Multivariable Control (ME-422) - Exercise session 3B

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In this set of exercises, you will learn to control a continuous stirred tank reactor (CSTR) around a given operating point.

CSTRs are very common in industry since they provide the adequate environment to make chemical reactions happen under precise system conditions (temperature, liquid concentration, etc.). Because of this, they need to be controlled to maintain the internal variables at the proper references despite the external disturbances. Precisely, the working behaviour of the considered system can be summarized as follows:

- The reactor receives an input liquid with a concentration c_f, a temperature T_f and a flow q. The concentration c_f is a variable that can be manipulated as a *control input* to the system. The temperature T_f and the flow q are not controlled. We first consider them as fixed and known parameters, but in a more general framework they could be considered as *external disturbances*, as we will do later in the course.
- The reactor walls are regulated to a temperature T_c which can be manipulated as a *control input* to the system.
- The internal reaction produces a fluid with a temperature T and the concentration c. The outflow rate is denoted by q.
- Both the concentration c and the temperature T in the reactor are measured variables.

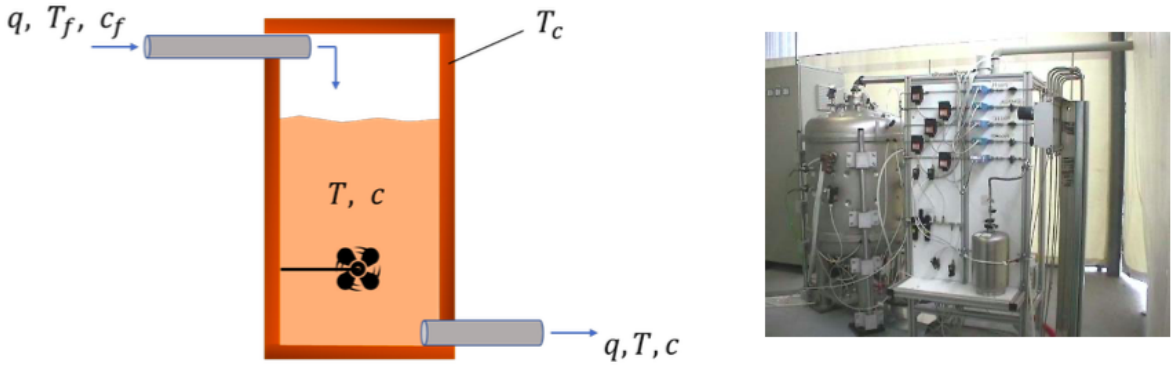

Figure 1: Schematic and realistic views of an industrial continuous stirred tank reactor.

Nonlinear system model The continuous-time nonlinear dynamics of the CSTR are described by the state-space equations:

$$\begin{aligned}\dot{c}(t) &= \frac{q}{V}\left(c_f(t) - c(t)\right) - k_0\, e^{-\frac{E}{RT(t)}} c(t)\,,\\ \dot{T}(t) &= \frac{UA}{V\rho c_p}\left(T_c(t) - T(t)\right) + \frac{q}{V}\left(T_f - T(t)\right) - \frac{\Delta H}{\rho c_p} k_0\, e^{-\frac{E}{RT(t)}} c(t)\,.\end{aligned} \tag{1}$$

Table 1 summarizes the physical meaning and values of the different parameters.

Parameter	Value	Description (see Figure 1)	Units
V	100	Reactor volume	L
k_0	7.2×10^{10}	Nonthermal factor	$\frac{1}{\text{min}}$
E	72747.5	Activation energy per mole	$\frac{\text{J}}{\text{mol}}$
R	8.314	Boltzmann's ideal gas constant	$\frac{\text{J}}{\text{mol K}}$
ΔH	5×10^4	Molar enthalpy (i.e. heat of reaction per mole)	J/mol
ρ	1000	Liquid density	g/L
c_p	0.239	Specific heat capacity of the liquid	$\frac{\text{J}}{\text{g K}}$
UA	5×10^4	Overall heat transfer coefficient multiplied by tank area	$\frac{\text{J}}{\text{min K}}$
q	100	Flow rate	$\frac{\text{L}}{\text{min}}$
T_f	350	Input temperature	K

Table 1: Parameters of the CSTR system.

1. Load the parameters from the `data_CSTR.m` file in the workspace. Then, implement the model given by (1) in Simulink.

2. Find the inputs $(\bar{c}_f, \bar{T}_c)$ that give the desired equilibrium at $\bar{c} = 0.5\,\frac{\text{mol}}{\text{L}}$, $\bar{T} = 350\,\text{K}$. Corroborate the results by simulating the system at the equilibrium.

3. Simulate the system when adding a 1% disturbance over the initial conditions around the equilibrium. Does the system converge to previously computed equilibrium?

4. Linearize the system around $(\bar{c}, \bar{T})$ with inputs $(\bar{c}_f, \bar{T}_c)$, i.e. the equilibrium obtained in point 2. Is the linearized system stable? Can you conclude anything about the stability of the equilibrium of the nonlinear system?
   Hint: Use the time-based linearization block, for obtaining the matrices of the linearized system.

5. Multiple equilibria exist when considering the constant inputs $(\bar{c}_f, \bar{T}_c)$:

   (a) Load the data from the `CSTR_eq2.m` and verify that the state pair (`ctilde`, `Ttilde`) is an equilibrium when considering the inputs $(\bar{c}_f, \bar{T}_c)$.

   (b) Linearize the system around the new equilibrium $(\tilde{c}, \tilde{T}) = (\texttt{ctilde}, \texttt{Ttilde})$. Is the linearized system stable? Can you conclude anything about the stability of this new equilibrium of the nonlinear system? Validate your answer by simulating the system when adding a 1% perturbation on the initial conditions.