Multivariable Control (ME-422) - Exercise session 8

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1. Consider the system

$$\begin{cases} x_1^+ = (1-\alpha)x_1 + \beta x_2 - u + d \\ x_2^+ = \alpha x_1 + (1-\beta)x_2 \end{cases}$$
$$y = x_1$$

where $d(k)$ is a disturbance. Using eigenvalue assignment, we want to design a controller for tracking a constant reference $y^0(k)$ without offset, when the disturbance is constant but unknown.

   (a) Check for which parameter values α, $\beta \in \mathbb{R}$ the problem can be solved.

   (b) For $\alpha = \frac{1}{2}$ $\beta = \frac{1}{2}$, design the controller.
   Hint: First include the integrator dynamics and formulate the dynamics of the augmented system, i.e., system that includes the plant and integrator dynamics. Then, for the augmented system, design a stabilizing state-feedback controller assuming that its states can be measured.

2. Consider the system in the previous problem, but now assume that the disturbance is measured. For $\alpha = \frac{1}{2}$, $\beta = \frac{1}{2}$, after verifying the necessary assumptions, design a feedforward compensator.

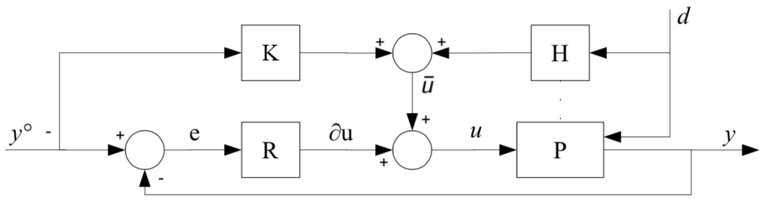

Figure 1: Feedforward Compensator Control Scheme

Figure 1 illustrates the control scheme you will need to design: first design K and H matrices, and then design stabilizing controller R to stabilize the system with inputs $(\partial u, d)$ and output e. To achieve the second objective, first build a Luenberger observer with eigenvalues in 0.5 and 0.6 in order to estimate the states ∂x and then design a state feedback controller $\partial u(k) = K_1 \partial \hat{x}(k)$ with eigenvalues in 0.2 and 0.3.

3. **Inverted Pendulum on Cart**
In this exercise, you will design a controller for offset-free tracking of output variable for the cart-pendulum system. This is the same system that you have seen in the previous exercise session. All the files that you will need for the simulation are provided on Moodle.

   Download the file *Inverted+pendulum+2018-3+no+solutions.zip* and unzip it. Consider the inverted pendulum described in the file *Inverted+Pendulum+description.pdf* and already used in the previous exercise session. See Figure 2 for the schematic description of the system.

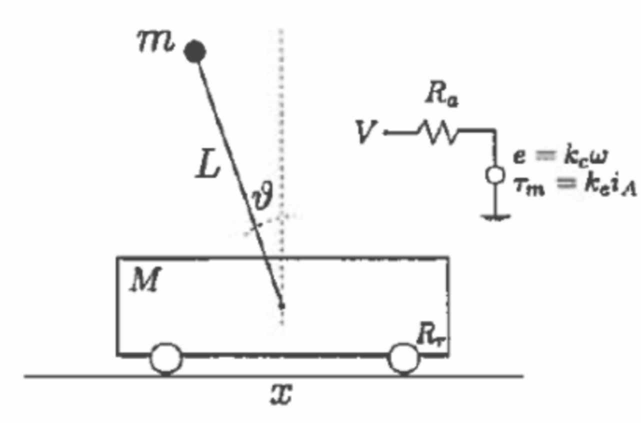

Figure 2: Cart-pendulum system.

By discretizing, with sampling period $T = 0.1s$, the model obtained through linearization about the equilibrium corresponding to the vertical position, one obtains

$$x^+ = \underbrace{\begin{bmatrix} 1 & 0.0824 & -0.0087 & -0.0003 \\ 0 & 0.6692 & -0.1652 & -0.0087 \\ 0 & 0.0178 & 1.0582 & 0.1019 \\ 0 & 0.3367 & 1.1652 & 1.0582 \end{bmatrix}}_{A_d} x + \underbrace{\begin{bmatrix} 0.0009 \\ 0.0165 \\ -0.0009 \\ -0.0168 \end{bmatrix}}_{B_d} u \tag{1}$$

(a) Set the angular position of the pendulum as the output of the system. Is it possible to track a setpoint $y^0(k) \in \mathbb{R}$, $k = 0, 1, \ldots$ by adding integral action? If not, why?

(b) Set the cart position as the output and design an offset-free controller for tracking a reference position $y^0(k) \in \mathbb{R}$, $k = 0, 1, \ldots$. Check the conditions guaranteeing that this goal is achievable. To stabilize the extended system, you can assume the whole state is measured, hence avoiding to introduce an observer, and design a controller assigning the closed-loop eigenvalues in e^{-2T}, $e^{-2.5T}$, e^{-3T}, e^{-4T}, and e^{-10T}.

(c) Implement the controller in simulink file *pole_placement_integrator_pendulum_anim_nosol.slx* and verify stability by running simulations for different combinations of the reference inputs. Remember to run the initialization file *pendulum_sys_init.m* prior to running the simulation. **Hint:** In order to implement the integrator dynamics in discrete time, you can use either *Discrete State-Space* block or *Discrete-Time Integrator* block in Simulink. If using the latter option, double-click the block to open the *Block Parameters* menu and make sure to select one of the *Accumulation* methods as the *Integrator Method* option.