Model Predictive Control : Exercise 5

Consider the discrete-time system

$$
\begin{aligned}
x_{k+1} &= \begin{bmatrix} 0.7115 & -0.4345 \\ 0.4345 & 0.8853 \end{bmatrix} x_k + \begin{bmatrix} 0.2173 \\ 0.0573 \end{bmatrix} u_k \\
y_k &= \begin{bmatrix} 0 & 1 \end{bmatrix} x_k + d
\end{aligned}
$$

where d is an unknown constant disturbance and x_0 is unknown. The goal of this exercise sheet is to design a controller able to track a constant output reference while fulfilling input constraints

$$-3 \le u_k \le 3$$

Prob 1 | Observer Design

Since state and disturbances are unknown at time zero, we need to design an observer to estimate them. We call $\hat{x}$ and $\hat{d}$ the estimate of x and d, respectively. Design an observer for the given system, and test it for the condition $x_0 = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$, $\hat{x} = \begin{bmatrix} 3 & 0 \end{bmatrix}^T$, $\hat{d} = 0$, $d = 0.2$ and $u = 0$.

Hints:

- You can use YALMIP to implement the MPC controller
- To estimate the disturbance you will have to augment the state as seen in class
- Note the eigenvalious of $(A + LC)$ are the same as those of $(A' + C'L')$
- The matlab function `K = place(A,B,F)` computes a state-feedback matrix K such that the eigenvalues of $A - BK$ are those specified in the vector F

Prob 2 | Steady-state target computation

Given the system above, and a reference r, use YALMIP to compute a steady state for the system that minimizes u^2.

Prob 3 | MPC tracking

Implement an MPC controller to track an output reference signal r.

- Confirm that the estimates converge to the true values, the output converges to the reference and that the input does not violate the constraints by plotting the result for references $r = 1$ and $r = 0.5$

Hints :

- Use a terminal set of $X_f = \mathbb{R}^n$ and a terminal cost of $V_f(x_N) = \Delta x_N' P \Delta x_N$ where P is the solution of $P - A'PA = Q$
  These correspond to a terminal controller of $u = 0$. Note that this is a valid terminal controller because the system is stable.
  You can use the function `P = dlyap(A,Q)` to compute P.

- Good values for the horizon and stage costs are: $N = 5$, $Q = I$, $R = 1$
- In the previous exercise you designed an observer specifying the eigenvalues of the estimation error state-update matrix. Eigenvalues with a small norm will speed up the estimation process, but may increase the initial overshoot of the estimate d. A large $\hat{d}$ can cause the problem of computing the set-point to be infeasible. Use moderate eigenvalues (e.g. $0.5, 0.6, 0.7$).