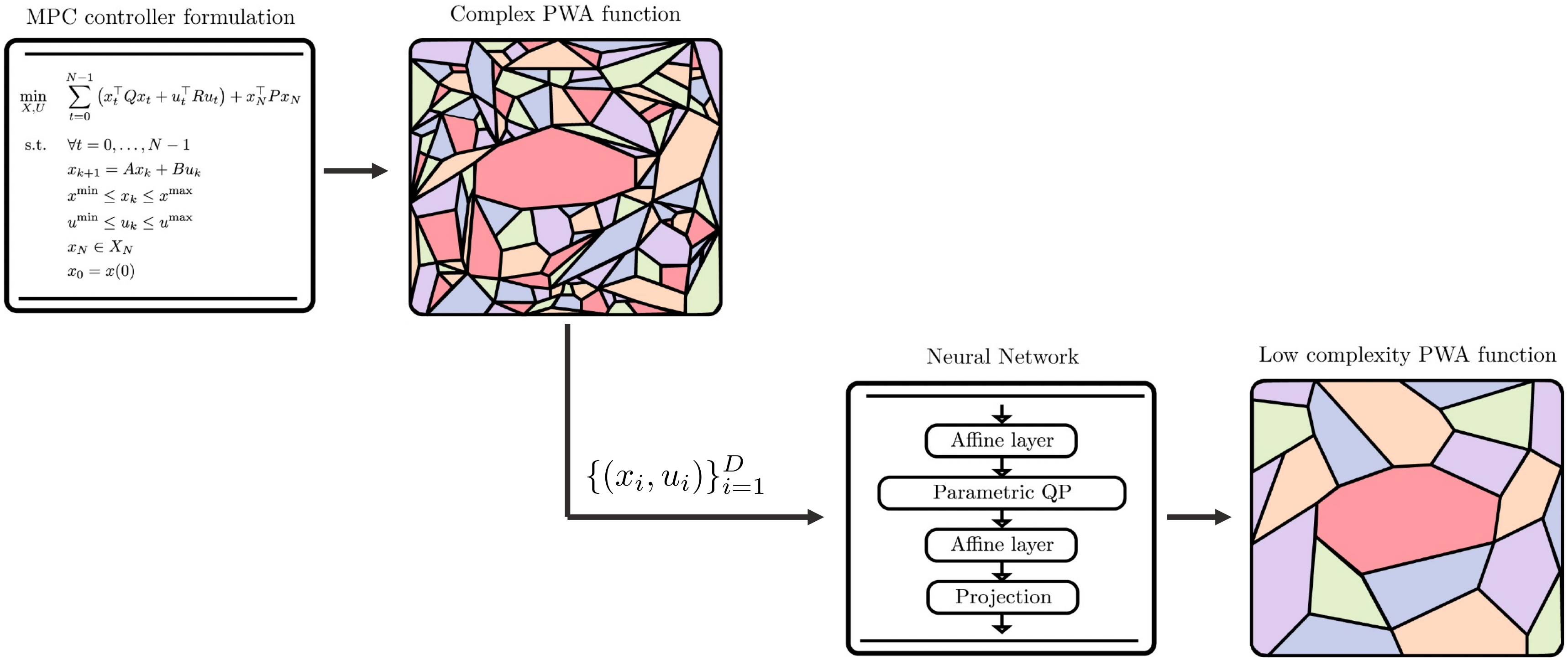
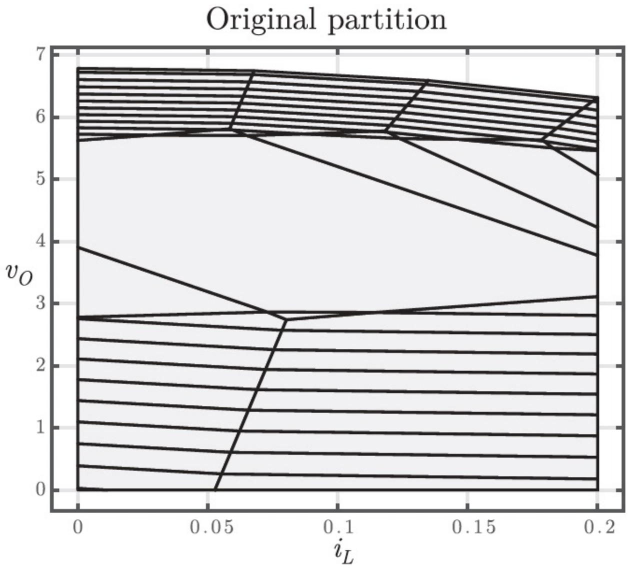
Preprocessing

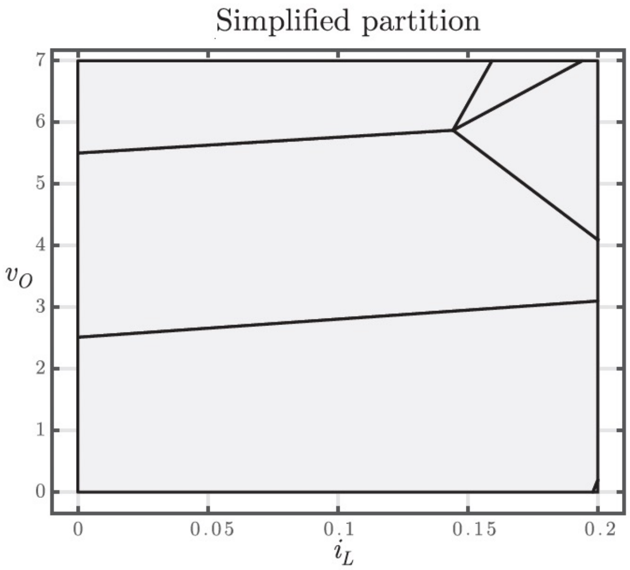


(2021) Embedded PWM Predictive Control of DC-DC Power Converters Via Piecewise-Affine Neural Networks. IEEE OJIES.
E.T. Maddalena, M.W.F. Specq, V.L. Wisniewski, C.N. Jones.

Preprocessing



153 regions

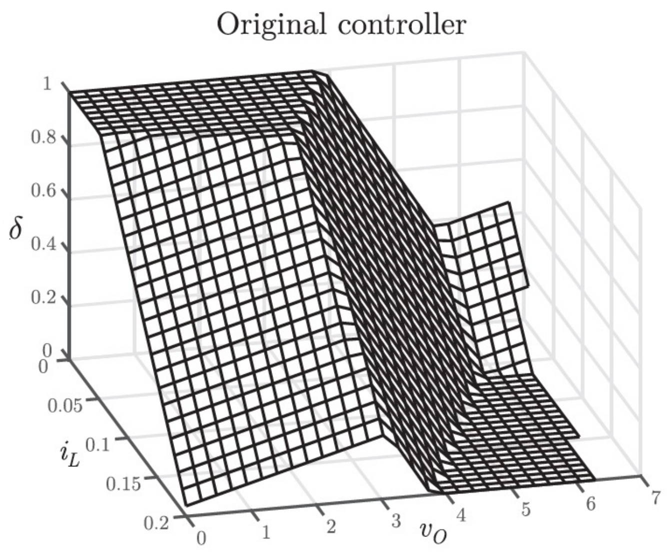


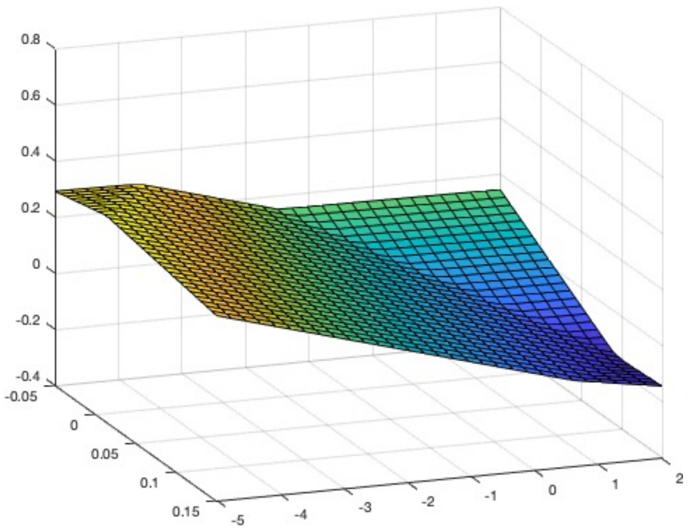
6 (5) regions

Training it once = 35 mins w/o GPU acceleration on a 3.1 GHz Intel Core i7 machine.

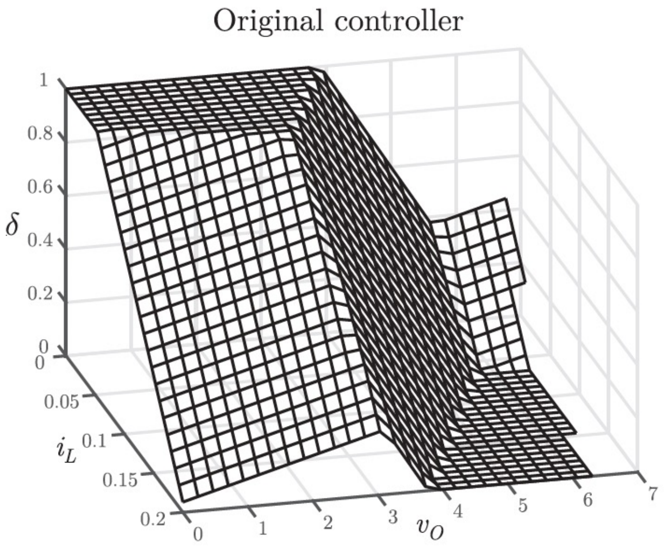
The final result yielded a 91% of reduction in the number of regions.

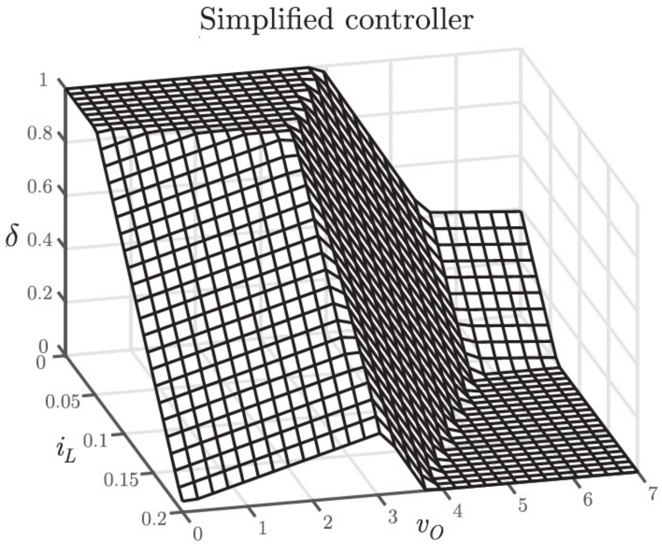
Preprocessing



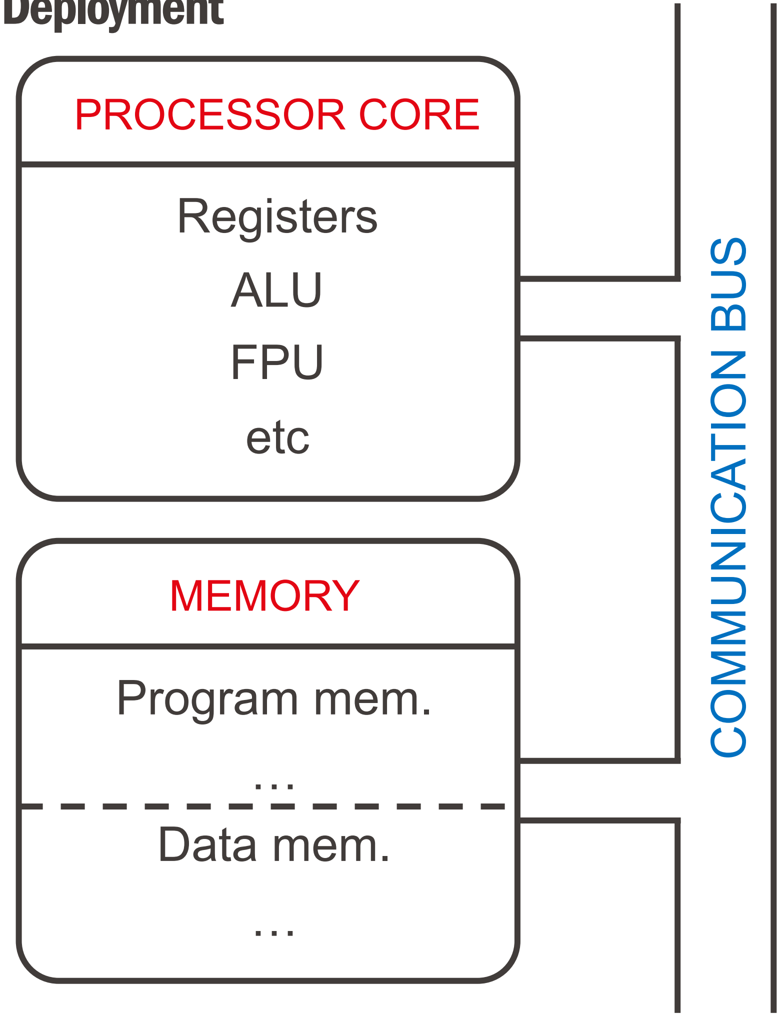


Preprocessing





Deployment


PROCESSOR CORE
Registers
ALU
FPU
etc
MEMORY
Program mem.
...
Data mem.
...
COMMUNICATION BUS

Deployment

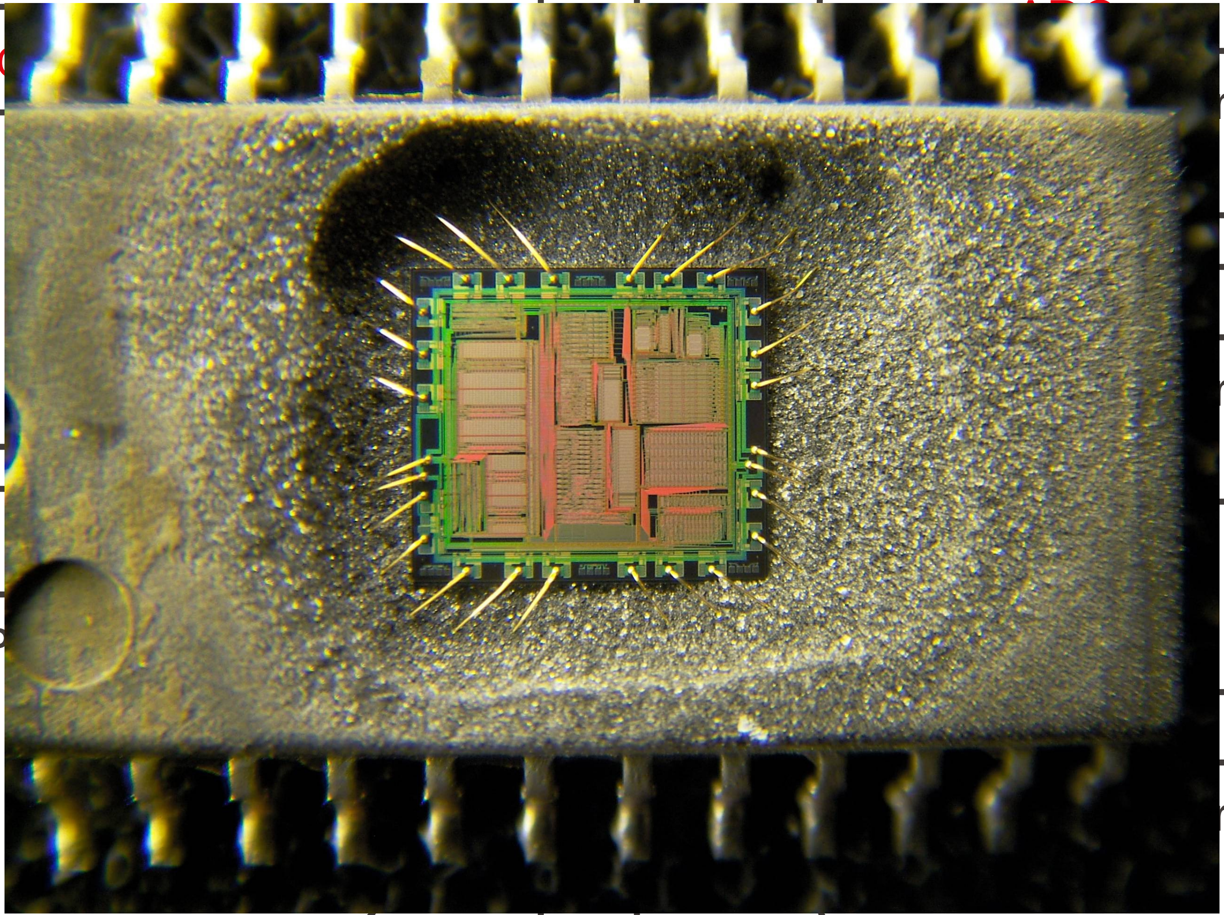
PRO

P

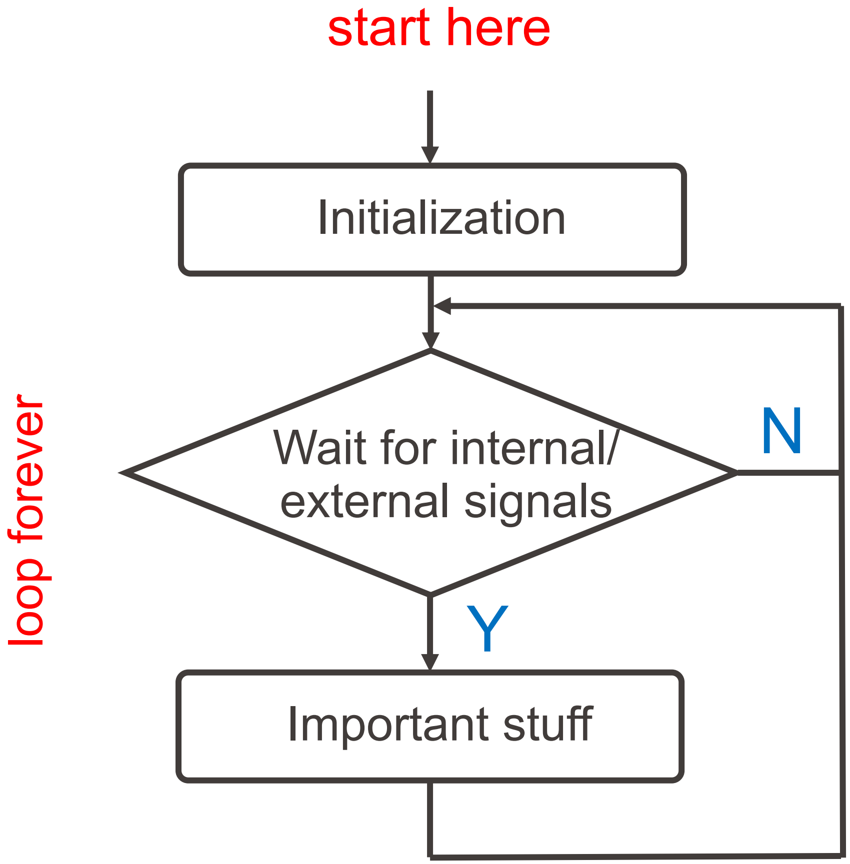
rs

rs

rs



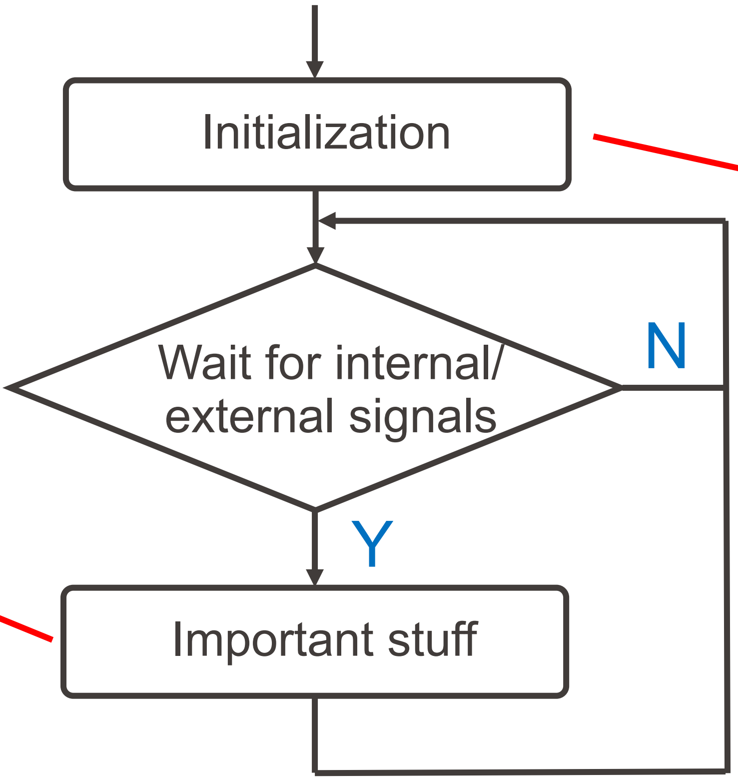
EXTERNAL WORLD


start here
Initialization
Wait for internal/
external signals
N
Y
Important stuff
loop forever

Deployment

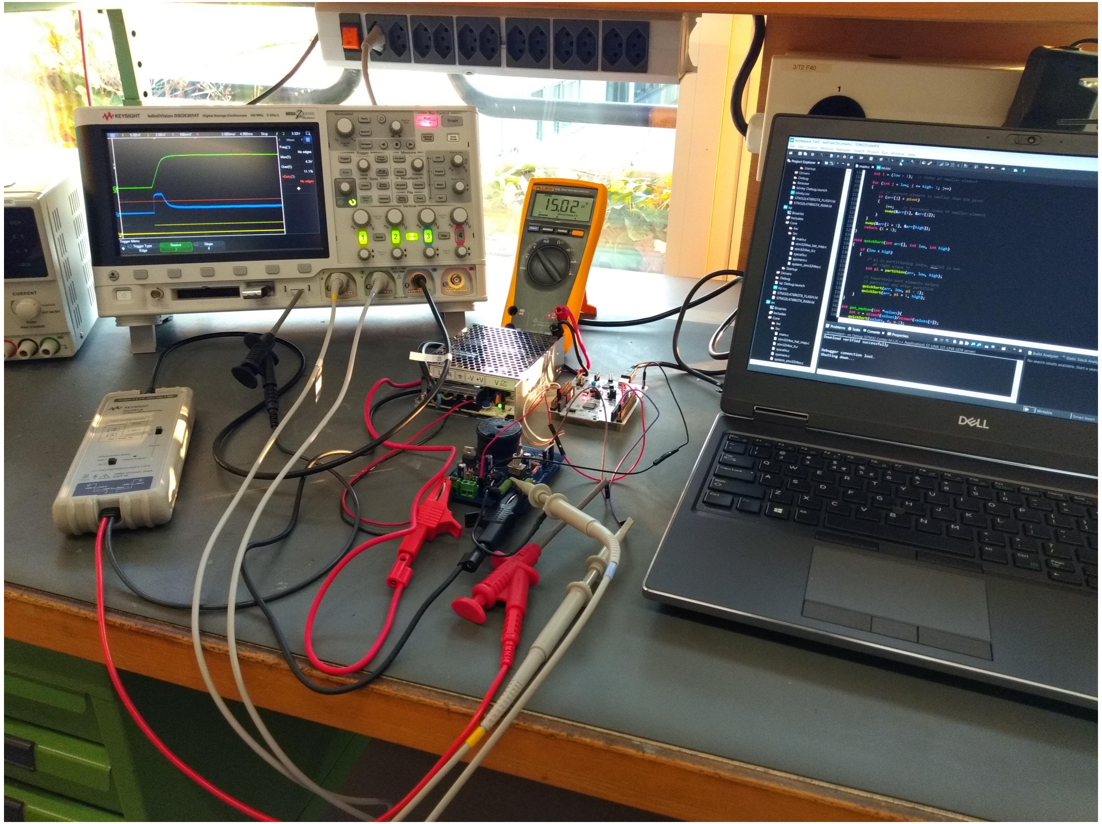
Timer ISR:

- Sample analog signals
- Scale signals
- Solve point location problem
- Compute control signal
- Scale it
- Update PWM duty cycle



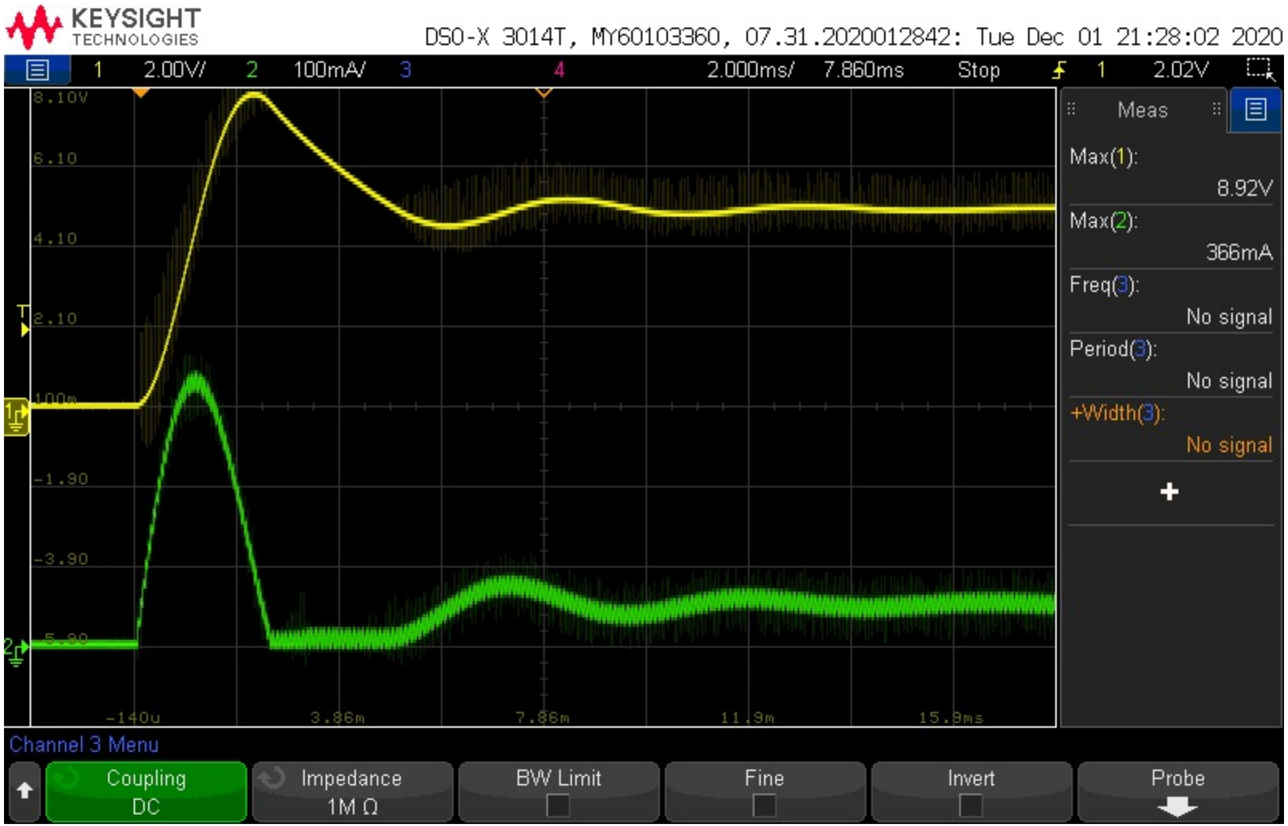
- Initialize timers
- Initialize ADCs
- Initialize PWM

Results



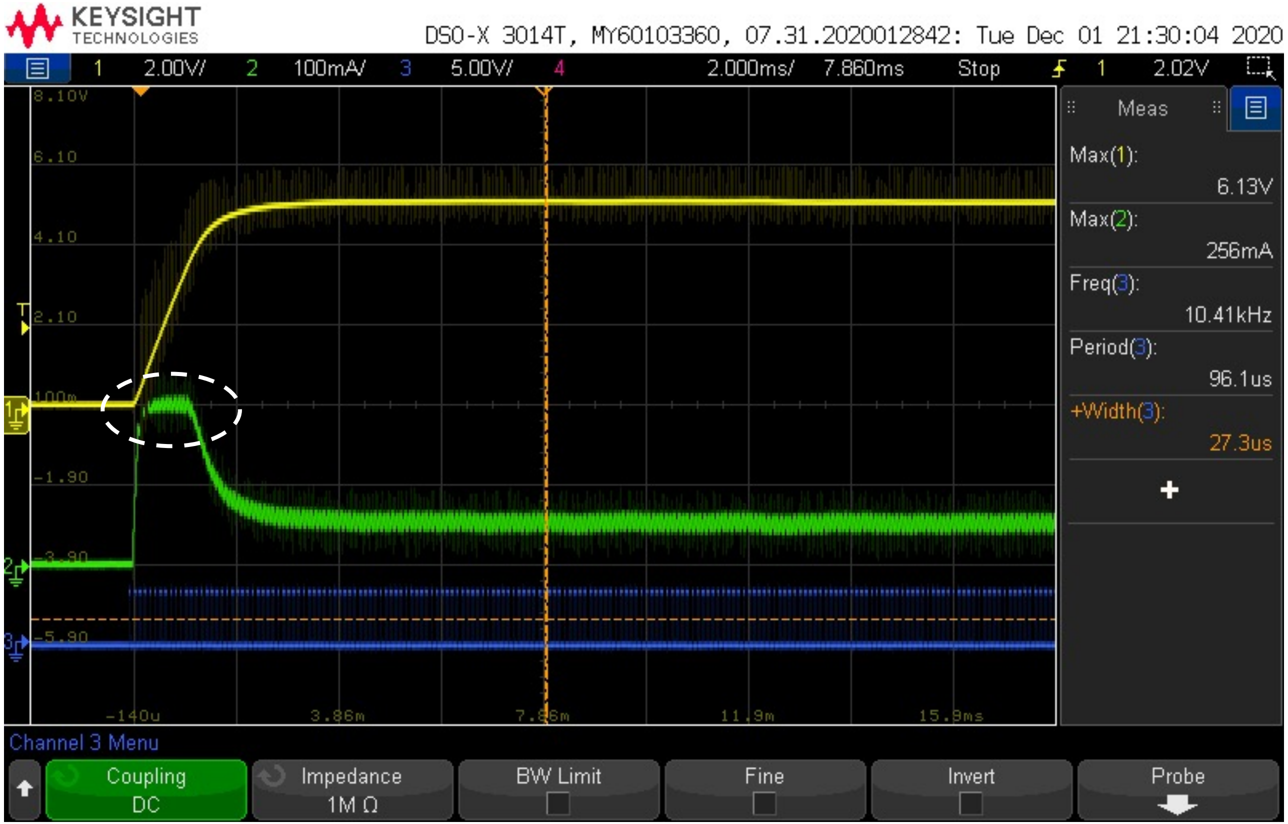
Interrupt-driven code (100 us timer), 2 ADCs + DMA memory transfers, averaging over 8 samples (filter), no floating-point variables, open-loop state estimator to ‘correct’ the first/second samples (diode barrier problem)

Results



KEYSIGHT TECHNOLOGIES
DSO-X 3014T, MY60103360, 07.31.2020012842: Tue Dec 01 21:28:02 2020
1 2.00V/ 2 100mA/ 3 4 2.000ms/ 7.860ms Stop 1 2.02V
8.10V
6.10
4.10
2.10
100m
-1.90
-3.90
5.90
-140u
3.86m
7.86m
11.9m
15.9ms
Meas
Max(1):
8.92V
Max(2):
366mA
Freq(3):
No signal
Period(3):
No signal
+Width(3):
No signal
Channel 3 Menu
Coupling
DC
Impedance
1M Ω
BW Limit
Fine
Invert
Probe

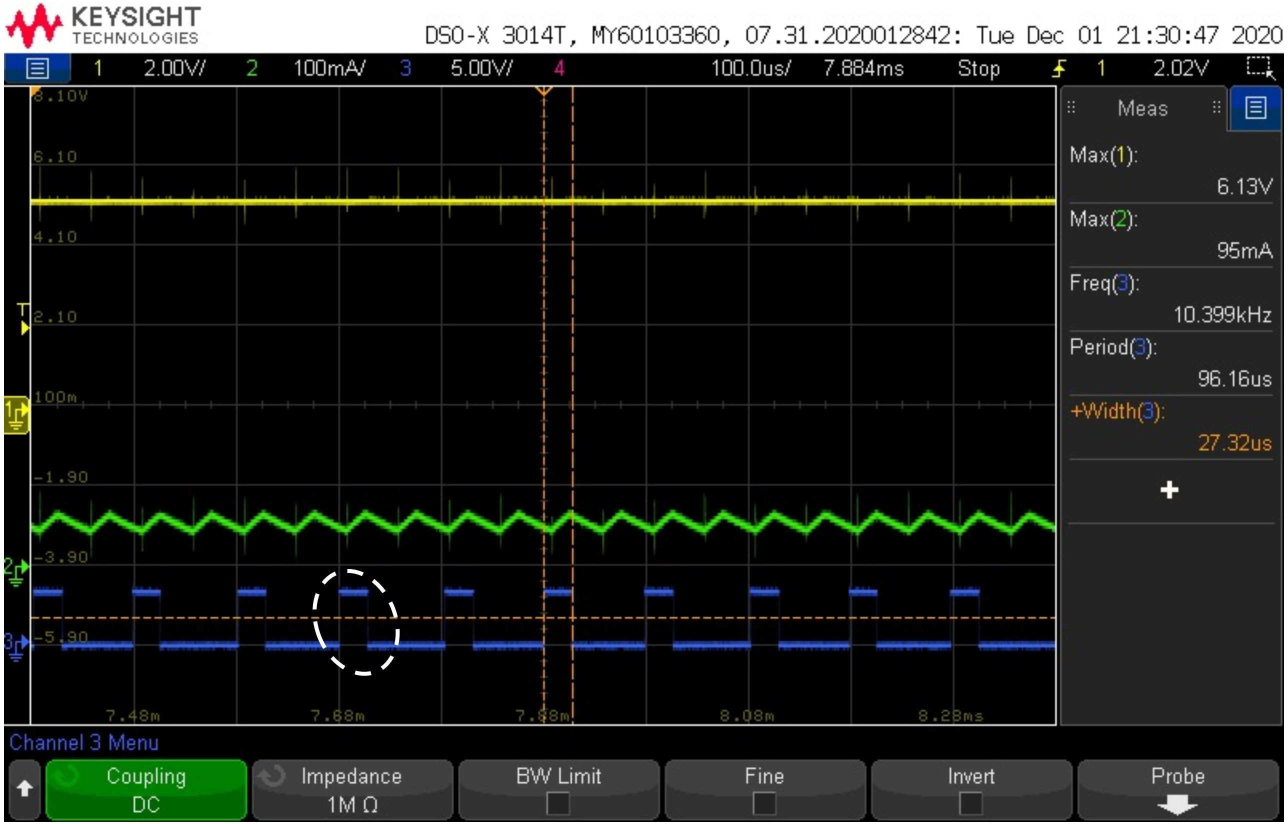
Results



200 mA current constraint being respected!

Milder voltage slope, but overall faster settling time

Results



The EMPC interrupt-service routine takes < 28 us to execute

Thank you for your attention!

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